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PRECISE TIME-INTERVAL GENERATOR

Dariusz Chaberski [1]

[1] Affiliation (Institute of Physics, Faculty of Physics, Astronomy and Informatics, Nicolaus Copernicus University, Grudziądzka 5, 87-100 Toruń, Poland), daras@fizyka.umk.pl

Abstract **-** The paper presents the design of high resolution TI (Time-Interval) generator implemented into FPGA (Field-Programmable-Gate-Arrays) device, their construction and experimental test results. The resolution improvement is guaranteed by a chain of digitally-controlled delay-elements. Presented TI generator posses two modes of operation. The first mode allows for precise time-interval generation. In the second mode TIs of specified Gaussian distribution may be generated.

Keywords: FPGA, time-interval, tapped-delay-line, inverse function of Gaussian cumulative distribution

1. INTRODUCTION

Precise TI generators find many applications in science, engineering and industry [1, 2, 3]. TI generators may create time-references in many physical experiments, prototyping and in many other different areas such as radar and telecommunication systems generally [4, 5].

Two different approaches are used to provide time-intervals (time-delays), namely analogue delay-lines or digital ones. Analogue delay-lines usually use passive components such as RLC circuits, micro-strips or optical fibbers in order to introduce the desired delay for the signal [3, 6]. Analogue delay-lines can neither be integrated nor their delay-values are stable and independent on the input signal frequency. The use of digital delay-lines allows integrating, obtaining independency on input-signal frequency and the control that targets most digital systems [1, 2].

However, digital delay-lines, especially those implemented into FPGA structures, posses relatively low resolution and high non-linearity. Higher resolution and smaller non-linearity can be achieved when ASIC (Application Specific Integrated Circuit) technology is used [1, 2, 7].

This article presents the method of TI generator resolution improvement to the level o single picoseconds. The resolution improvement is obtained by the serial connection of delay-elements of different characteristics. TI generator takes advantages of digital-lines implemented in FPGA.

2. THE IDEA

The single DCDE (Digitally-Controlled-Delay-Element) may be created as a TDL (Tapped-Delay-Line) with its outputs connected to the multiplexer data inputs (Fig. 1). The address inputs (Control) of the multiplexer select the TDL output to be passed out. Higher control-value usually chooses paths of higher delay between $\mathrm{CLK_{IN}}$ input and $\mathrm{CLK_{OUT}}$ output. The characteristic of DCDE is always non-linear and very often may not be even monotonic.

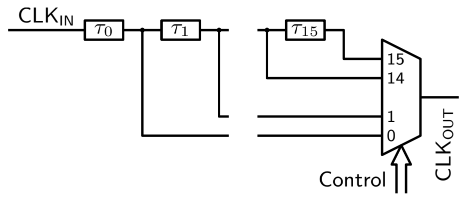

Fig. 1. The single digitally-controlled delay-element.

The chain of many DCDEs allows obtaining higher resolution due to mentioned non-linearities. Different real DCDEs may produce slightly different delays for the same control value. These differences are getting smaller when the number of DCDE increases. Because of non-linearity and non-monotonicity slightly different delay values may also be obtained for different control values in different DCDEs.

Let us suppose there are two four-stage DCDEs of characteristics presented in Fig. 2. These characteristics are non-linear and monotonic. The average delay change per index change is equal to about 1τ for both DCDEs.

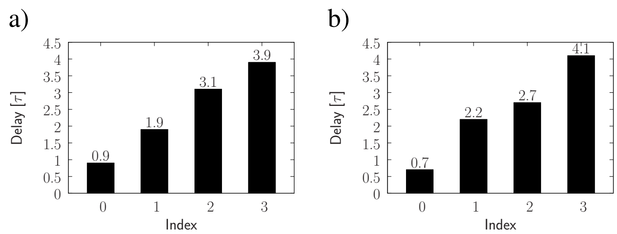

Fig. 2. The characteristic of the first (a) and the second (b) digitally-controlled-delay-element.

Additionally all sorted delay values create the curve that is very similar in shape to the inverse function of Gaussian cumulative distribution (Fig. 4). The similarity is higher the larger number of DCDEs is used. It has been experimentally found that four such elements are enough.

Applying random variables of uniform distribution to DCDE control inputs one can obtain TIs of Gaussian distribution. Controlling the ranges of these random variables allows obtaining TIs of Gaussian distributions of specific standard deviation and average values [1]. The independent random variables of uniform distributions

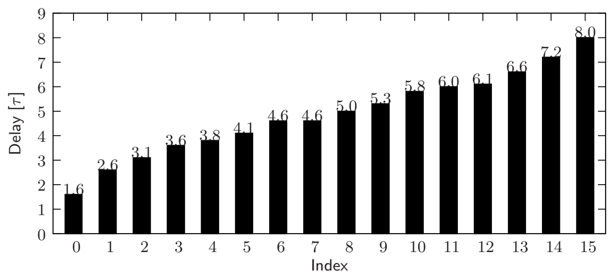

Fig. 3. The characteristic of two serially connected digitally-controlled-delay-elements.

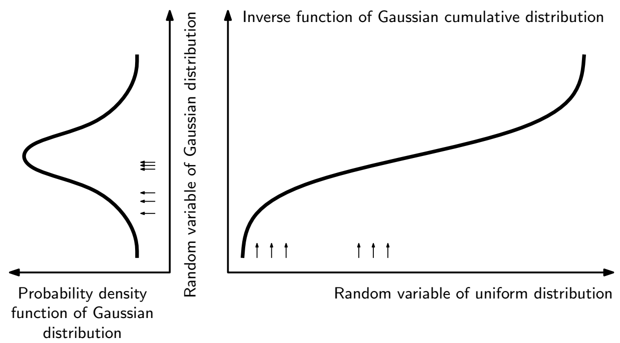

Fig. 4. The idea of obtaining random variable of Gaussian distribution.

(control values - Fig. 1) are easy accessible in computers and do not demand a lot of hardware.

Fig. 4 shows the process of generation of random variable of Gaussian distribution by the use of random variable of uniform distribution and inverse function of Gaussian cumulative distribution. Vertically placed arrows (input argument) represent the random variable of uniform distribution - they are equally distanced. Horizontal arrows, that represent the output values, are being thickened when the Gaussian-cumulative-distribution-inverse-function rises slower and are dispersed when this function rises faster. Consequently the horizontally placed arrows posses Gaussian distribution.

The fact that the delay should have Gaussian distribution results from the central-limit-theorem. The serial connection of several (four or more) DCDEs allows obtaining the delay of Gaussian distribution while each DCDE generates delay of uniform distribution and these delays are independent.

3. TIME-INTERVAL GENERATOR

Time-interval generator has been designed to be able to generate either high-precision TIs of specified length or TI of specified Gaussian distribution. Precise TIs of specified length are generated when control-vector values are fixed. Whereas TIs of specified Gaussian distribution can be generated by changing control values randomly in specified ranges by the use of limited random number generator.

3.1. Generating time-intervals of specified length

To be able to generate TIs of specified length one has to evaluate the table of structures where every structure-element contains the information about generated TI and the control-values (see table at the bottom of Fig. 5). The first field of every structure-element contains measured TI, the four remaining fields contain the control values. The structure elements are put in order with respect to TI values.

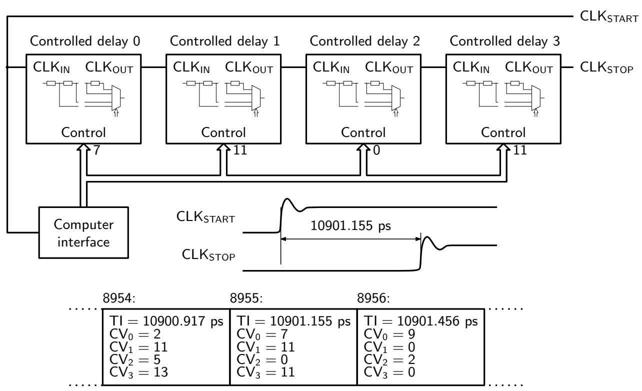

Fig. 5. Specified length time-interval generator.

To generate TI of specified length one has to find the nearest TI value in the table of the structures, feed control values to all DCDEs control-inputs and then trigger. Triggering consists of rising edge of $\mathrm{CLK_{START}}$ generation (see waveforms at the middle part of Fig.5). Every rising-edge passes through the chain of DCDEs in time dependent from FCDEs control values. The TI is measured from the rising-edge of signal $\mathrm{CLK_{START}}$ to the rising-edge of signal $\mathrm{CLK_{STOP}}$.

3.2. Limited random number generator

Limited random number generator is responsible for providing random numbers of uniform distribution in specified range. To obtain random numbers one can seize counter CNT values asynchronously to its incrementing clock $\mathrm{CLK_i}$ (Fig 6). When counter CNT reaches the TOP value then the comparator output sets up to 1. Then the next rising edge of $\mathrm{CLK_i}$ causes setting the counter with the BOTTOM value instead of incrementing it. In this way counter CNT values are hold in range $[\mathrm{BOTTOM}, \mathrm{TOP}]$.

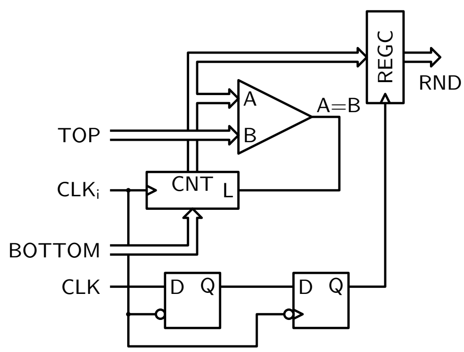

Fig. 6. Limited random number generator.

Every rising-edge of signal CLK after synchronisation (two bottom elements: the latch and the flip-flop) with the falling-edge of $\mathrm{CLK_i}$ registers the counter state in REGC. The REGC keeps the random value RND till the next sampling.

3.3. Specified Gaussian distribution time-interval generator

Fig. 7 shows diagram of TI generator of hardware implementation of Gaussian distribution. The idea has been explained in section 2. To obtain specific Gaussian distribution one has only to set the limit values (TOP and BOTTOM) for all DCDEs and enable all clock generators.

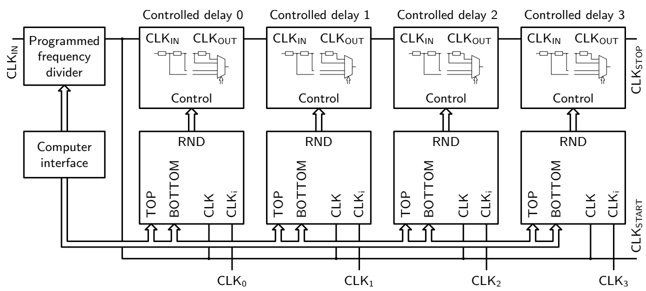

Fig. 7. Specified Gaussian distribution time-interval generator.

Every DCDE delay changes randomly with uniform distribution but the whole delay that is the sum of all DCDE delays posses, as mentioned in section 2, very similar to Gaussian distribution. To increase the standard deviation of generated TIs one has to increase the distance between the BOTTOM and TOP values. When additionally the sum $TOP + BOTTOM$ is not changed then the average value of Gaussian distribution remains constant. By changing the sum $TOP + BOTTOM$ one can control the average value of generated TIs. The sum is higher, the average value increases. When the difference $TOP - BOTTOM$ is constant the standard-deviation of generated TI is also constant.

4. EXPERIMENTAL RESULTS

This section contains experimental results that confirms the rightness of the ideas presented in section 2. Following subsections demonstrate TI generator characteristic, the possibility of resolution increase by serial connection of four DCDEs as well as the possibility of obtaining TIs of Gaussian distribution. The imperfections of the method are also discussed.

4.1. Time-interval generator characteristic

Figure 8 shows characteristics of all four DCDEs. One can see that these characteristics are non-linear and non-monotonic. To make the characteristics of DCDEs monotonic their Control inputs were connected to LUTs providing index conversion. For example the 9-th delay of the first DCDE (Fig. 8a) index should become the last one and the 2-nd delay of the third DCDE (Fig. 8d) should become the 1-st one.

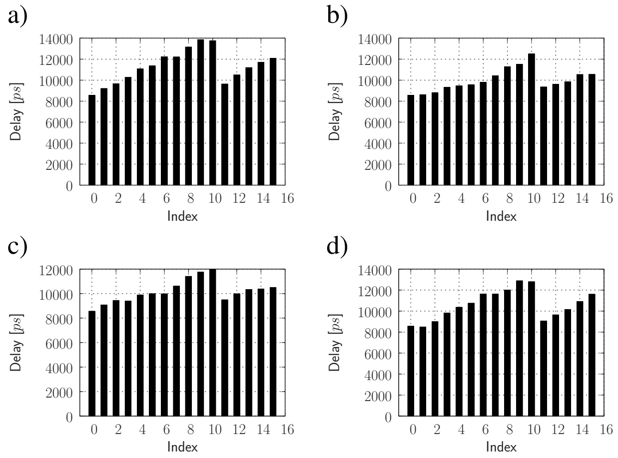

Fig. 8. The characteristic of the first (a), second (b), third (c) and fourth (d) digitally-controlled-delay-element.

Simulated distributions

Figure 9 shows delay simulated from DCDEs characteristics (Fig. 8) in full spectrum of control-values, i.e. from $(0, 0, 0, 0)$ to $(15, 15, 15, 15)$ (index from 0 to $16^4 - 1$). The delay changes from about $8\ ns$ to about $26\ ns$. The characteristic is almost linear in range of indexes from 10000 to 50000. In this range delay could be controlled with resolution of about $\frac{18\ ns - 14\ ns}{50000 - 10000} = 0.1\ ps$. Of course FPGA structure temperature fluctuations as well as disturbances and supply voltage noise makes that single TI posses much greater uncertainty than this resolution (results presented in Fig. 9 have been obtained by averaging 2^{24} measurements).

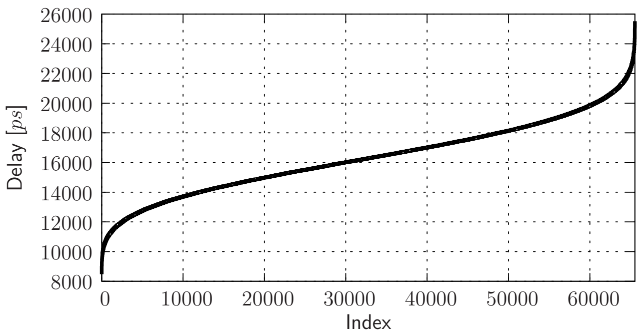

Fig. 9. DCDEs delay dependency from control-values index (control-values were changing in full range).

By applying random-variable of uniform distribution as an index value (Fig. 9) one can simulate the distribution of TIs generated by all DCDEs (Fig. 10). The obtained distribution (Fig. 10) is sufficiently similar to the Gaussian one. Here the control-variables have been changed in full range (from $(0, 0, 0, 0)$ up to $(15, 15, 15, 15)$). Changing control-values in limited ranges one can obtain Gaussian distributions of desired average value and standard-deviation, of course, in limited range.

Fig. 11 shows delay characteristics dependency from index, parametrised with control-values limits. For all curves

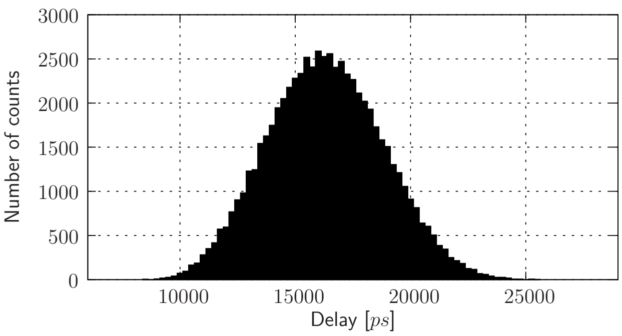

Fig. 10. Simulated distribution of time-intervals obtained from Fig. 9 analogically as it was presented in Fig. 4

limits distances are the same. For example, the distance for $(2, 2, 2, 2, 8, 8, 8, 8)$ is equal to $(8-2)+(8-2)+(8-2)+(8-2) = 24$ and the distance for $(9, 8, 8, 4, 15, 14, 14, 10)$ is equal to $(15-9)+(14-8)+(14-8)+(10-4) = 24$ and so on. That is why all these delays have similar spread (standard-deviation) versus index change. However, for all curves the control-values start and stop at different points. The 1-st curve control-values start at $(2, 2, 2, 2)$ and the 6-th curve ones start at $(9, 8, 8, 4)$. This explains the vertical displacement (different average values) of the curves.

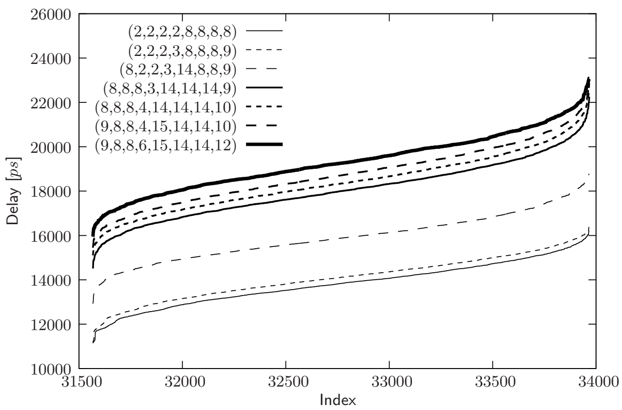

Fig. 11. DCDEs delay dependency from control-values. Control-values changes in such ranges that guarantee constant standard-deviation and variable average value.

Fig. 12 shows the case when the average-values for all delay curves are the same. All curves crosses the same point (average-value) and spread in both directions dependently from the limits of control-values. The length of the front-view of the curve determines distribution standard-deviation. When control-values changes in ranges from $(2, 2, 3, 3)$ to $(12, 12, 11, 11)$ TIs posses the largest standard deviation (Fig. 12). Here TIs changes from about 11 ns to about 19 ns (peak-to-peak), so standard-deviation is equal to about $\frac{19\ ns - 11\ ns}{6} \simeq 1.33\ ns$. Taking into account the curve with the shortest front-view $(5, 5, 5, 5, 9, 9, 9, 9)$ one can see that TIs posses meaningfully smaller dispersion. TIs changes from about 14 ns to about 17 ns, so their standard-deviation is equal to about 500 ps.

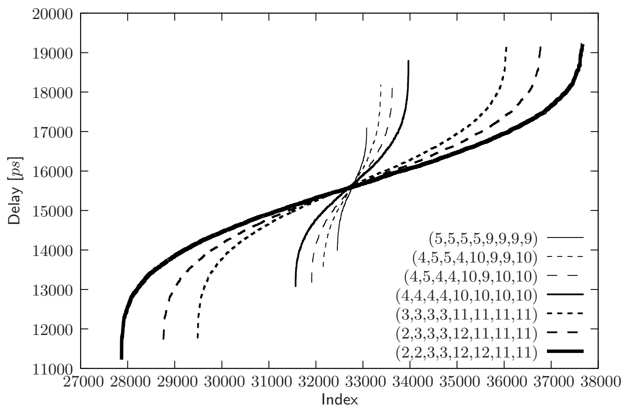

Fig. 12. DCDEs delay dependency from control-values. Control-values changes in such ranges that guarantee constant average-value and variable standard-deviation.

Fig. 13 and 14 show simulated PDFs (Probability-Density-Function) of TIs presented respectively in Fig. 11 and 12. PDFs presented in Fig. 13 should posses the same standard-deviations, in fact they slightly differ. These differences are caused by TIs generator non-linearities. PDFs average-values increases (Fig. 13) when average-value of both limits increases. This is true for all PDFs.

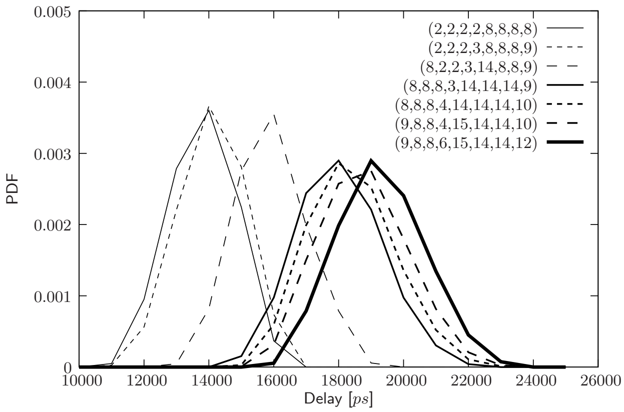

Fig. 13. Probability density functions of time-intervals obtained from serially connected digitally-controlled-delay-elements that characteristics have been presented in Fig. 11

In case when average values of both limits are equal (Fig. 14) the average value does not change. The limit changes cause only average value changes.

4.2. Time-intervals of specified length

Fig. 15 shows histograms of specified length TIs. Eight different TIs sets have been generated. Histograms obtained for shorter TIs (the upper histogram in Fig. 15) are narrower than in case when TIs are longer (the bottom histogram). The simplest explanation of the difference is the assumption that the delay-element introduces time-noise proportional to delay-value. In fact the time-noise visible in histograms was also produced by LeCroy 804Zi oscilloscope and was not negligible. When both probes

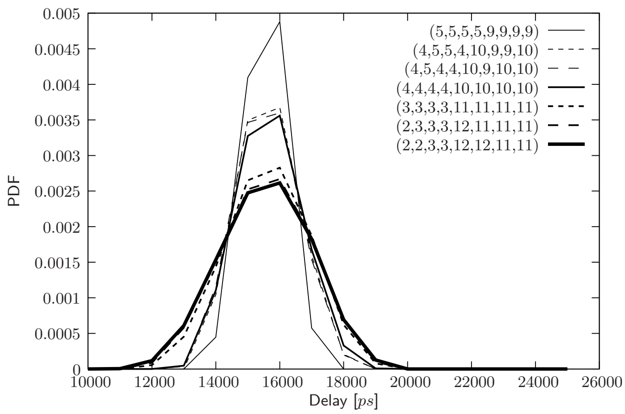

Fig. 14. Probability density functions of time-intervals obtained from serially connected digitally-controlled-delay-elements that characteristics have been presented in Fig. 12

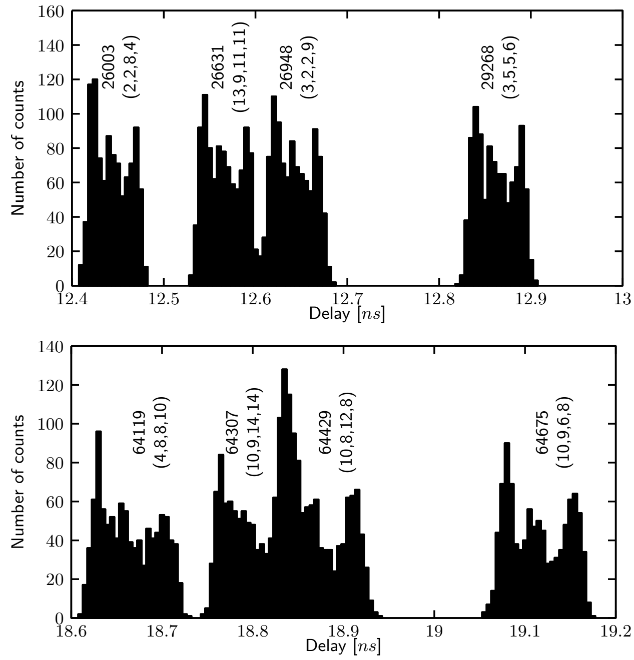

Fig. 15. Histograms of time-intervals of specified length obtained for eight different control-values. Control-values are placed in parenthesis with indexes above them.

of the oscilloscope were shorted then TIs of Gaussian distribution of standard deviation of about 20 ps were registered. The standard-deviation of 8-th peak (the most wide one whose index is equal to 64675) equals to about $\frac{120\ ps}{\sqrt{12}} \simeq 35\ ps$. Removing the oscilloscope influence ($\sqrt{(35\,ps)^2 - (20\,ps)^2}$), the standard deviation of TIs generated by TIs generator is less than 30 ps.

Fig. 15 also shows how high is the resolution of TI generator. Taking into account two first peaks of indexes 26003 and 26631 one can notice that there are 628 different possible TIs to be generated in time-range from about 12.44 ns to about 12.58 ns. This means that the average resolution in that range is equal to about than 0.23 ps per index. However in range from about 18.88 ns to about 19.12 ns the resolution is equal to about 0.97 ps.

According to Fig. 10 TIs generator can generate TIs in range from about 10 ns to about 25 ns. Summing up previous paragraphs, one can notice that the average resolution of generated TIs can be equal to 1 ps at the standard-deviation better than 30 ps.

4.3. *Time-intervals of Gaussian distribution*

Fig. 16 shows TIs histograms obtained in case when distances of control-values limits were the same ($5 - 2 = 8 - 5 = 9 - 6$). Generated TIs posses similar standard-deviations, while the average-values of TIs depend on the average-value of control-value limits.

Fig. 17 shows TIs histograms obtained in case when the arithmetic average-values of both control-value limits were the same ($5 + 2 = 4 + 3 = 3.25 + 3.75$). Generated TIs posses similar average-value. The standard-deviation of TIs distributions depends on control-value limits distance.

There are some imperfections in standard-deviation value, average-value and shapes of the histograms. However the correlation between control-value limits and obtained Gaussian histogram parameters are impressive. Some kind of linearisation and calibration is needed but the results are promising.

The range of generated TIs in this mode depends strongly on their standard-deviation. When standard deviation is higher, the range of generated TIs is smaller. Fig. 10 allows estimating maximum value of TIs standard-deviation (σ_t) dependency on its length (t) and vice versa. The rule is quite simple. The generated TI length t can vary (in both directions) from the Gauss center position (about 17 ns) as much as its threefold standard-deviation $3\sigma_t$ differs from the threefold standard deviation of the Gauss presented in 10 ($\frac{25\ ns - 9\ ns}{2} \simeq 2.67\ ns$). For example when $t = 20\ ns$ then maximal σ_t is equal to about $\frac{25\ ns - 20\ ns}{3} \simeq 1.67\ ns$, and when $t = 12\ ns$ then its standard-deviation σ_t can not be greater that $\frac{12\ ns - 9\ ns}{3} \simeq 1\ ns$. When standard-deviation would be equal to $1.5\ ns$ then TI could be generated in range from $10\ ns + 7.5\ ns - (7.5\ ns - 3 \times 1.5\ ns) = 14.5\ ns$ to $10\ ns + 7.5\ ns + (7.5\ ns - 3 \times 1.5\ ns) = 20.5\ ns$ but when $\sigma_t = 2\ ns$ then $t \in [16, 19]\ ns$.

5. CONCLUSIONS

Two methods of resolution improvement and hardware Gaussian distribution generation appeared to be very useful. TI generator can operate in two modes. In high-resolution mode it is possible to generate TIs with resolution of 1 ps and standard deviation of 30 ps. In the other mode TIs of specified Gaussian distribution can be generated. Gaussian distribution parameters can be adjusted in wide range.

Some kind of TDLs linearisation process would be needed to improve TI generator parameters. To avoid ambient-temperature change influence on generated TI parameters the process of the supply-voltage regulation should be implemented.

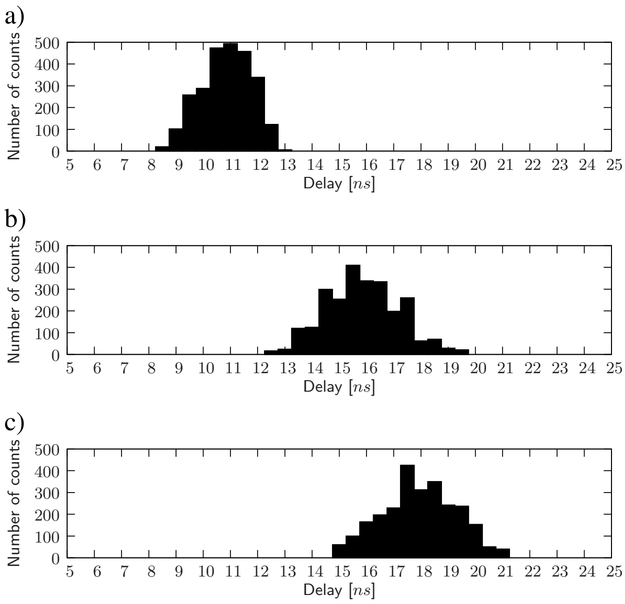

Fig. 16. Histograms of time-intervals of specified Gaussian distribution with constant standard-deviation. Control-values change in ranges (a) (2,2,2,2,5,5,5,5), (b) (5,5,5,5,8,8,8,8) and (c) (6,6,6,6,9,9,9,9)

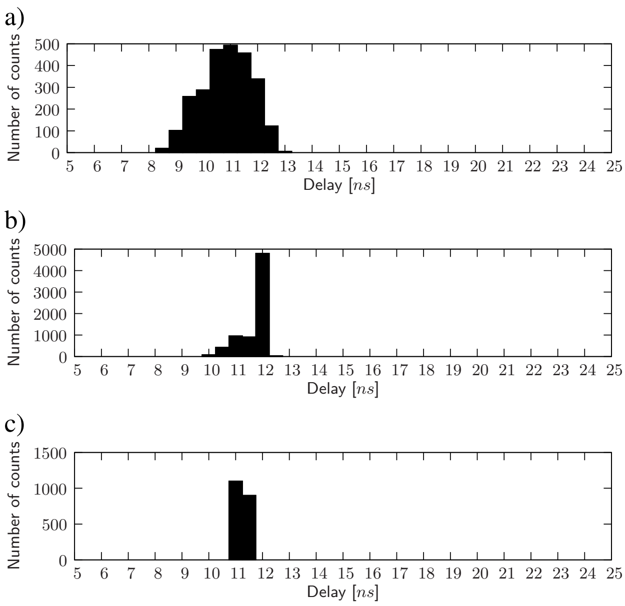

Fig. 17. Histograms of time-intervals of specified Gaussian distribution with constant average-value. Control-values change in ranges (a) (2,2,2,2,5,5,5,5), (b) (3,3,3,3,4,4,4,4) and (c) (3,3,3,4,3,4,4,4)

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