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# ANOTHER APPROACH ANALYZING THE CROSSTALK MATRIX FOR A PRECISE FORCE-TORQUE TRANSDUCER

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***Abstract*** – A new multi-component facility for force and torque, which can simultaneously realize precise axial force ($F_z$) and torsional moment (torque, $M_z$), is being developed in PTB. Moreover, precise two-axis measuring devices (force-torque transducers) as transfer standards must be also developed for the establishment of the measurement traceability of screw testing machines. Authors have calibrated a new-developed force-torque transducer in each direction of force and torque, separately. The crosstalk matrixes have been obtained by ordinal way, where the normal square-inverse matrix (2 by 2) was used. In this paper, authors tried to obtain the crosstalk matrix by means of another approach, pseudo-inverse matrix, where the data was used as many as possible (2 by 18 or 2 by 26), and the useful crosstalk matrix could be determined in one value.

***Keywords***: multi-component measurement, force, torque, screw testing machine, pseudo-inverse matrix

## 1. INTRODUCTION

In order to establish the measurement traceability of screw testing machines, a new multi-component facility for force and torque, which can simultaneously realize precise axial force ($F_z$) and twisting moment (torque, $M_z$), is being developed in PTB [1-2]. The facility utilizes an existing 1 MN Force Standard Machine (FSM), and has two deadweight loading stacks at both ends of a lever-arm with changing the loading directions from vertical to horizontal (Fig. 1).

Moreover, precise two-axis measuring devices (force-torque transducers) as transfer standards must be also developed for the calibration of screw testing machines. First, one of authors calibrated a new-developed force-torque transducer in each direction of force and torque, separately [3]. Second, crosstalk matrixes were obtained by ordinal way, where the normal square-inverse matrixes (2 by 2) were used. Finally, in this paper, authors tried to obtain a specific crosstalk matrix by means of another approach, pseudo-inverse matrix. The data was used as many as possible. It would not be the square matrix despite it has an inverse matrix (2 by 18 or 2 by 26). Then, a useful matrix could be obtained in spite of the case that the capacities of force and torque in a transducer were quite different.

## 2. CALIBRATION RESULTS [3]

### *2.1. Transducer*

A new developed compression-torsion transducer (c-t transducer, manufactured by GTM GmbH) has been used in the calibration experiment. Such a precise two-axis transducer is expected to be used as a reference standard for the calibration of a screw testing machine. A picture of the c-t transducer is shown in Fig. 2. DMP40 (manufactured by HBM GmbH) was also used as an indicator/amplifier. The aimed relative expanded uncertainty of calibration is less than $1\cdot10^{-4}$. The rated capacities of compressive force $F_z$ and torsional moment (torque) $M_z$ are 500 kN and 500 N·m, respectively.

### *2.2. Force component calibration*

Compressive force component calibration of the c-t transducer has been conducted by using the 1 MN Force Standard Machine (FSM) in PTB. A picture of the appearance of force calibration by using the 1-MN-FSM is shown in Fig. 3. The calibration has been carried out according to ISO 376, 10 calibration steps by 50 kN pitch, up to the maximum force of 500 kN.

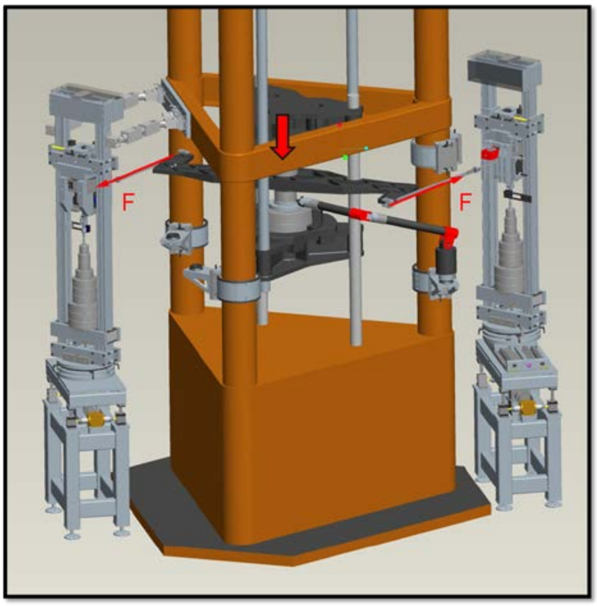


Fig. 1. A multi-component facility for force and torque[1-2].

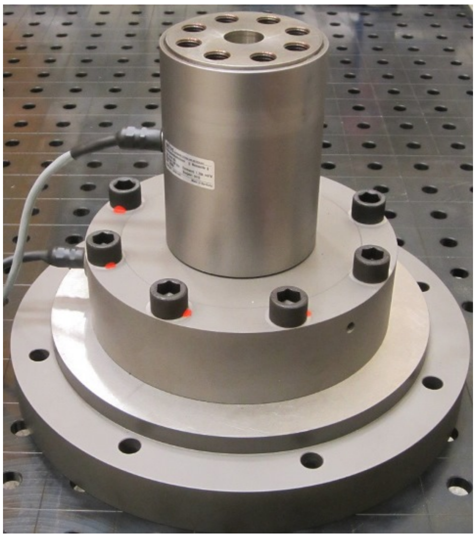

Fig. 2. Picture of the compression-torsion transducer.

Calibration results are shown in Fig. 4. Precise calibration result could be obtained for the compressive force measurement, whereas a certain scattering were observed for the torque measurement. It might be caused by the influence of weight oscillation in the torsional direction.

Fig. 3. Picture of the force calibration by using the 1-MN-FSM.

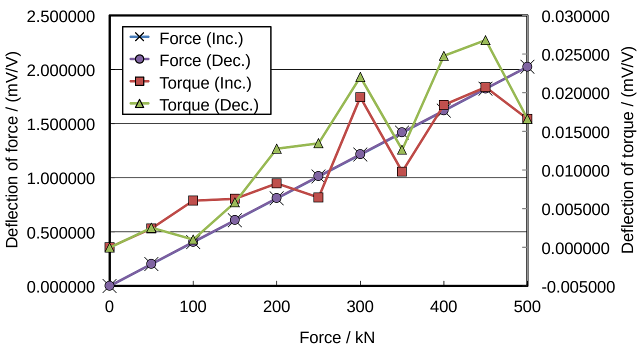


Fig. 4. Calibration results for the compressive force direction.

### *2.3. Torque component calibration*

Torque component calibration of the c-t transducer has been conducted by using the 1 kN·m torque Standard

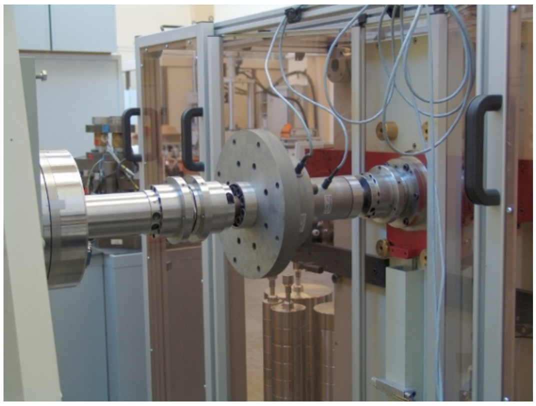

Fig. 5. Picture of the torque calibration by using the 1-kN·m-TSM.

Machine (TSM) in PTB. A picture of the appearance of torque calibration by using the 1-kN·m-TSM is shown in Fig. 5. The calibration has been carried out according to DIN 51309, 8 calibration steps by 10 %, 20 %, 30 %, 40 %, 50 %, 60 %, 80 %, 100 % of the maximum torque, 500 N·m. The c-t transducer was calibrated for the direction of clockwise (CW) and anticlockwise (ACW) torques, respectively.

Calibration results are shown in Figs. 6(a) and 6(b). Precise calibration results could be obtained for both CW and ACW torque measurements, whereas a certain eccentric curves were observed for the force measurement. It has been thought that the curve of parasitic component was just the inherent characteristic in this c-t transducer.

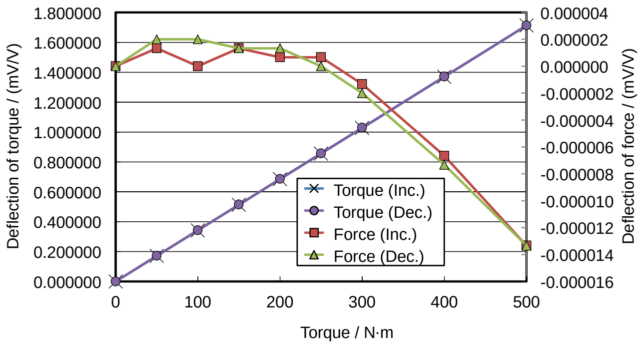


Fig. 6(a). Calibration results for the CW torque direction.

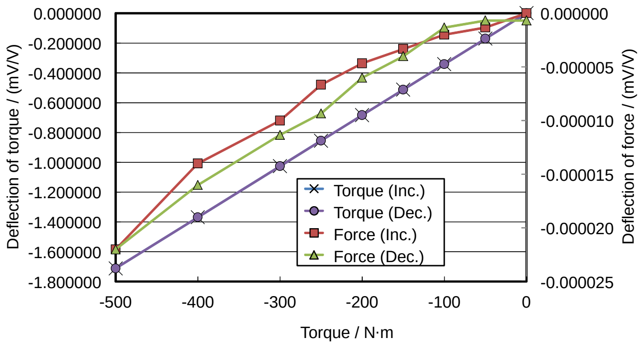


Fig. 6(b). Calibration results for the ACW torque direction.

### *2.4. Crosstalk matrix*

Based on the calibration results, polynominal interpolation equations of 3rd grade for the output deflections have been calculated for the compressive force $F_z$ and the clockwise torque $+M_z$. After that, the 2 by 2 crosstalk matrixes were evaluated for increasing and decreasing force and torque, respectively. The results were shown in Table 1 [3].

The crosstalk matrixes are able to be usefully used for the afterward calibration of screw testing machines and so on at the loading points other than the calibration points of the c-t transducer when the interpolation equations are used. However, the crosstalk matrix becomes the function of loaded force and torque in this case, that is, the crosstalk matrix for the c-t transducer is not only one, but variable.

## 3. PSEUDO-INVERSE MATRIX

In order to determine one value of the crosstalk matrix and to use data points as many as possible, the authors tried to introduce the pseudo-inverse matrix method [4-5].

The inverse matrix of the non-square matrix could not be generally obtained. However, when there is a matrix **V** of line $n$ by row $p$ ($n \neq p$), an inverse matrix of **V** as defined in the following equation is called "pseudo-inverse matrix."

$$\mathbf{V}^{+} = \mathbf{V}^{t}(\mathbf{V}\cdot\mathbf{V}^{t})^{-1} \tag{1}$$

The matrix **A** is called crosstalk matrix or calibration matrix when the following relationship between force $F_z$ and torque $M_z$ occurred on the c-t transducer and corresponding output deflections is formed:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} V_{\mathrm{FZ}} \\ V_{\mathrm{MZ}} \end{bmatrix} = \begin{bmatrix} F_{\mathrm{Z}} \\ M_{\mathrm{Z}} \end{bmatrix}, \tag{2a}$$

$$\mathbf{A} \times \mathbf{V} = \mathbf{F}\,. \tag{2b}$$

When the force vector is $\mathbf{F}\{F_{z\_s}, 0\}$ (unit of $F_{z\_s}$ is kN) and the output vector is $\mathbf{V}\{V_{FZ1}, V_{MZ1}\}$ (units are mV/V) during force calibration by using the FSM, the following equation is formed:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} V_{\mathrm{FZ1}} \\ V_{\mathrm{MZ1}} \end{bmatrix} = \begin{bmatrix} F_{\mathrm{Z\_s}} \\ 0 \end{bmatrix}, \tag{3}$$

When the force vector is $\mathbf{F}\{0, M_{z\_s}\}$ (unit of $M_{z\_s}$ is N·m) and the output vector is $\mathbf{V}\{V_{FZ2}, V_{MZ2}\}$ (units are mV/V) during torque calibration by using the TSM, the following equation is formed:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} V_{\mathrm{FZ2}} \\ V_{\mathrm{MZ2}} \end{bmatrix} = \begin{bmatrix} 0 \\ M_{\mathrm{Z\_s}} \end{bmatrix}, \tag{4}$$

From eqs. (3) and (4), a simultaneous linear equation with four unknowns is formed. It can be expressed by the following equations:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} V_{\mathrm{FZ1}} & V_{\mathrm{FZ2}} \\ V_{\mathrm{MZ1}} & V_{\mathrm{MZ2}} \end{bmatrix} = \begin{bmatrix} F_{\mathrm{Z\_s}} & 0 \\ 0 & M_{\mathrm{Z\_s}} \end{bmatrix}, \tag{5a}$$

$$\mathbf{A} \times \mathbf{V} = \mathbf{F}_{\mathrm{s}}\,. \tag{5b}$$

Because the matrix **V** is the square matrix, the inverse matrix $\mathbf{V}^{-1}$ can be solved. Then:

$$\mathbf{A} = \mathbf{F}_{\mathrm{s}} \times \mathbf{V}^{-1}\,, \tag{6a}$$

$$\mathbf{A} = \frac{1}{V_{\mathrm{FZ1}}V_{\mathrm{MZ2}} - V_{\mathrm{FZ2}}V_{\mathrm{MZ1}}} \begin{bmatrix} F_{\mathrm{Z\_s}}V_{\mathrm{MZ2}} & -F_{\mathrm{Z\_s}}V_{\mathrm{FZ2}} \\ -M_{\mathrm{Z\_s}}V_{\mathrm{MZ1}} & M_{\mathrm{Z\_s}}V_{\mathrm{FZ1}} \end{bmatrix}. \tag{6b}$$

Each component of the matrix **V** is obtained by using interpolation equations, although **A** becomes the function of output deflections, that is, the load of force and torque.

Here, the non-square matrix **V** can be introduced as follows:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \left[ \begin{matrix} V_{\mathrm{FZ1\_50kN}} & V_{\mathrm{FZ1\_100kN}} & V_{\mathrm{FZ1\_150kN}} & V_{\mathrm{FZ1\_200kN}} \\ V_{\mathrm{MZ1\_50kN}} & V_{\mathrm{MZ1\_100kN}} & V_{\mathrm{FZ1\_150kN}} & V_{\mathrm{FZ1\_200kN}} \end{matrix} \right.$$

$$\begin{matrix} V_{\mathrm{FZ1\_250kN}} & V_{\mathrm{FZ1\_300kN}} & V_{\mathrm{FZ1\_350kN}} & V_{\mathrm{FZ1\_400kN}} & V_{\mathrm{FZ1\_450kN}} \\ V_{\mathrm{MZ1\_250kN}} & V_{\mathrm{MZ1\_300kN}} & V_{\mathrm{MZ1\_350kN}} & V_{\mathrm{MZ1\_400kN}} & V_{\mathrm{MZ1\_450kN}} \end{matrix}$$

$$\begin{matrix} V_{\mathrm{FZ1\_500kN}} & V_{\mathrm{FZ2\_+50Nm}} & V_{\mathrm{FZ2\_+100Nm}} & V_{\mathrm{FZ2\_+150Nm}} & V_{\mathrm{FZ2\_+200Nm}} \\ V_{\mathrm{MZ1\_500kN}} & V_{\mathrm{MZ2\_+50Nm}} & V_{\mathrm{MZ2\_+100Nm}} & V_{\mathrm{FZ2\_+150Nm}} & V_{\mathrm{FZ2\_+200Nm}} \end{matrix}$$

$$\left. \begin{matrix} V_{\mathrm{FZ2\_+250Nm}} & V_{\mathrm{FZ2\_+300Nm}} & V_{\mathrm{FZ2\_+400Nm}} & V_{\mathrm{FZ2\_+500Nm}} \\ V_{\mathrm{MZ2\_+250Nm}} & V_{\mathrm{FZ2\_+300Nm}} & V_{\mathrm{FZ2\_+400Nm}} & V_{\mathrm{FZ2\_+500Nm}} \end{matrix} \right] =$$

$$\left[ \begin{matrix} F_{\mathrm{Z\_s\_50kN}} & F_{\mathrm{Z\_s\_100kN}} & F_{\mathrm{Z\_s\_150kN}} & F_{\mathrm{Z\_s\_200kN}} & F_{\mathrm{Z\_s\_250kN}} & F_{\mathrm{Z\_s\_300kN}} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \right.$$

$$\begin{matrix} F_{\mathrm{Z\_s\_350kN}} & F_{\mathrm{Z\_s\_400kN}} & F_{\mathrm{Z\_s\_450kN}} & F_{\mathrm{Z\_s\_500kN}} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{\mathrm{Z\_s\_+50Nm}} & M_{\mathrm{Z\_s\_+100Nm}} \end{matrix}$$

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ M_{\mathrm{Z\_s\_+150Nm}} & M_{\mathrm{Z\_s\_+200Nm}} & M_{\mathrm{Z\_s\_+250Nm}} & M_{\mathrm{Z\_s\_+300Nm}} & M_{\mathrm{Z\_s\_+400Nm}} \end{matrix}$$

$$\left. \begin{matrix} 0 \\ M_{\mathrm{Z\_s\_+500Nm}} \end{matrix} \right], \tag{7a}$$

where the all data for compressive force points (10 points) and clockwise torque points (8 points) was used. The form of the matrix **V** is 2 by 18 (called matrix $\mathbf{V}_{18}$). Moreover, if the data of anticlockwise torque points (8 points) is added, another matrix of the form of 2 by 26 can be obtained (called matrix $\mathbf{V}_{26}$) as follows:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \left[ \begin{matrix} V_{\mathrm{FZ1\_50kN}} & V_{\mathrm{FZ1\_100kN}} & V_{\mathrm{FZ1\_150kN}} & V_{\mathrm{FZ1\_200kN}} \\ V_{\mathrm{MZ1\_50kN}} & V_{\mathrm{MZ1\_100kN}} & V_{\mathrm{FZ1\_150kN}} & V_{\mathrm{FZ1\_200kN}} \end{matrix} \right.$$

$$\begin{matrix} V_{\mathrm{FZ1\_250kN}} & V_{\mathrm{FZ1\_300kN}} & V_{\mathrm{FZ1\_350kN}} & V_{\mathrm{FZ1\_400kN}} & V_{\mathrm{FZ1\_450kN}} \\ V_{\mathrm{MZ1\_250kN}} & V_{\mathrm{MZ1\_300kN}} & V_{\mathrm{MZ1\_350kN}} & V_{\mathrm{MZ1\_400kN}} & V_{\mathrm{MZ1\_450kN}} \end{matrix}$$

$$\begin{matrix} V_{\mathrm{FZ1\_500kN}} & V_{\mathrm{FZ2\_+50Nm}} & V_{\mathrm{FZ2\_+100Nm}} & V_{\mathrm{FZ2\_+150Nm}} & V_{\mathrm{FZ2\_+200Nm}} \\ V_{\mathrm{MZ1\_500kN}} & V_{\mathrm{MZ2\_+50Nm}} & V_{\mathrm{MZ2\_+100Nm}} & V_{\mathrm{FZ2\_+150Nm}} & V_{\mathrm{FZ2\_+200Nm}} \end{matrix}$$

$$\begin{matrix} V_{\mathrm{FZ2\_+250Nm}} & V_{\mathrm{FZ2\_+300Nm}} & V_{\mathrm{FZ2\_+400Nm}} & V_{\mathrm{FZ2\_+500Nm}} \\ V_{\mathrm{MZ2\_+250Nm}} & V_{\mathrm{FZ2\_+300Nm}} & V_{\mathrm{FZ2\_+400Nm}} & V_{\mathrm{FZ2\_+500Nm}} \end{matrix}$$

$$\begin{bmatrix} V_{\mathrm{FZ2\_-50Nm}} & V_{\mathrm{FZ2\_-100Nm}} & V_{\mathrm{FZ2\_-150Nm}} & V_{\mathrm{FZ2\_-200Nm}} & V_{\mathrm{FZ2\_-250Nm}} & V_{\mathrm{FZ2\_-300Nm}} & V_{\mathrm{FZ2\_-400Nm}} & V_{\mathrm{FZ2\_-500Nm}} \\ V_{\mathrm{MZ2\_-50Nm}} & V_{\mathrm{MZ2\_-100Nm}} & V_{\mathrm{FZ2\_-150Nm}} & V_{\mathrm{FZ2\_-200Nm}} & V_{\mathrm{MZ2\_-250Nm}} & V_{\mathrm{FZ2\_-300Nm}} & V_{\mathrm{FZ2\_-400Nm}} & V_{\mathrm{FZ2\_-500Nm}} \end{bmatrix} =$$

$$\begin{bmatrix} F_{\mathrm{Z\_s\_50kN}} & F_{\mathrm{Z\_s\_100kN}} & F_{\mathrm{Z\_s\_150kN}} & F_{\mathrm{Z\_s\_200kN}} & F_{\mathrm{Z\_s\_250kN}} & F_{\mathrm{Z\_s\_300kN}} & F_{\mathrm{Z\_s\_350kN}} & F_{\mathrm{Z\_s\_400kN}} & F_{\mathrm{Z\_s\_450kN}} & F_{\mathrm{Z\_s\_500kN}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{\mathrm{Z\_s\_+50Nm}} & M_{\mathrm{Z\_s\_+100Nm}} & M_{\mathrm{Z\_s\_+150Nm}} & M_{\mathrm{Z\_s\_+200Nm}} & M_{\mathrm{Z\_s\_+250Nm}} & M_{\mathrm{Z\_s\_+300Nm}} & M_{\mathrm{Z\_s\_+400Nm}} & M_{\mathrm{Z\_s\_+500Nm}} & M_{\mathrm{Z\_s\_-50Nm}} & M_{\mathrm{Z\_s\_-100Nm}} & M_{\mathrm{Z\_s\_-150Nm}} & M_{\mathrm{Z\_s\_-200Nm}} & M_{\mathrm{Z\_s\_-250Nm}} & M_{\mathrm{Z\_s\_-300Nm}} & M_{\mathrm{Z\_s\_-400Nm}} & M_{\mathrm{Z\_s\_-500Nm}} \end{bmatrix}. \quad (7b)$$

When $\mathbf{V}_{26}$ is used, both the clockwise and anticlockwise torques are able to be evaluated by one crosstalk matrix **A**.

Again, eqs. (7a) and (7b) can be expressed by the matrix equations:

$$\mathbf{A}_{18} \times \mathbf{V}_{18} = \mathbf{F}_{\mathrm{s\_18}} \text{ or } \mathbf{A}_{26} \times \mathbf{V}_{26} = \mathbf{F}_{\mathrm{s\_26}}. \quad (7c)$$

After solving the pseudo-inverse matrix $\mathbf{V}^{+}_{18}$ or $\mathbf{V}^{+}_{26}$, crosstalk matrix $\mathbf{A}_{18}$ or $\mathbf{A}_{26}$ can be calculated.

$$\mathbf{A}_{18} = \mathbf{F}_{\mathrm{s\_18}} \times \mathbf{V}^{+}_{18} \text{ or } \mathbf{A}_{26} = \mathbf{F}_{\mathrm{s\_26}} \times \mathbf{V}^{+}_{26}. \quad (8)$$

Table 1 shows the results. Results of $\mathbf{A}_{18}$ are similar to that of $\mathbf{A}_{26}$.

Finally, the actual axial compressive force and torsional torque can be estimated by using the c-t transducer with determined crosstalk matrix **A** when screw testing machines are calibrated with the c-t transducer as the reference standard by using eqs. (2).

Table 1. Calculation results of crosstalk matrixes.

| | $A_{11}$ / $A_{21}$ | $A_{12}$ / $A_{22}$ |
|---|---|---|
| $\mathbf{A}_{18}$ | 2.469E+02 | 9.477E-04 |
| | -3.644E+00 | 2.921E+02 |
| $\mathbf{A}_{26}$ | 2.469E+02 | -8.253E-04 |
| | -3.644E+00 | 2.922E+02 |

The values in Table 1 do not involve interpolation equations, so that an arbitral loading other than force and/or torque at the calibration points would be strictly difficult to be estimated. In addition, only increasing force and torque but not decreasing ones were evaluated in this paper because it might be considered that the actual screw testing machine hardly use decreasing values of force and torque in the test of screw elements and so on. They all are future subjects to be solved.

The authors think the crosstalk matrix obtained by using the pseudo-inverse matrix would be useful because all data points are able to be used for the calculation. However, the accuracy of the matrix will be able to be evaluated after the establishment of the multi-component facility for force and torque in PTB (Fig. 1).

## 4. CONCLUSIONS

In order to obtain the crosstalk matrix of precise two-axis compressive force and torsional moment (clockwise and anticlockwise torques) transducer, the method using pseudo-inverse matrix was proposed. This method can determine the one value of crosstalk matrix, comparing with the ordinal method using interpolation equation. However, the problem that arbitral loading values other than the calibration points of the c-t transducer are not strictly able to be estimated should be solved as a future subject.

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