

## GEOMETRIC MEASUREMENT OF THE VOLUME OF OIML CLASSES E AND F WEIGHTS WITHOUT SURFACE CONTACT

*Omar J. Purata*<sup>1</sup>, *José de la O.*<sup>2</sup>, *Elvia Funes*<sup>2</sup>

<sup>1</sup> Universidad de Guanajuato, Guanajuato, México, opurata@ugto.mx

<sup>2</sup> CIATEC, A.C., León, México, jdelao@ciatec.mx, efunes@ciatec.mx

**Abstract** – A noncontact test method for the volume determination of weights from (1 to 1 000) g is described. The method uses an optical comparator to size cylindrical weights, and allows to compute the volume of the weight according to OIML R 111-1 Method E. The volume values obtained for the weights were compared against hydrostatic weighing volume results for reference. The volume relative uncertainty levels reached with the optical comparator are small enough for calibration operations of even OIML Class E<sub>2</sub> weights.

**Keywords:** volume, weights, optical comparator, geometric measurement

### 1. INTRODUCTION

In mass measurement of weights in air the density or volume of the weight must be known, in order to compute the buoyancy correction of air. OIML R111-1 [1] describes six accepted methods for the determination of the density of weights, including four methods that involve the immersion of the weight in some liquid (Methods A to D), one geometric method (E) and one no-experimental method (F).

Even when hydrostatic weighing (Method A) is the most accurate method [1, 2, 3] it is complex and it implies to immerse the weight in a liquid bath, which make it unsuitable for weights with cavities. Additionally this method is time consuming if the volumes of many weights are to be measured [4, 5].

There are also non immersion methods other than the Methods E and F from [1], like the based on weighing in air at different densities [5, 6]. This method however, needs the setup of comparator balances under airtight enclosures in order to modify the pressure and hence the air density. Even when this option yields uncertainties as good as hydrostatic weighing, is more an option for National Metrology Institutes (NMI's) mass standards and/or OIML class E<sub>1</sub> weights.

Another method extensively studied [1, 4, 7, 8] uses an acoustic volumeter. It has good performance for volume measurements of standard weights, even for OIML R 111-1 class E<sub>1</sub> weights. However, to apply the acoustic volumeter measurement at least one reference weight of known volume and similar shape to that of the test weight is needed [4].

When the immersion of the weight in a liquid bath is not acceptable OIML R111-1 recommends the use of Method E. However the recommendation points in paragraph B.7.8.1.1: "...there is a risk of scratching the surface during the

measurement, and therefore, test method E should not be used on class E and F weights." [1]. The reason for this restriction is grounding on the fact that the apparatus suggested to do the geometric measurements of Method E are Vernier caliper (or digital caliper), micrometer and radius gauge (see page 56 of [1]), all of them are surface contact instruments.

In this work, an alternative for the application of the OIML R111-1 Method E is proposed. An optical comparator with noncontact measurement was used to do the geometric measurement of weights in order to determine its volume. The volume of weights from 20 g to 1 kg mass were determined and the results obtained allow to satisfy the uncertainty requirements for the OIML class F<sub>1</sub> and even class E<sub>2</sub> weights.

### 2. GEOMETRIC MEASUREMENT WITH OPTICAL COMPARATOR

The measurement system used in this work consists of an optical comparator with a 20X lens and measuring range (0 to 250) mm in X-axis and (0 to 150) mm in Y-axis. The optical comparator display has a resolution of 0.001 mm. The geometric measurements were made following a procedure of a laboratory accredited under ISO/IEC 17025:2005 [9].

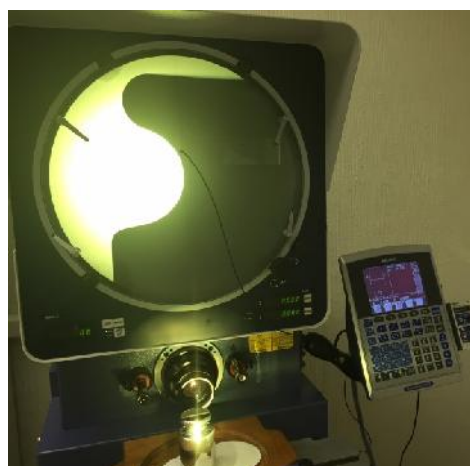


Fig. 1. An OIML class E<sub>2</sub> weight being geometrically characterized with the optical comparator.

## 2.1. Optical comparator and geometric dimensions

Fig. 1 shows a picture of the optical comparator arrangement with a weight in position to be measured without surface contact. No Vernier calipers, digital calipers or micrometers are needed.

The measurement procedure basically involves handling carefully the weight to take three noncontact measurements of each height, diameter and radius needed, according to Fig. 2. For the radii at least three different spots must be chosen in order to get a regression on the comparator computer.

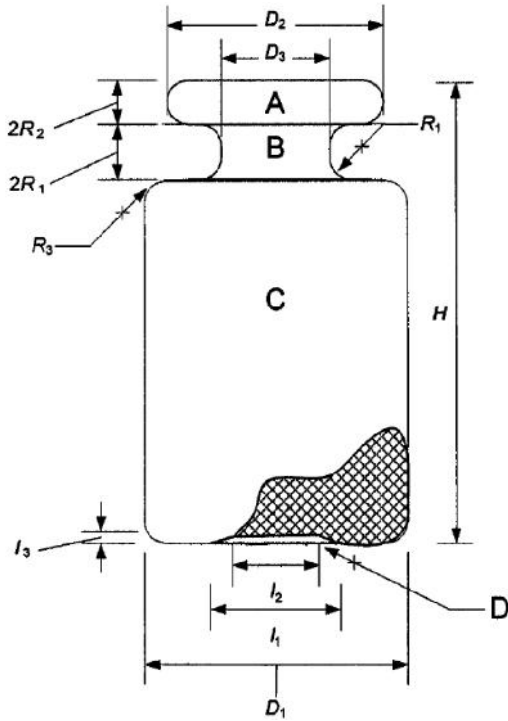


Fig. 2. The four different sections for volume determination of a cylindrical weight (Figure B.8 in [1]).

It is important to note the similarity between the Fig. 2 and the actual shape of a weight that can be seen on Fig. 1. As will be stated later the method E for volume determination by geometric measurement is not suitable for all range long of OIML R111-1 weights, even when an optical comparator is used.

## 2.2. Mathematical model

The volume of each of the four sections in Fig. 2 (A, B, C and D) and hence the volume of the whole weigh can be computed from its measured dimensions according to the following standard formulae [1]:

$$V_A = 2 R_2 \left( \frac{D_2^2}{4} - R_2 D_2 + R_2^2 + \frac{R_2 D_2}{4} - \frac{R_2^2}{2} + \frac{2R_2^2}{3} \right) \quad (1)$$

$$V_B = R_1 \left( \frac{D_3^2}{2} + 2R_1 D_3 - \frac{R_1 D_3}{2} - \pi R_1^2 + \frac{10R_1^2}{3} \right) \quad (2)$$

$$V_C = \frac{D_1^2}{4} \left[ H - 2(R_1 + R_2) \right] - \pi R_3^2 \left( 2D_1 - \frac{10R_3}{3} - \frac{D_1}{2} + R_3 \right) \quad (3)$$

$$V_D = \frac{l}{l_2} l_3 (l_1^2 + l_1 l_2 + l_2^2) \quad (4)$$

$$V_{weight} = V_A + V_B + V_C - V_D \quad (5)$$

## 2.3. Uncertainty of the volume measurement

The uncertainty budget for the measurement of the four volume sections described in (1) to (4) involves the following contributors:

- The calibration certificate of the optical comparator (reference uncertainty).
- Optical comparator resolution.
- Temperature difference between the optical comparator and the weight.
- Thermal expansion coefficient difference between the optical comparator and the weight.
- Temperature difference between the weight and the environment.
- Thermal expansion coefficient of the weight.
- Repeatability of the geometric measurements.

Then for the use of (5) the partial derivatives of the four weight sections are taken into account in the final weight volume uncertainty calculation.

## 2.4. Maximum relative uncertainty for the volume of weights

It can be demonstrated that there is a maximum value of the volume relative uncertainty that allows to keep in control the uncertainty contribution of the buoyancy correction of air, due to the volume of the weight. Of course, value of the volume relative uncertainty is practically the same that the value of the density relative uncertainty. An expression available for the calculation of that maximum value of the volume relative uncertainty is [10]:

$$u(V)_{rel}^{max} = 341.230 \frac{\delta m}{m} \quad (6)$$

where  $u(V)_{rel}^{max}$  is the volume relative uncertainty;  $um$  is the maximum permissible error for the weight, in milligrams as stated in Table 1 of [1]; and  $m$  is the nominal mass value of the weight, with the same units as  $um$ . A value of 8 000 kg/m<sup>3</sup> as an average value for stainless steel density was assumed in equation (6). This formula will allow to assess the volume uncertainty (in its relative form) reached when the optical comparator was used for volume determination of weights.

## 3. RESULTS

The measured volume results for two sets of weights class E<sub>2</sub> of different trademarks are shown below. The volumes obtained for weights from (1 to 1 000) g, along with their combined standard uncertainties are shown in Tables 1 and 2. The Set 1 of weights has been calibrated more than ten times by CENAM (the Mexican NMI) or another accredited laboratory that uses the hydrostatic weighing method. Whereas the Set 2 is more recent but it also has been calibrated by an accredited laboratory using the hydrostatic weighing method.

It is important to note the geometric differences between the two set of weights under study. Set 1 (older) has a different adjust to shape of Fig. 2 than Set 2 (the newer), as can be seen in Fig. 3.

Table 1. Uncertainties of the volume measurement of Set 1 of weights using the optical comparator for Method E [1].

Nominal mass / g	V <sub>weight</sub> / cm <sup>3</sup>	Combined standard uncertainty / cm <sup>3</sup>
1 000	125.118	0.078
500	62.305	0.014
200	24.915	0.025
100	12.412 8	0.009 2
50	6.301 4	0.005 2
20	2.543 9	0.004 1
10	1.283 9	0.001 7
5	0.648 2	0.002 0
2	0.266 02	0.000 85
1	0.141 9	0.001 5

Table 2. Uncertainties of the volume measurement of Set 2 of weights using the optical comparator for Method E [1].

Nominal mass / g	V <sub>weight</sub> / cm <sup>3</sup>	Combined standard uncertainty / cm <sup>3</sup>
1 000	123.47	0.12
500	62.354	0.036
200	25.060	0.021
100	12.470 6	0.008 4
50	6.320 0	0.006 8
20	2.546 1	0.002 8
10	1.278 4	0.002 8
5	0.647 8	0.001 1
2	0.265 89	0.000 51
1	0.142 35	0.000 16



Fig. 3. Projection of two weights with different trademark and same nominal value (10 g) and OIML R 111-1 classification (E<sub>2</sub>).

The calculated volume relative uncertainties for the two sets of weights studied are compared against the values calculated with (6) for OIML R 111-1 weights classes E<sub>2</sub> and F<sub>1</sub> in Tables 3 and 4. Volume relative uncertainties obtained with hydrostatic weighing are also included as a reference.

Table 3. Volume relative uncertainties of Set 1.

Nominal mass / g	Volume relative uncertainty for class E <sub>2</sub> / %	Volume relative uncertainty for class F <sub>1</sub> / %	Volume relative uncertainty with hydrostatic weighing / %	Volume relative uncertainty with optical comparator / %
1 000	0.126	0.393	0.079	0.063
500	0.126	0.393	0.080	0.023
200	0.118	0.393	0.080	0.100
100	0.126	0.393	0.079	0.074
50	0.157	0.471	0.080	0.083
20	0.314	0.982	0.080	0.162
10	0.471	1.571	0.080	0.131
5	0.786	2.514	0.080	0.314
2	1.571	4.714	0.079	0.320
1	2.357	7.857	0.079	1.068

Table 4. Volume relative uncertainties of Set 2.

Nominal mass / g	Volume relative uncertainty for class E <sub>2</sub> / %	Volume relative uncertainty for class F <sub>1</sub> / %	Volume relative uncertainty with hydrostatic weighing / %	Volume relative uncertainty with optical comparator / %
1 000	0.126	0.393	0.038	0.094
500	0.126	0.393	0.020	0.058
200	0.118	0.393	0.038	0.083
100	0.126	0.393	0.075	0.067
50	0.157	0.471	0.144	0.108
20	0.314	0.982	0.094	0.109
10	0.471	1.571	0.075	0.221
5	0.786	2.514	0.368	0.171
2	1.571	4.714	0.913	0.191
1	2.357	7.857	1.852	0.115

## 4. DISCUSSION

Considering the exposed in Section 3, the following findings could be drawn:

- a) Volume standard uncertainty obtained with the geometric measurement in the optical comparator for weights of nominal mass in the range of (1 to 1 000) g, was between ( $16 \times 10^{-5}$  and  $12 \times 10^{-2}$ )  $\text{cm}^3$ , while volume relative uncertainty ranges between  $23 \times 10^{-5}$  and  $10.68 \times 10^{-3}$ .
- b) From the results of Tables 3 and 4 it can be stated that the volume relative uncertainty obtained with the optical comparator satisfy the requirements of [1] for weights classes  $E_2$  and  $F_1$ . This finding is very important because [1] establishes that no geometric method must be used for weights 1 kg and below for classes  $E_2$  and  $F_1$ , see paragraph B.7.8.1.1 and Table B.8 from [1]. Even with that, it is important to note that the principal concern in [1] is related with the possible damage that  $E_2$  and  $F_1$  weights could suffer when using Vernier calipers (or digital calipers) and micrometers.
- c) The volume relative uncertainties obtained with hydrostatic weighing are similar to those obtained with the geometric measurement for weights above (20 to 50) g. This could be explained by the fact that those are the boundary values at which the average density in Table 5 from [1] keeps almost constant.

An important thing to note is that according to paragraph B.7.8.4 of [1], the bigger contributor to the uncertainty in Method E is the deviation of the actual shape of the weight from the mathematical model –equations (1) to (5). However, the choose of the rank of weights in Set 1 and Set 2, was assuring that the deviation of the weights regard Figure 2 was negligible.

## 5. CONCLUSIONS

An optical comparator has been applied to the measurements of the volume of standard weights. The method involves noncontact geometric measurements that eliminate the risk of scratching the surface of the weights, and allows the determination of volume when the immersion of the weight in a liquid is not acceptable.

From the uncertainty evaluation of the measurement, all the relative volume uncertainties of the weights in the nominal mass range between (1 to 1 000) g satisfy the requirements for OIML classes  $F_1$  and  $E_2$ .

The method proposed here is applicable to the volume measurements of weights whose shape is similar to that of the Figure 2.

Future works of this research include the quantitative analysis of the lack of adjust of the actual shape of the weight versus the mathematical model in Figure 2 and equations (1) to (5). Another research branch could be the modification of mathematical model to include minor variations of shape in sections of Figure 2.

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