XXI IMEKO World Congress “Measurement in Research and Industry”
August 30 – September 4, 2015, Prague, Czech Republic

A 2-DOF MODEL FOR BACK-TO-BACK SHOCK TRANSDUCERS

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Abstract – By applying a two-degree-of-freedom system model (2-DOF) for an Endevco type 2270 back-to-back reference accelerometer, systematic deviations between measured and fitted 1-DOF transfer functions could be improved. Some key information for the identification of the model parameters is found in the frequency range beyond the first resonance, a range which was rarely considered in the past.

Keywords: acceleration, shock transducer, parameter identification

1. INTRODUCTION

The calibration of shock accelerometers approaches the method of a model description, where instead of the specification of a peak signal ratio, the accelerometer is characterized by a model and its parameters [1, 2]. The usually proposed and most simple model consists of a single-degree-of-freedom system (1-DOF) shown in Fig. 1 with its transfer function $S_{qa1}(\omega)$ given in (1).

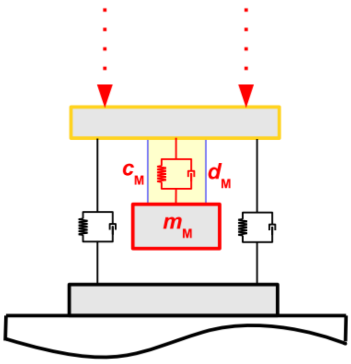

Fig. 1: 1-DOF model of a B2B transducer, the red dotted lines indicate the measuring beams of two laser vibrometers probing the reference surface .

$$S_{qa1}(\omega) = \frac{S_0 {\omega_0}^2}{{\omega_0}^2 + \mathrm{i}\omega\delta^2 - \omega^2} \qquad (1)$$

Here, S_0 is the sensitivity, ω_0 the resonance frequency and δ the damping.

Figure 2 shows measured shock transfer functions of an Endevco type 2270 back-to-back (B2B) reference transducer. This type was chosen because the vast majority of high intensity shock transducers calibrated in our laboratory are of this type. Each coloured line represents the mean of five shock measurements using two laser vibrometers aimed at diametrical opposite positions at the reference surface. The orientation is indicated by the colours [4]. The fitted 1-DOF transfer function is plotted as a dotted line. A detailed view of the lower frequency range, additionally including a measured sine transfer function, is shown in Fig. 3. Both figures illustrate a systematically lower sensitivity at increasing excitation frequencies of the transfer function of the 1-DOF model. Therefore, fitting the 1-DOF model only in the range below 20 kHz would result in a non-matching (too low) resonance frequency.

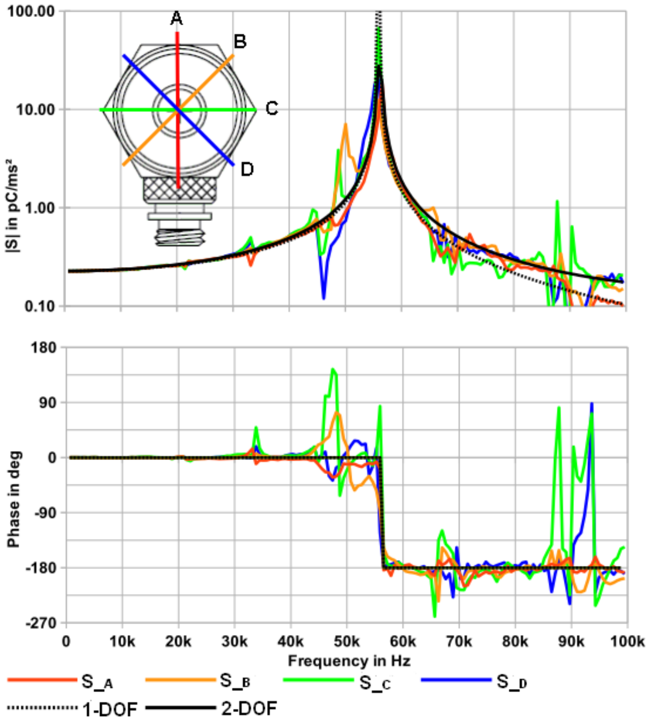

Fig. 2: Measured shock transfer functions of an Endevo type 2270 transducer. The colours indicate laser beam positions. The dotted line is the 1-DOF fit, the black solid line the 2-DOF fit.

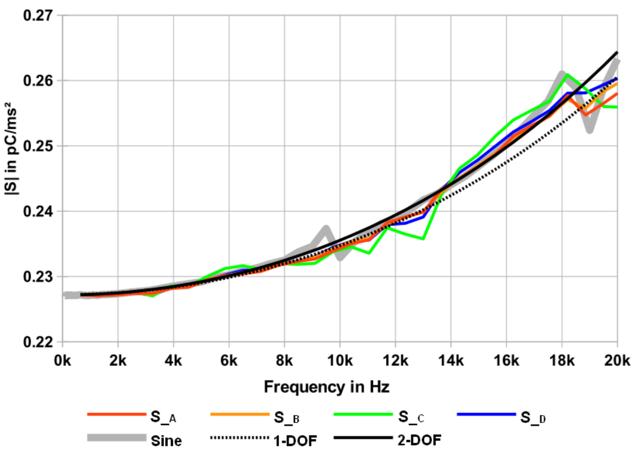

Fig. 3: Zoomed portion of Fig. 2 including sine calibration data.

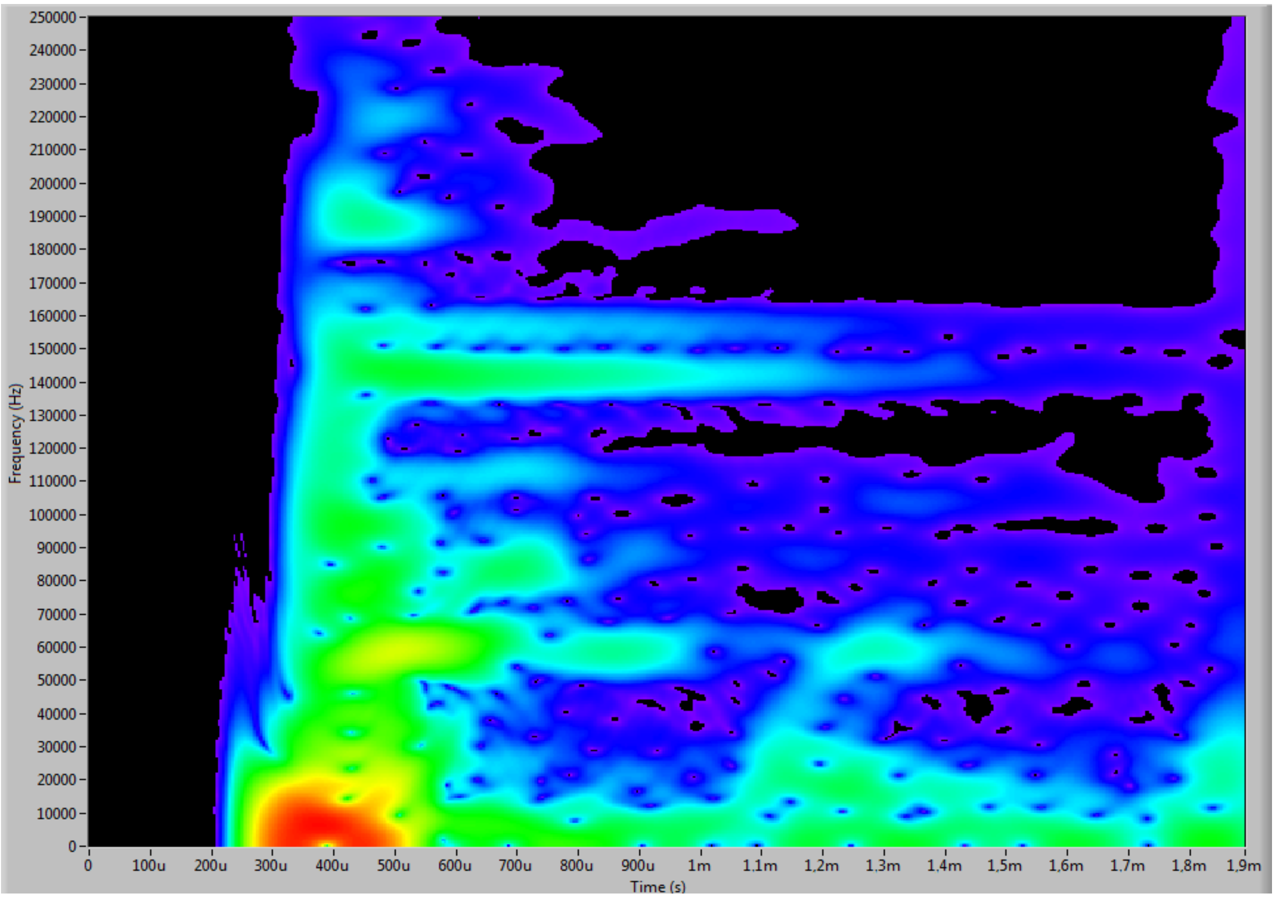

Fig. 4: Screenshot of a power spectrogram of th e output signal an Endevco type 2270 transducer excited with a Hopkinson bar dipole shock pulse of a peak acceleration of 80 km/s² and a duration of 84 µs. The intensity is colour coded in arbitary logarithmic scale for accentuated contrast.

2. THE 2-DOF SYSTEM

Reference [3] proposed a 2-DOF model for a single-ended accelerometer explaining the influence of different material of the armature. A corresponding model adaptation to a back-to-back transducer is shown in Fig. 5. An additional linear mass-spring-damper system (m_2, c_2, d_2) couples the seismic system (m_1, c_1, d_1) to the reference surface and the mounting base.

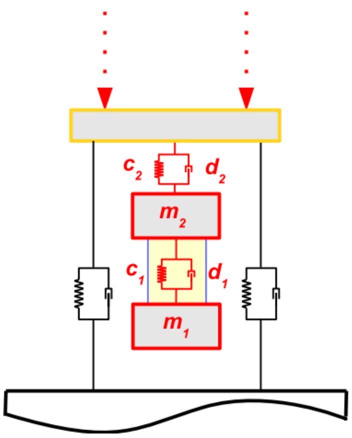

Fig. 5: Proposed 2-DOF model.

The 2-DOF transfer function results to:

$$S_{qa2}(\omega) = S_0 \cdot [\omega_1{}^2 \cdot (\mathrm{i}\omega\delta_2 + \omega_2{}^2)] \cdot \left[\left((\omega_2{}^2 + \eta\omega_1{}^2) + \mathrm{i}\omega(\delta_2 + \eta\delta_1) - \omega^2\right) \cdot (\omega_1{}^2 + \mathrm{i}\omega\delta_1 - \omega^2) - \eta(\mathrm{i}\omega\delta_1 + \omega_1{}^2)^2\right]^{-1} \quad (2)$$

Here, S_0 denotes the sensitivity, $\omega_1 = \sqrt{c_1/m_1}$ the resonance frequency and $\delta_1 = d_1/m_1$ the damping of the seismic system, $\omega_2 = \sqrt{c_2/m_2}$ and $\delta_2 = d_2/m_2$ the corresponding parameters of the coupling system, and $\eta = m_1/m_2$ denotes the mass ratio. It should be noted that for a given accelerometer the ratio η is constant.

Additional indicators for the necessity of a more complex than a 1-DOF model and for information about reasonable start values of a nonlinear fit process are given in Fig. 4. The figure shows a power spectrogram of the output of a type 2270 transducer excited with a Hopkinson bar dipole pulse of 80 km/s² and 84 µs pulse duration. The spectral power is coloured in an arbitrary logarithmic scale for accentuated contrast. The first resonance could be identified at about 58 kHz and higher resonances at about 145 kHz, 156 kHz and 185 kHz.

3. IMPROVEMENT OF THE SIGNAL PROCESSING

The spectrogram in Fig. 4 also indicates that some signal power is even found at frequencies up to 220 kHz. Most of our equipment like charge amplifiers and the signal processing components of the vibrometer raw signals were designed and calibrated for a bandwidth of only up to 100 kHz. Fortunately, the preamplifier output of our BK2525 signal conditioner could be used up to 250 kHz, see Fig. 6. The numerical filters, demodulation and differentiation routines of the vibrometer raw signal processing were adjusted to a bandwidth of 300 kHz.

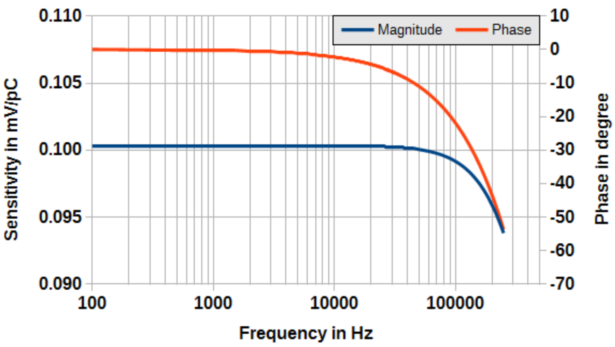

Fig. 6: Transfer function of the preamplifier BK2525.

4 SHOCK MEASUREMENTS

The transducer was excited using an Hopkinson bar [4] providing a peak acceleration of 70 km/s² and a pulse length of 45 µs. The acceleration was measured on the reference surface at opposite positions together with the voltage output of the charge amplifier. Seven measurements, each at the horizontal and the vertical position (see Fig. 2), were performed. The offset of the charge amplifier output was eliminated, both signals were slightly windowed with a flat-top window only affecting the first and last 10% of the signal, a DFT was applied, and the charge amplifier was compensated in the frequency domain.

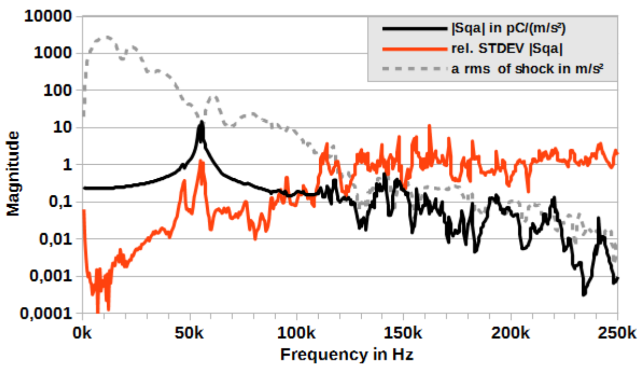

Fig. 7 Magnitude of the transducer sensitivity, its relative standard deviation and shock acceleration

The resulting transfer function calculated as the complex mean of the 14 shock measurements is shown in black in Fig. 7 up to a frequency of 250 kHz. The red plot shows the relative standard deviation of the sensitivity magnitude and the dotted plot shows the magnitude of the shock acceleration. At frequencies >110 kHz, the relative standard deviation reaches 100%, resulting in a noisy transfer function.

Further methods for excitations with higher amplitudes at higher frequencies to achieve a better signal to noise ratio in the transfer function without the risk of damaging the transducer are currently under investigation.

5 RECIPROCAL SENSOR EXCITATION

The reciprocal excitation of the piezoelectric system of the accelerometer with an AC current sweep and the measurement of its impedance is easily possible in frequency ranges well above 100 kHz. Fig. 8 shows the schematic circuit of a transformer with a centre-taped secondary winding. A generator feeds the primary winding forcing a current in the secondary winding. The trim capacitors Tr1 and Tr2 compensate minor imbalances of the realized device. If the impedance of the sensor matches the impedance of the trimmer Tr3, the center-tap voltage U_c approaches zero. The original purpose was a device that allows an 'all ground-referenced' impedance match of a sensor cable assembly with a charge amplifier calibration source at arbitrary frequencies, since our LCR meter works with fixed frequencies. Recently our electrical department kindly measured the transducers impedance in the frequency range up to 300 kHz. Both results are shown in Fig. 9. The transducer impedance is plotted in black and the voltage ratio U_c/U_g with the right scale is plotted in red. Both traces indicate resonances at f_{rc1}≈62 kHz, f_{rc2}≈143 kHz, f_{rc3}≈156 kHz, f_{rc4}≈186 kHz and f_{rc5}≈222 kHz which are in good agreement with the observed resonances in Fig. 4.

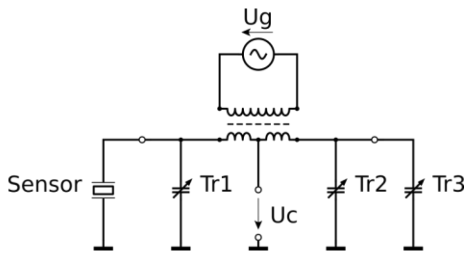

Fig. 8 Transformer-based circuit to match the capacity of the sensor and Tr3.

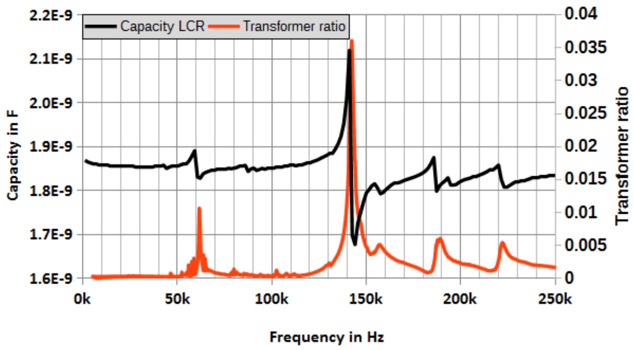

Fig. 9 Impedance of an Endevco type 2270 incl. cable measured with an LCR bridge (at left) and output ratio of the transformer setup (at right).

6. MODEL FITTING

The measured data presented in section 4 (see Fig. 7, 10) was now used to apply a complex weighted Levenberg-Marquardt fit on the 2-DOF transfer function (2). Data points with a relative standard deviation greater than 10% were excluded and the squared inverse of the relative standard deviation was used as the weighting function. Fig. 10 displays the data used for the fit in yellow and the resulting transfer function of the 2-DOF model in red.

As start parameters, the following values were used: The first two resonances of the impedance measurement f_{s1}=62 kHz, f_{s2}=143 kHz, the sensitivity $S_0 = 0.226$ pC/ms², $\eta_s = 1$, $\delta_{s1,2} = 0.01$ are used. The resulting calculated fit parameters are:

$S_0 = 0.2266$ pC/ms²,
f_1=62.07 kHz,
$\delta_1 = 0.03$,
f_2=148.4 kHz,
$\delta_2 = 20000$ and
$\eta_s = 1.33$.

The fit process is currently not sufficiently robust, as the variation of the start parameters shows stable results for S_0 and ω_1 whereas ω_2 mostly lock to frequencies near the found higher resonances for η ranging from 0.4 to 2.2 and very high values for δ_2.

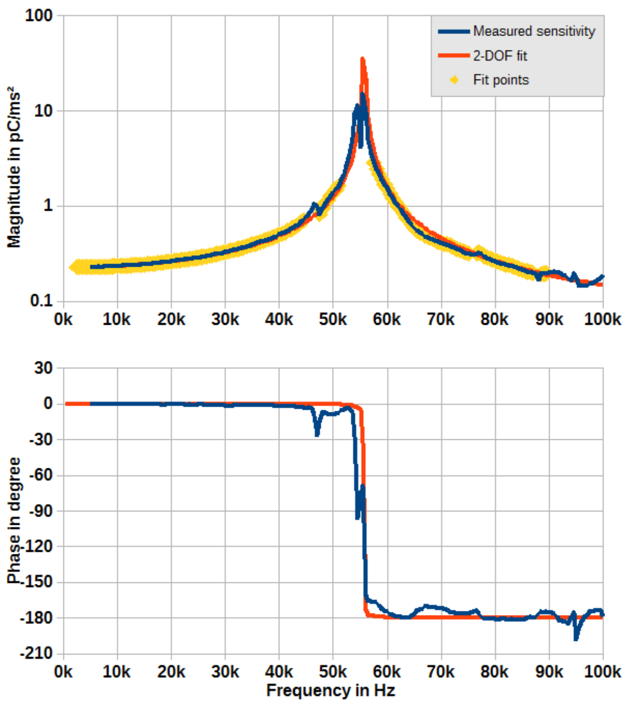

Fig. 10: 2-DOF fit of the magnitude and phase of the frequency response of an Endevco type 2270 transducer.

4. CONCLUSIONS AND OUTLOOK

While the actual acceleration response of an Endevco type 2270 transducer could (currently) only be exited and measured up to 100 kHz, impedance measurements up to 250 kHz indicate more than two degrees of freedom of the actual transducer. However, a 2-DOF model can be fitted that gives a very good match up to a frequency of 100 kHz. An improved 2-DOF model capable characterizing the acceleration and electrical response might allow to identify some of the model parameters by simple electrical measurements. This new approach is currently investigated and will be presented in the future.

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