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UNCERTAINTY VALIDATION IN THE CALIBRATION OF INDUSTRIAL PLATINUM RESISTANCE THERMOMETERS (IPRT) USING MONTE CARLO METHOD

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Abstract– Monte Carlo method by simulation has been done to validate calibration of IPRT. The work is based on the Callendar van Dusen equation for 0 °C to 500 °C calibration range. How to derive the mathematics model for running the simulation is also described. Results are compared with results obtained by the propagation of uncertainty (GUM) which shows good agreement and coherent.

Keywords: thermometers, calibration, simulation, Monte Carlo, Uncertatinty

1. INTRODUCTION

Industrial Platinum Resistance Thermometers (IPRTs Pt-100) are widely used for industrial applications in Indonesia. Their traceability is provided by KIM LIPI through a Pt-25 Standard Platinum Resistance Thermometer (SPRT) calibrated on fixed-point cells. The calibration process usually uses liquid baths as the medium. The facility can cover -40 °C to 500 °C calibration range. Two calibration methods can be conducted depending on what unit is the IPRT output. A method that yields Callendar van Dusen (CVD) equation [1] for the IPRT is done if the IPRT is connected to an Ohmmeter while a method to get correction value for the IPRT's output is applied if the IPRT is connected to a device displaying value in degree Celsius (digital thermometer with IPRT sensor). The correction value is usually formed in a polynomial equation.

The uncertainty of measurements for each method is always mentioned in the IPRT's calibration certificate. However, up to now, the uncertainty analysis was still based on the law of propagation of uncertainty described by the GUM (Guide to the Expression of Uncertainty in Measurement) [2] even though there is another method introduced also by JCGM, called propagation of distributions using a Monte Carlo method [3].

The presence of this supplement is very useful for validation of the uncertainty obtained by GUM. Comparison between calibration uncertainty of a platinum resistance thermometer based GUM and Monte Carlo has been proposed [4]. However, the resistance thermometer used is a Pt-25 not a Pt-100 and does not take into account the uncertainty contributed from the fitting (standard error estimate) which is presence when determining the coefficient of CVD equation and making interpolation. And for most of our cases, it gives significant impact to the calibration uncertainty. In this study, Monte Carlo simulation is applied to the IPRT calibration particularly based on CVD's equation. The result is compared to that of the uncertainty analysis based on propagation of uncertainty (GUM). The calibration range is from 0 °C to 500 °C as generally demanded by our customer

2. CALIBRATION METHOD AND UNCERTAINTY ANALYSIS

The calibration process is conducted by comparing the IPRT agains calibrated SPRT in the liquid bath as shown in Fig1. Three liquid baths with different spesification are used to cover all calibration range. A DC Resistance bridge is used to measure the output of IPRT and also SPRT as the standard and the data are stored to the computer. Twenty three temperature measurement points are choosed spread over range of 0 °C to 500 °C. The resistance output of SPRT are convert to temperature using its certificate calibration, therefore from the measurement points, 23 data pairs of temperature standar of SPRT and the resistance output of IPRT are collected as it is shown in Table 1. The pairs of data are used to generate the coefficient of CVD equation (*A* and *B*) as (1). Listquare method in MS.Excel function is used for regresion calculation.

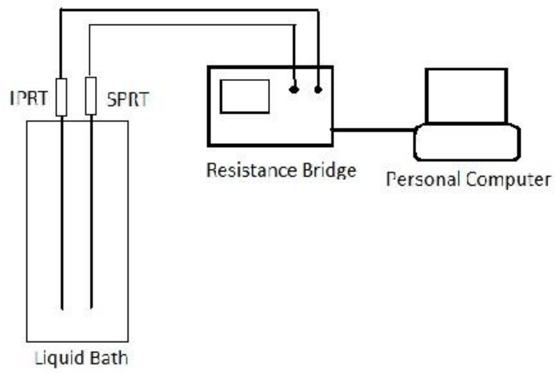

Fig. 1. Calibration setup.

The CVD equation used in this study is as follows:

$$R_t = R_0(1 + At + Bt^2) \quad (1)$$

Where R_t is the output of the IPRT in Ω, R_0 is the resistance of the IPRT in the ice point in Ω, and t is the temperature coming from the SPRT in °C.

Table 1. Data pairs used in this work

Resistance Ω	Temperature °C	Resistance Ω	Temperature °C
100.0300	0.00	168.5923	180.09
105.7783	14.73	174.1286	195.08
111.5967	29.71	183.2959	220.05
117.3913	44.69	192.3803	245.00
123.1564	59.66	201.2333	269.48
128.9006	74.65	210.1839	294.44
134.6254	89.65	219.0678	319.40
140.3177	104.62	228.1728	345.18
145.9816	119.61	262.7958	445.19
151.8235	135.13	271.3796	470.62
157.4417	150.12	281.4977	500.78
163.0284	165.11		

2.1. Uncertainty by GUM

The uncertainty analysis based on GUM is completed by using the matematical model as (2). t_x is temperature that provided by (1) for such measured output resistance of IPRT, R_t. The t_x is departed from the ITS-90 temperature scale, t_{90}, due to several uncertainty components which are given by Table 2.

$$\begin{aligned} t = t_{90} &+ ¢ts + ¢ts_{Drift} + ci \times (¢Rs + ¢Rs_{Drift}) + \\ &ci \times ¢RU + ci \times ¢RU_{Drift} + ci \times ¢H + \\ &ci \times ¢R_{res} + ci \times ¢A_{repeat} + ci \times ¢A_{SH} + \\ &¢M_{insta} + ¢M_{uni} + ¢M_{es} + ci \times ¢SEE \end{aligned} \quad (2)$$

2.2. Uncertainty calculation by Monte Carlo Method

Matematical model for Monte Carlo analysis is based on (1). The coefficient A and B obtained from the listsquare method are considered as constants which are 3.908425 x 10^{-3} °C^{-1} and -5.70594 x 10^{-7} °C^{-2} By introducing the standard uncertainty of each measured quantity, the model to run Monte Carlo simulation can be written as (3):

$$R_t + uR_t = (R_0 + uR_0)[1 + A(t + ut) + B(t + ut)^2] \quad (3)$$

Where

$$\begin{aligned} uR_t = &¢RU + ¢RU_{Drift} + ¢H + ¢R_{res} + ¢A_{repeat} + \\ &¢A_{SH} + ¢SEE \end{aligned} \quad (4)$$

$$\begin{aligned} uR_0 = &¢RU + ¢RU_{Drift} + ¢A_{SH} + \\ &¢_{repeat} + ¢_{res} + \frac{¢M_{es}}{2.5} \end{aligned} \quad (5)$$

$$\begin{aligned} ut = &¢ts + ¢ts_{Drift} + 10(¢Rs + ¢Rs_{Drift}) + \\ &¢M_{insta} + ¢M_{uni}; \end{aligned} \quad (6)$$

Rearrange (3) and yields (7):

$$\begin{aligned} &Bt^2 + (A + 2B(ut))t + B(ut)^2 + A(ut) - \\ &(R_t + uR_t)/(R_0 + uR_0) + 1 = 0 \end{aligned} \quad (7)$$

To find the uncertainty of temperature, a quadratic formulation (8) is used.

$$t = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad (8)$$

Where

$$a = B; b = A + 2But; c = B(ut)^2 + Aut - \frac{R + uR_t}{R0 + uR_0} + 1$$

The model is run repeatedly 1000000 steps for every set point and then analyzed to obtain standard devition,upper and lower limits, expanded uncertainty, coverage factor, and the shape of final distribution.

Table 2. Uncertainty budget of IPRT Calibration

Uncertainty Component	Symbol	Distribution	Type	Unit	Estimation	Division	Sensitivity coefficient (ci)	Standard uncertainty (°C)
Standard Resistance Thermometer								
1.Trueness SPRT	$¢ts$	Normal	B	°C	0.005	2	1	0.0025
2. SPRT Drift	$¢ts_{Drift}$	Rectangular	B	°C	0.01	√3	1	0.00577
3. Trueness bridge	$¢Rs$	Normal	B	Ω	0.00082	2	10	0.0041
4. Drift of bridge	$¢Rs_{Drift}$	Rectangular	B	Ω	0.00071	√3	10	0.0041
Resistance Thermometer under Calibration								
5. Trueness bridge	$¢RU$	Normal	B	Ω	0.00142	2	2.5	0.00071

6. Drift of bridge	¢RU_{Drift}	Rectangular	B	Ω	0.0006	√3	2.5	0.00087
7. Hysteresis	¢H	Rectangular	B	Ω	0.002	√3	2.5	0.00289
8. Resolution	¢R_{res}	Rectangular	B	Ω	0,0005	√3	2.5	0.00072
9. Repeatability	¢A_{repeat}	Normal	A	Ω	0.00013	1	2.5	0.00033
10. Self heating	¢A_{SH}	Rectangular	B	Ω	0.002	√3	2.5	0.00289
Calibration Media								
11. Uniformity	¢M_{uni}	Rectangular	B	°C	0.03	√3	1	0.01732
12. Stability	¢M_{insta}	Rectangular	B	°C	0.004	√3	1	0.00231
13. ice point	¢M_{es}	Rectangular	B	°C	0,002	√3	1	0.00115
Interpolation								
14. SEE	¢SEE	Normal	A	Ω	0.0086	1	2.5	0.02158
Expanded uncertainty (k=2)	**U=0.059**							

3. RESULT AND DISCUSSION

3.1. The calibration uncertainty

For the GUM based uncertainty anaysis, the calculation for getting expanded uncertainty is stright forward. All of the standard uncertainties are combined then multiplying the result with the coverage factor k which is generally k=2 coused by assumption that the combine uncetainty has normal distribution . For this case, the expanded uncertainty U for the temperature is found to be 0.059 °C.

From the Monte Carlo simulation, the expanded uncertainty is attached to each set point as it is shown in Fig 2. The calibration uncertainty value starts from 0.057 °C to 0.065 °C with k=1.94 to k=1.96. Compare to GUM result, Monte Carlo method can see the non-linear effect of the model which begins to appear at temperatures above 200 °C proved by the linear line in Fig 2. However, if only one significant digit is considered then two method gives the same value which is U=0.06 °C.

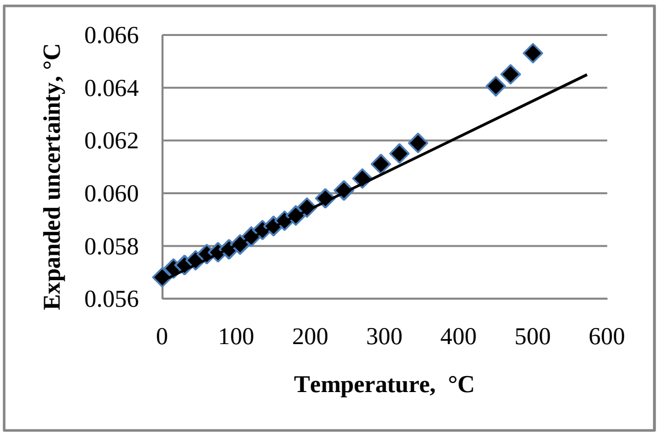

Fig. 2. The expanded uncertainty obtained by Monte Carlo Method

3.2 Finding the influence of each uncertainty component

Determining how big the influence of an uncertainty component is essential to improve our CMC (calibration measurement capabilities). This analysis requires variance. For GUM method, by following Table 2 the variance that would be obtained is simple calculated by dividing the square of the standard uncertainty with the square of the combined uncertainty multiplied by 100 percent. For Monte Carlo method, the calculation is more complex. To have the variance of a single component, the following step should be conducted [5]:

a. Calculate the combined standard uncertainty, for given all of uncertainty components.
b. Calculate the combined standard uncertainty for given a single component while the others are set to be zero.
c. Calculate the contribution of the component by dividing the square of the combined standard uncertainty obtained by (b) with that obtained by (a) multiplied by100 percent.

The example analysis of the influence of each uncertainty component can be seen in Table 3. It is showed that the most significant component is the standard error estimate from the interpolation to get CVD coefficient A and B. In second place is occupied by the bath uniformity. Thus, to improve our CMC, these components should be reduced.

Table 3. The influence of each uncertainty component at 500 °C

Uncertainty Component	Symbol	Value based on GUM (%)	Value based on Monte Carlo (%)
1.Trueness SPRT	¢t_S	0.7	0.6
2. SPRT Drift	¢ts_{Drift}	3.8	3
3. Trueness bridge	¢Rs	1.9	1.5

4. Drift of bridge	¢Rs_{Drift}	1.9	1.5
5. Trueness bridge	¢RU	0.4	1.3
6. Drift of bridge	¢RU_{Drift}	0.1	0.3
7. Hysteresis	¢H	1.0	1.1
8. Resolution	¢R_{res}	0.1	0.2
9. Repeatability	¢A_{repeat}	0.0	0.0
10. Self heating	¢A_{SH}	1.0	3.5
11. Uniformity	¢M_{uni}	**34.6**	**26.8**
12. Stability	¢M_{insta}	0.6	0.5
13. ice point	¢M_{es}	0.2	1.4
14. SEE	¢SEE	**53.7**	**59.6**

3.3 The shape of distribution

Fig. 3 presents the shape of temperature distribution resulted by Monte Carlo method for temperature 500 °C. The shape is close to a normal distribution which is exactly the same with the assumption under GUM method.

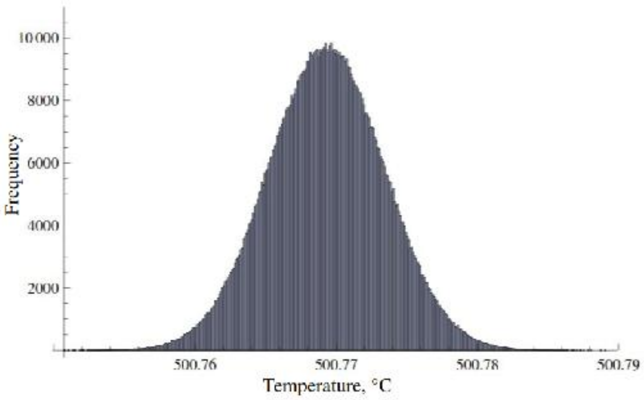

Fig. 3. The distribution of temperature determined by Monte Carlo method at 500 °C

4. CONCLUSIONS

Applying Monte Carlo Simulation to the IPRT Pt-100 calibration is useful to validate the uncertainty analysis based on GUM. It gives more information for the calibration uncertainty such as the calibration uncertainty in every point within the calibration range, and the distribution shape. In this case, the calibration uncertainty for IPRT between GUM method and Monte Carlo method shows agreement for one significant number. Thus, writing the uncertainty in the IPRT calibration certificate should follow this way. Moreover, choosing the coverage factor k=2 is also an appropriate choice as it is showed by Monte Carlo simulation that the distribution of the combined uncertainty is normal distribution. Lastly, to improve CMC, it is necessary to reduce ¢SEE and ¢M_{uni} since both of them are the most significant components in the calibration uncertainty.

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