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UNCERTAINTY EVALUATION OF SINUSOIDAL FORCE MEASUREMENT

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Abstract – Periodical force measurement is used for the primary calibration of force transducers. This abstract describes models for an uncertainty determination of the sensitivity, which is the main measurand. Influences like the mechanical coupling of test masses as well as their rocking during a measurement are the dominating contributions to the uncertainty of measurement. The achievable uncertainties will be demonstrated on the basis of intercomparison periodical force measurements between CEM and PTB as well as the comparison between standard GUM and Monte Carlo based uncertainty evaluation

Keywords: dynamic force measurement, acceleration measurement, uncertainty evaluation, key comparison, Monte Carlo method.

1. INTRODUCTION TO THE METHOD

Periodical force measurement is one of the primary methods for the realization of dynamic force, whereby the force is determined according to Newtons Law, mass times acceleration. The traceability is thereby realized by an acceleration measurement based on a laser vibrometer and a mass measurement. The laser vibrometer is ussually operated with a laser wave length of 632.8 nm. The vibrometer used here at PTB was calibrated against the national acceleration standard for vibrometer calibration. The masses are determined with a balance, calibrated with a set of reference masses calibrated against the national mass standard. The measurand which is determined in a periodical force calibration is the dynamic sensitivity. The sensitivity is thereby the ratio of the output signal of the force transducer and the dynamic force. To obtain the pure calibration of the force sensor, the frequency response of the conditioning amplifier has to be unfolded from the electrical signal. For the dynamic calibration of bridge amplifiers a special bridge calibration standard is used, which was calibrated against the national voltage standard. In the case of piezoelectric transducers the charge amplifiers are calibrated against a reference capacity. It should be noted that all calibrations have to be done for the frequency range used in the force measurement. The sinusoidal calibration is performed in such a way that all frequencies which are used for a calibration are measured separately. Thereby exactly a certain number of sinusoidal periods will be used to avoid filter windows for the determination of the amplitudes in the FFT analysis of the measured time signals. The excitation signal is generated via an arbitrary waveform generator module, which is then directly fed in the power amplifier of an electrodynamic shaker, where the force transducer is mounted. The transducer itself is equipped with an additional loading mass to generate, together with the adjusted acceleration, a certain dynamic force. More details of the method and signal analysis can be found in [1].

2. MEASUREMENT MODELS

For the determination of the uncertainty of the sensitivity the following model is used:

$$S_f = \frac{\overline{U} \cdot k_U}{(M + m_i) \cdot \overline{a} \cdot k_a \cdot K_{corr}} \tag{1}$$

Thereby S_f is the frequency-dependent sensitivity given in mV/V/N in case of a strain gauge force transducer, and in pC/N in case of a piezoelectric transducer. The output signal of the conditioning amplifier is U, and k_U is the calibration factor of the amplifier at the certain excitation measuring frequency. The denominator contains M the mass of the force initiating test mass or load mass, and m_i is the head mass or internal mass of the force transducer. The acceleration measured on the surface of the loading mass is a, and k_a is the calibration factor for the acceleration of the vibrometer at a certain frequency. Finally K_{corr} is a correction factor of the acceleration, taking into account the vertical acceleration gradient of the loading mass.

It should be noted that this model is only valid if the loading mass is directly screwed onto the force transducer. As mentioned above, if one is using an adapter to dock the loading mass on the transducer, also the acceleration of the adapter has to be measured, due to the relative motion of the adapter in respect to the loading mass. In this case the following model has to be applied:

$$S_f = \frac{\overline{U} \cdot k_U}{(m_a + m_i) \cdot \overline{a}_1 \cdot k_{a1} + M \cdot \overline{a}_2 \cdot k_{a2} \cdot K_{corr}} \tag{2}$$

Thereby a_1 is the acceleration measured on the adapter and a_2 the acceleration measured on the loading mass, whereby k_{a1} and k_{a2} are the associated calibration factors. The mass of the adapter is given by m_a.
It should be noted that the accelerations as well as the dedicated force signal will be obtained by averaging several scan points, distributed over the surface of the whole mass block or adapter.
To distinguish the influence of the adapter from the test mass, it is useful to separate the sensitivity into two contributions.

$$S_1 = \frac{\overline{U}}{(m_a + m_i) \cdot \overline{a}_1} \qquad S_2 = \frac{\overline{U}}{M \cdot \overline{a}_2} \tag{3}$$

Thereby S_1 is the sensitivity which is measured on the adapter and S_2 the sensitivity determined on the mass block. As seen later on, the behaviour of these two values will be quite different. Both sensitivities can then be combined to the final sensitivity for the force transducer:

$$S_f = \frac{1}{\underbrace{\frac{(m_a + m_i) \cdot \overline{a}_1}{\overline{U}}}_{\frac{1}{S_1}} + \underbrace{\frac{M \cdot \overline{a}_2}{\overline{U}}}_{\frac{1}{S_2}}}; \qquad S_f = \frac{S_1 . S_2}{S_1 + S_2} \tag{4}$$

For more clearness the calibration constants and the correction factor for the vertical acceleration gradient are not written in either part of equations (4), to both parts of equation (3), but have to be considered in the analysis. As seen from the structure of the final sensitivity, *S*, is just a parallel connection of the individual sensitivities, S_1 and S_2.

On the other hand the sensitivity can also be expressed in terms of the parameters, stiffness k_f and damping factor b_f of the force transducer. In the simplest case the transducer can be approximated by two masses (bottom and top mass), which are connected by a spring. In this case the sensitivity is given by:

$$S_f = \frac{S_{f0}}{\sqrt{1 + \left(\frac{b_f}{k_f} \cdot \omega \right)^2}} \tag{5}$$

Thereby S_{f0} is the statically determined sensitivity. For practical use equation (5) can be developed in a Taylor series:

$$S_f = S_{f0} \cdot \left(1 - \frac{1}{2} \lambda \omega^2 + \frac{3}{8} \lambda^2 \omega^4 - \frac{5}{16} \lambda^4 \omega^6 + \ldots \right) \tag{6}$$

Thereby $\lambda = b_f / k_f$, the ratio of stiffness to damping factor. As seen later on, this kind of polynomial can be used to fit the measured data of the sensitivity.

3. MEASURED SENSITIVITY

To illustrate the different contributions according to equations (3), an example of measurement is given in Fig. 2. The measurement was done with a strain gauge transducer with a nominal force of 2.2 kN. The loading mass of 6 kg was mounted onto the transducer with a mechanical adapter (mass approx. 1 kg). The adapter is based on two clamping bushes which clamp the mass block to the transducer, see Fig1.
The acceleration was measured with the aid of a scanning vibrometer. The scanning vibrometer is able to measure, on a certain given area, many acceleration points in a sequential way. Usually a grid of points is defined on the surface of the object to be measured. In our case, this was one grid on the ring surface of the mass block and another one on the circular surface of the inner shaft of the adapter. Thereby the mass block grid had 28 points and the adapter grid 9 points. The measuring procedure is the following; in a first scanning run all points are measured one after the other. Afterwards the software will check if really all points are valid, if not, invalid points will be measured again. A invalidated measurement may for instance be an noisy signal, where the software cannot really determine the acceleration amplitude. The whole procedure will be repeated three times, and as the final acceleration value the mean value is taken.

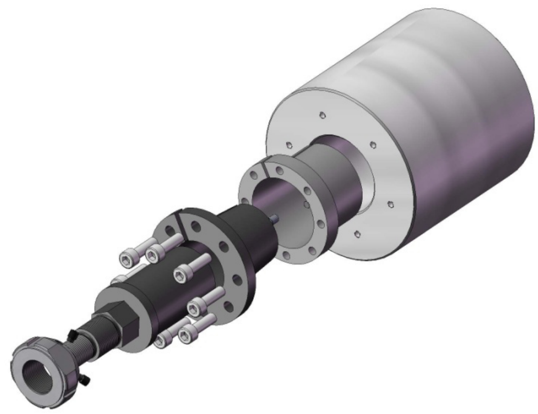

Fig. 1. Mechanism to clamp a mass block with a mechanical adapter to a force transducer. One clamping bush clamps the mass block, the other one clamps both to a cylindrical shaft, which is screwed onto the force transducer.

In Fig. 2 one can see that the behaviour of the two sensitivities is quite different.
The sensitivity S_1 on the adapter is more or less constant with increasing frequency; in contrast the sensitivity measured on the mass block decreases as a function of frequency. The combined sensitivity, S, is at lower frequencies, which is in accordance with the statically determined sensitivity. Figure 2 shows also that in the case where an adapter is used, it is necessary to take into account the adapter behaviour. If one only measured on the mass block, the sensitivity would be overestimated.

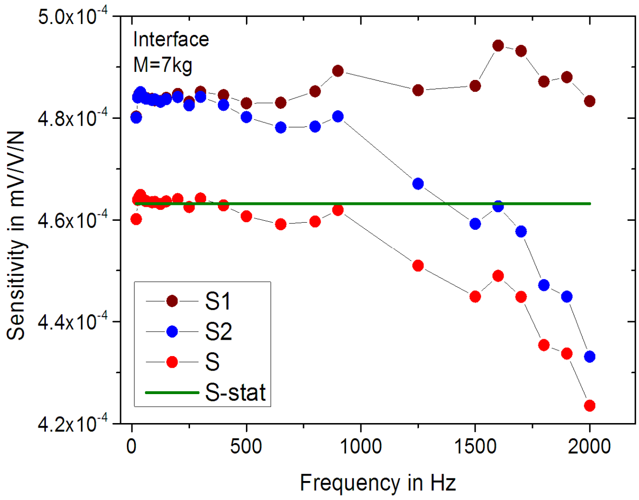

Fig. 2. The sensitivity of a strain gauge transducer which was equipped with a 7 kg loading mass is shown. The sensitivity was devided to the two contributions according equation (3). In addition the statically measured sensitivity is shown as a constant line.

In the framework of the EMRP Programme "Dynamic Measurement of Mechanical Quantities" [2] a comparison of sinusoidal force measurement was performed between CEM and PTB; thereby the measurement setups were quite similar. The main difference was that CEM used a single point vibrometer. Nevertheless, also at CEM there were several acceleration measurements on the surface of the mass block by moving the vibrometer. In Fig. 3 the result comparing CEM and PTB regarding the sensitivity measurement is shown for the case of the transducer described above. For the coupling of the 6 kg test mass the same adapter was used. It should be noted that the analysis of the data was undertaken with different data acquisition systems.

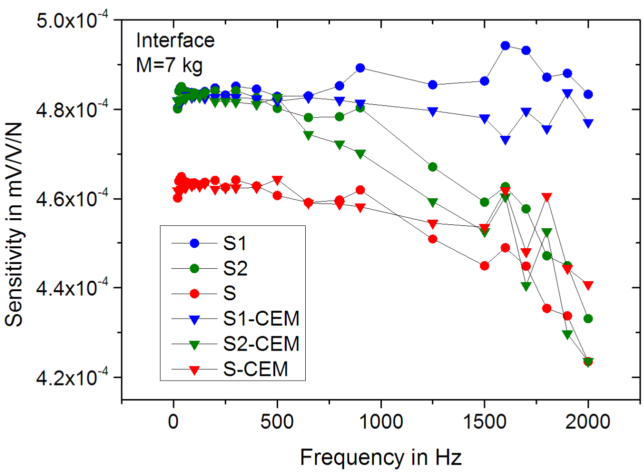

Fig.3. Comparison between CEM and PTB of the sensitivity from the transducer described already in Fig.2.

According to Fig. 3 the behaviour of the sensitivity curves is quite similar for CEM and PTB. At frequencies above 1 kHz a bigger deviation can be seen. This can be shown even

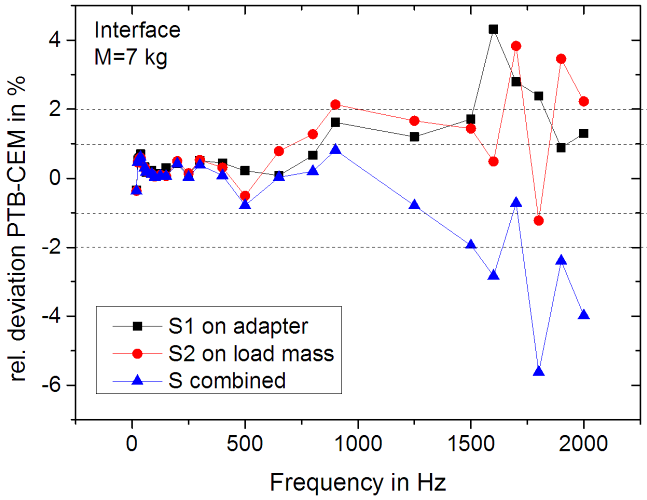

Fig.4. Relative deviation of the sensitivity in percent between CEM and PTB.

more clearly, if the relative deviation of the sensitivity between both laboratories is compared. According to Fig. 4 one can summarize that for the investigated case the deviation of the combined sensitivity is ≤1% up to frequencies of ≈1.5 kHz.

The origin of the bigger deviations can be seen in mechanical influences between the force transducer on one side and the adapter and the test mass on the other side. As shown in [3] there is a greater probability for the rocking of the test mass against the transducer body. This rocking may be included in the standard deviation of the acceleration measurement if the surface of the mass body is scanned with several acceleration points. As also shown in [3] the direction of rocking may change with frequency and can be also quite asymmetric. Because of the fact that the CEM was only able to measure the acceleration on a few selected points (4-5), the effect of rocking cannot be detected carefully. From this point of view it is not surprising that bigger deviations occur at higher frequencies.

4. UNCERTAINTIES BASED ON STANDARD GUM

Figure 5 shows the uncertainties of the two sensitivities S1 and S2, the uncertainty of the force value as well as the uncertainty of the combined sensitivity S. The main contribution to the uncertainty arises from the averaged accelerations. The mass value of the test mass and the adapter was determined with a relative uncertainty of $5\cdot 10^{-3}$ %, whereby the internal mass was determined with 1 %. As shown in [1] the relative uncertainty connected with the calibration of the frequency response of the bridge amplifier used can be given with ≈0.03 %. The vibrometer calibration contributes 0.2 % to the measured acceleration values. The propagation of uncertainty was derived according to the standard GUM using the model in accordance with equations 2-4. As shown in Fig. 5 the main contribution to the whole sensitivity originate from the uncertainty S_2, connected with the mass block. This is consistent with the fact that the mass block is more influenced by rocking

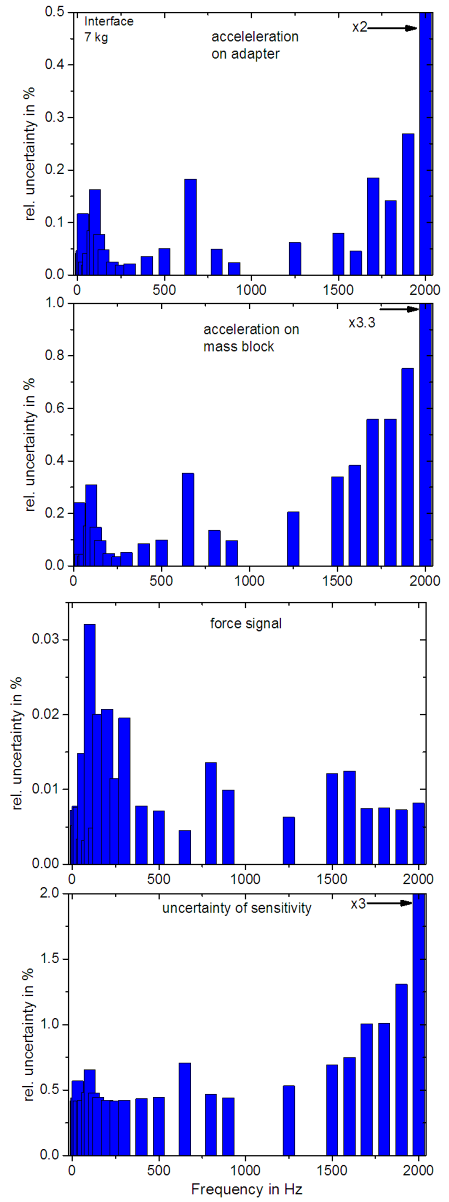

Fig. 5. Uncertainty distributions of the main measurement components which are the contributions S1 and S2, the force signal, as well as the final uncertainty of the combined sensitivity.

motions compared with the adapter, which is more rigid and more compact in the middle of the excitation axis. The uncertainty contribution of the force signal is small compared with the acceleration uncertainty.

4. APPROXIMATION OF THE SENSITIVITY

Calibration data are more comfortable to handle if they can be described by an analytical function. Often the calibration for a sensor has to be deposited in the software of a machine or incorporated in measuring programs. For that reason, a polynomial fit of the calibration data may be provided in accordance with equation (6). In Fig. 6 the following polynomial was fitted to the data:

$$y = a_0 \cdot \left(1 - a_1 x^2 + a_2 x^4 - a_3 x^6\right) \qquad (7)$$

Thereby x is the frequency and y the sensitivity. In Fig. 6 the fit (red) is shown together with his confidence (95%)

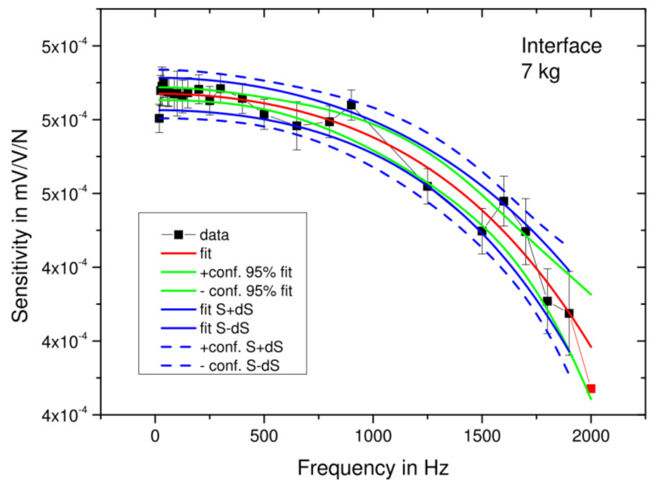

Fig. 6. Sensitivity data together with a polynomial fitting curve according equation (7) in red. In addition the confidence band of the fit function for 95% confidence (green) is shown. The solid blue line is a polynomial fit through the error bars of the data points and the dashed blue line the lower and upper confidence level (95%) of this fit.

band (green). The same fit can be applied for the error bars of the data (solid blue) which also gives a confidence (95%) band (dashed blue).

5. UNCERTAINTY BASED ON MC-SIMULATION

According the GUM Supplement1 the output uncertainty of a model can also be obtained by the propagation of the Probability Density Functions (PDF) of the input quantities. This approach has the advantage, that the uncertainty of the output quantity can derived in a more natural way from the standard deviation of the output PDF. The k-expansion factor is not needed because a coverage interval can be obtained, with the ad of the output PDF. The amount of output quantities which fall in a certain probability interval can be counted. In the case of a non-symmetric output PDF the coverage interval is normally also non symmetric and is more realistic then the error region obtained with the standard GUM approach.

For the model shown in eq.1 different kind of input PDFs were used. In the cases, where repeated measurements were performed a t-Distribution was applied. The reason is, that in the case of a finite number of measurements the estimate of the standard deviation can be more realistic described with a t-distribution. As shown above, the acceleration measured on the mass block were obtained by averaging 28 scanning points, the acceleration on the adapter by 9 points and the force signal by the sum of both, 37 measurements. In addition the uncertainty connected with the calibration of the vibrometer and the bridge amplifiers were taken into account with a rectangular distribution. The other input values, the masses of the mass block, the adapter and the internal mass of the force transducer were also modelled with a rectangular distribution.

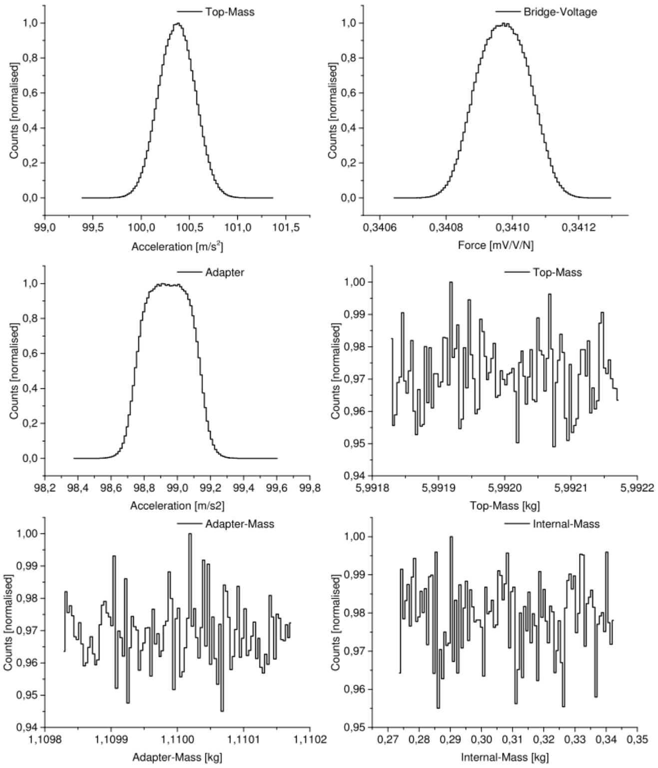

Fig. 7. Input PDFs for the Monte Carlo evaluation of the uncertainty of the sensitivity. The PDFs were created with $1.0 \cdot 10^6$ events. The histograms were accumulated with a binsize of 100.

According Fig. 7 the symmetric distributions of the accelerations are different. In the case of the PDF obtained for the acceleration of the adapter the influence of the rectangular distribution can be seen. The reason is that the standard deviation derived from this measured points are less than in the case of the acceleration measured on the top mass. The physical origin of this behaviour is the different tendency to undergo rocking motion. The adapter is compact concentrated in the middle of the excitation axis, whereby the top mass can be more easily excited by transverse motions. The distribution of the force signal is already converging a Gaussian distribution due to the highest number of measurements (37).

The MC simulation was performed in such a way, that first all input PDFs for a certain frequency were calculated with $1.0 \cdot 10^6$ events. After there, the output PDF was calculated according eq. 1. The calibration factors for the amplifier and the accelerations were taken with unity and the uncertainties connected with the calibration (bridge amplifier and vibrometer) were taken into account by adding a rectangular distribution to the input t-distributions of the voltage and the accelerations. From each output PDF the mean value and standard deviation were calculated (see Table 1). In addition a discrete representation G of the output distribution function $G_Y(\eta)$ (whereby η is the variable describing the possible values of the output quantity) for the output quantity Y was constructed according GUM Supplement1.

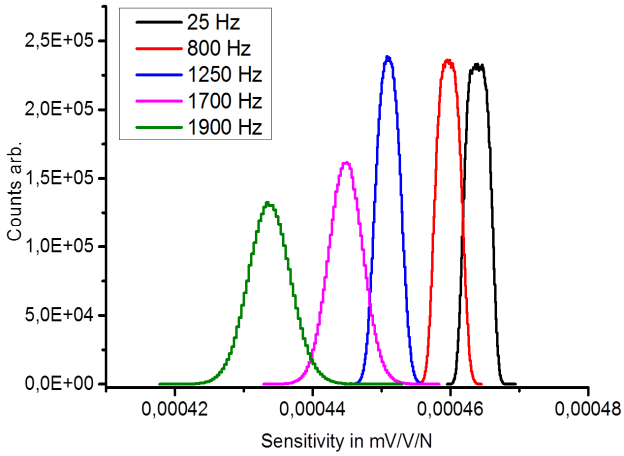

Fig. 8. Output PDFs of the sensitivity obtained at different frequencies.

This distribution can be used to determine a coverage interval in such a way, that with a certain probability p, all output values can be found in this interval. By using this interval also the coverage factor can be obtained; if the coverage interval is divided through the standard uncertainty obtained from the output PDF. In Figure 8 examples of output PDFs for different frequencies are plotted. According this plot, one can see a shift of the distributions as a function of the frequency which is due to the loss of sensitivity with increasing frequencies. In addition an increasing broadening of the distributions can be seen at higher frequencies. The main reasons are mechanical influences like rocking modes, which lead to a higher standard deviations of the acceleration input PDFs.

In Table 1 the uncertainty analysis according the standard GUM procedure is compared with the Monte Carlo method of GUM Supplement1. Thereby in the columns 2-3 the measured sensitivities with their dedicated relative uncertainties determined according the standard GUM are given.
The partial differentiation of the model (eq.1) was done with the free available software Maxima [4] to get the sensitivity coefficients. The software provides a Computer Algebra System which can be flexible linked to any programming language. The results of the software were validated punctual with the commercial software GUM-Workbench [5]. In column 4 and 5 the mean value and the relative standard uncertainty based on a coverage interval of 95,45% are given. In addition the coverage interval was used to calculate the k-factor, which is given in the last column. In column 6 the relative deviation of the measurand (col. 2) and the mean value based on the MC simulation (col. 4) are given. This difference is very small which is a proof that the number of trials (one million) used in the MC procedure is quite sufficient. In column 7 the relative deviation of the relative uncertainties is shown; also based on the standard GUM value of column 3.

1 Frequency	2 S-Measure	3 dS/S k=2	4 S-MC	5 dS/S	6 ΔS (Meas.-MC)	7 Δ(dS/S) (Meas.-MC)	8 k MC
Hz	mV/V/N	%	mV/V/N	%	%	%	
18,75	4,602E-04	0,43	4,600E-04	0,52	0,04	-19,04	1,80
25	4,639E-04	0,46	4,640E-04	0,55	-0,01	-18,70	1,83
31,25	4,645E-04	0,59	4,650E-04	0,68	-0,11	-16,33	1,93
37,5	4,649E-04	0,44	4,650E-04	0,52	-0,02	-19,03	1,80
56,25	4,638E-04	0,44	4,640E-04	0,52	-0,04	-19,05	1,80
62,5	4,637E-04	0,44	4,640E-04	0,52	-0,06	-19,03	1,80
87,5	4,635E-04	0,50	4,630E-04	0,59	0,10	-18,01	1,87
100	4,635E-04	0,67	4,640E-04	0,77	-0,10	-14,40	1,96
125	4,631E-04	0,50	4,630E-04	0,59	0,03	-18,17	1,87
150	4,636E-04	0,46	4,640E-04	0,55	-0,08	-18,66	1,83
200	4,641E-04	0,44	4,640E-04	0,52	0,03	-18,87	1,80
250	4,626E-04	0,44	4,630E-04	0,52	-0,09	-19,00	1,80
300	4,642E-04	0,44	4,640E-04	0,53	0,04	-18,82	1,81
400	4,629E-04	0,45	4,630E-04	0,54	-0,03	-18,67	1,83
500	4,607E-04	0,46	4,610E-04	0,55	-0,06	-18,45	1,83
650	4,592E-04	0,73	4,590E-04	0,82	0,04	-13,20	1,97
800	4,597E-04	0,49	4,600E-04	0,57	-0,07	-17,65	1,86
900	4,620E-04	0,46	4,620E-04	0,54	0,00	-18,00	1,83
1250	4,510E-04	0,55	4,510E-04	0,63	-0,01	-15,71	1,91
1500	4,449E-04	0,71	4,450E-04	0,80	-0,02	-12,10	1,97
1600	4,489E-04	0,77	4,490E-04	0,85	-0,01	-11,24	1,98
1700	4,448E-04	1,03	4,450E-04	1,11	-0,04	-8,60	2,00
1800	4,354E-04	1,03	4,350E-04	1,11	0,09	-8,29	2,00
1900	4,337E-04	1,33	4,340E-04	1,43	-0,06	-7,17	2,01

Table 1 Comparison of the sensitivity and their uncertainty obtained with the standard GUM procedure and the MC-method.

Here one can see that, the uncertainty according the MC method is roughly 10-20 % higher than the values obtained with the conventional standard GUM method. The reason is that in the standard GUM approach the standard uncertainty of the mean value is calculated by the division of the standard deviation with the square root of the number of measurements. If only a limited number of measurements can be performed, which is normally the case, rather the scaled and shifted t-distribution must be applied, where the standard uncertainty u of the distribution is calculated from the standard deviation s by:

$$u = \sqrt{\frac{n-1}{n-3}} \cdot \frac{s}{\sqrt{n}} \qquad (8)$$

Only in the case of infinite number of measurements this uncertainty converges to the Gaussian case.

5. CONCLUSIONS

The frequency dependent sensitivity is the main output of a periodical force calibration. Mostly adapters have to be used for coupling test masses to the transducer. The proper sensitivity can only obtained if the behaviour of such adapters is taken into account during the calibration. The main uncertainty contribution comes from the acceleration of the test mass. Comparisons between CEM and PTB show a good agreement for the sensitivity for frequencies up to 1 kHz. The results of the uncertainty evaluation with the standard GUM approach and the MC method deviate from each other by 10 %- 20 % which is due to the assumption of different input distributions. All together one can reach uncertainties of ≤1% for frequencies below 1.5 kHz.

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