THE SETUP AND PERFORMANCE EVALUATION OF ACCELEROMETER COMPARISON CALIBRATION SYSTEM IN NML TAIWAN

Tsung Hsien Tu, Kuang-Yih Tsuei, Jiun-Kai Chen, Yu-Chung Huang, Pei-Yao Yu, Kuang-I Chang

National Measurement Laboratory, Center for Measurement Standards, Industrial Technology Research Institute, Hsinchu, Taiwan

thtu@itri.org.tw

Abstract – Accelerometer comparison calibration technique involves the back-to-back coupling the accelerometer under test directly to a traceable reference standard accelerometer and both accelerometers will be experienced the same motion from shaker. Thus the sensitivity of standard accelerometer can be transferred to the test accelerometer via a calibration constant from the output signals. The results of the relative expand uncertainty for the voltage sensitivities for accelerometer are less than 1.4 %, 1.9 % and 2.9 % within frequency range from 50 Hz to 1.5 kHz, from 1.5 kHz to 5 kHz, and from 5 kHz to 7 kHz, respectively. This paper will present the construction, the performance evaluation of the calibration system in NML, CMS/ITRI.

Keywords: accelerometer calibration, ISO 16063-21

1. INTRODUCTION

Precision vibration measurements depend on accurate, repeatable and stable calibration method. Standardization of calibration test equipment and measurement techniques are ensuring more accurate and repeatable measurement. Vibration calibration is the art about the measurement the sensitivity of accelerometer. The uses of periodically oscillatory (sinusoidal) excitation provided by an electromechanical exciter or shaker and the measurement of the accelerometer sensitivity are described by the ISO 16053-21, Method for the calibration of vibration and shock transducers – Part 21: Vibration calibration by comparison to a reference transducer[1]. Accelerometer calibration using reference (or transfer) standard is the most common and efficiency method for most typical accelerometer users. The accelerometer under test is directly back-to-back mounted to a reference accelerometer and both experience the same sinusoidal motion driven by shaker under specific amplitude under certain frequency. The ratio of the output signals provide the calibration constant for the accelerometer under test, thus the sensitivity of the sensor under test (SUT) is calculated through the calibration constant and the sensitivity of the reference accelerometer[2]. This paper firstly presents the principle of accelerometers back-to-back calibration technique. Secondly, the components within the accelerometer sensitivity measuring system are introduced. Thirdly, the procedure and information for the performance evaluation of the calibration system in NML is illustrated.

2. HARDWARE DESCRIPTION

For accelerometer sensitivity comparison calibration, the sensor under test, mounted “back-to-back” to the secondary standard is excited sinusoidally, steeping through the desired frequencies at the desired acceleration amplitudes. The measurement components are divided into two distinct subsystems: excitation and measurement system as shown in Fig. 1 and Fig. 2. The excitation subsystem consists of electrodynamic shaker and associated amplifier is designed to produce the test conditions (constant vibration level at a specified amplitude and frequency). The measurement subsystem is composed of computer data acquisition and control, back-to-back reference standard, and signal conditioners for different type of accelerometers. This subsystem will measure the voltage output ratio of the two accelerometers and calculated the sensitivity of the sensor under test.

Fig. 1. Accelerometer calibration system module blocks

<table>
<thead>
<tr>
<th>Standards or instruments</th>
<th>Mfr.</th>
<th>Model</th>
<th>Serial Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Checking Standard/Signal conditioner</td>
<td>PCB</td>
<td>353B04/442A102</td>
<td>LW173019/365</td>
</tr>
<tr>
<td>Data Acquisition card</td>
<td>NI</td>
<td>PCI-4461</td>
<td>1918PC4</td>
</tr>
<tr>
<td>Reference Standard/Signal conditioner</td>
<td>PCB</td>
<td>080A200/442A102</td>
<td>170048/366</td>
</tr>
<tr>
<td>Vibration Exciter</td>
<td>PCB</td>
<td>396C10</td>
<td>772</td>
</tr>
</tbody>
</table>

Table 1 Standards and/or instrument of calibration instruments
3. SYSTEM EVALUATION

The main purpose of system evaluation is for the estimation and calculation of measurement uncertainty. Such uncertainty will express dispersion of the measurand. The description of measurement results must include the expression of uncertainty; otherwise such measurement process would not be a complete one. The uncertainty of a typical accelerometer calibration system falls into three primary categories: the voltage ratio measurement, the transfer standard accelerometer, and the exciter performance.

3.1. Measurement principle

There are many factors contributing to accelerometer calibration uncertainty. Consider the measurement equation for accelerometer calibration in Eq.(1)

\[ S_u = \frac{V_u}{V_r} \times S_r = V_R \times S_r \]  

(1)

where \( S_u \) is the voltage sensitivity of the accelerometer set to be calibrated (mV/(m/s²)), \( S_r \) is the voltage sensitivity of the reference standard accelerometer set (mV/(m/s²)), \( V_u \) is the output voltage of the accelerometer set to be calibrated measured (mV), \( V_r \) is the output voltage of the working standard accelerometer set measured (mV), \( V_R \) is ratio of \( V_u \) and \( V_r \).

Equation (1) states the ratio of output voltages of both reference standard accelerometer and SUT which must be equal to the ratio of their respective sensitivities. Because of the ratio nature of the voltage measurement, it is easy to see that if an external factor effects the voltage measurement.

3.2. Uncertainty analysis

We take the check standard accelerometer (PCB 353B04) to evaluate the vibration calibration system. If each input \( X_i \) of \( S_u \) and \( V_R \) is uncorrelated, the variance of measurement result \( S_u \) can be calculated in the following way:

\[ u^2(S_u) = \sum \left( \frac{\partial S_u}{\partial X_i} \right)^2 u^2(X_i) \]

\[ = \left( \frac{\partial S_u}{\partial V_R} \right)^2 u^2(V_R) + \left( \frac{\partial S_u}{\partial S_r} \right)^2 u^2(S_r) \]

(2)

In Eq(2), \( u(S_u) \) and \( u(V_R) \) is the standard uncertainty of the working standard accelerometer and the affection to output voltage ratio from the checking standard accelerometer and reference standard accelerometer. Each sensitivity coefficient related to the partial derivative with each input \( X_i \) is deduced as follows:

\[ \frac{\partial S_u}{\partial S_r} = V_R, \quad \frac{\partial S_u}{\partial V_R} = S_r \]

(3)

3.3. Error Sources and uncertainty contributors

The error sources in the uncertainty estimation include the random error, shaker transverse and rocking motion, sensitivity uncertainty from the primary standard and from drift of the reference standard accelerometer, and the uncertainty in the voltage ratio measurement. The evaluation of measurement uncertainty in this document comprises Type A and Type B evaluations of uncertainty, which are defined as follows:

(A) Standard uncertainty is the uncertainty associated with the measurement result is expressed by using one standard deviation.

(B) Type A standard uncertainty (random error) is results in deviations from calibration to calibration from a series of measurements on a check standard accelerometer. Influences contribute to the random error included: (i) cabling and connector, (ii) mounting torque variation, (iii) operator technique, (iv) temperature and humidity fluctuations, (v) electrical hum and noise in the measurement chain, in the statistical analysis by using one standard deviation.

(C) Type B standard uncertainty (systemic error) is obtained by an assumed probability density function based on the degree of belief that an event will occur and is estimated by scientific judgement coupled with all available information using non-statistical means. The transverse and rocking motion from the shaker \( u(V_R\_rocking) \), the standard uncertainty from the reference standard accelerometer calibration report, and the voltage ratio error from the voltage measuring device (National Instruments, PCI-4461).

3.3.1 Type A relative standard uncertainty from the voltage ratio measurement

The type A standard uncertainty is combined of voltage ratio repetition \( u_{A1}(V_R) \) and \( u_{A2}(V_R) \) from long term and short term, respectively. This type A uncertainty is also called random error, and is determined by iterative test of a check standard on different days (almost 50 days in this case). Over frequency range 50 Hz to 7 kHz, 39 test \( (n_t=39) \) were performed on the check standard accelerometer were performed on the calibration system. Each calibration consisted of 19 sensitivities determined at 19 different exciting frequencies.

(i) Relative standard uncertainty from the long term repeatability, \( u(V_{R\_j}) \)
The relative deviation error from the 39 repeatedly voltage ratio readings, $V_{Rep,1}$, is determined through following equation

$$V_{Rep,1} = \frac{V_{SDV,Rep,1}}{n_1} \times 100$$

where $V_{SDV,Rep,1}$ is the standard deviation of voltage ratio in the $n_1$ repeated measurements and $n_1 = 39$.

Next, the relative standard uncertainty of the long term repeatability $u(V_{Rep,1})$ could be estimated by (5). The measuring and caluating results of $V_{SDV,Rep,1}$ and $u(V_{Rep,1})$ are listed in Table 2.

$$u(V_{Rep,1}) = \frac{V_{Rep,1}}{\sqrt{8}}$$

(ii) Relative standard uncertainty from the short term repeatability, $u(V_{Rep,2})$

The short term repeatability is determined from 5 measurements. The uncertainty range of short term repetition, $V_{Rep,2}$, is determined following:

$$V_{Rep,2} = \frac{V_{SDV,Rep,2}}{\sqrt{n_2}} \times 100$$

In Equation (6) the $V_{SDV,Rep,2}$ is the standard deviation of voltage ratio in the short term calibration tests, and $n_2 = 5$. Then the relative standard uncertainty of the short term repeatability $u(V_{Rep,2})$ could be estimated

$$u(V_{Rep,2}) = \frac{V_{Rep,2}}{\sqrt{8}}$$

Table 2. The uncertainties from repetitions measurement

<table>
<thead>
<tr>
<th>Frequency Range</th>
<th>50 to 1.5k</th>
<th>1.5k to 5k</th>
<th>5k to 7k</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{SDV,Rep}$</td>
<td>0.0031</td>
<td>0.0111</td>
<td>0.0230</td>
</tr>
<tr>
<td>$u(V_{Rep,1})$</td>
<td>0.0286</td>
<td>0.1020</td>
<td>1.0890</td>
</tr>
<tr>
<td>$V_{SDV,Rep,2}$</td>
<td>0.0012</td>
<td>0.0007</td>
<td>0.0004</td>
</tr>
<tr>
<td>$u(V_{Rep,2})$</td>
<td>0.0294</td>
<td>0.0162</td>
<td>1.0890</td>
</tr>
</tbody>
</table>

3.3.2 Type B standard uncertainty in the voltage ratio measurement

The type B uncertainty is estimated from every possible error contributions of VR from test standards or equipment specifications. The type B standard uncertainty will be the combination of all the individual systemic contributors. In this system evaluation they are (i) the uncertainty from the traceable reference standard calibration, (ii) the uncertainty from the voltage ratio measuring device (iii) the uncertainty from the shaker transverse and rocking motion.

(i) Relative standard uncertainty from the reference standard traceability to primary standard

$$u(S_{Trace}) = S_e \times U_{Sen,Trace}$$

where $S_e$ is the voltage sensitivity calibration result

$U_{Sen,Trace}$ is the relative expanded uncertainty

$K$ is the coverage factor, $K=2$

The voltage sensitivity of the reference standard accelerometer, $S_e$, the relative expanded uncertainty $U_{Sen,Trace}$ and the cover factor, $K$, could refer to the calibration report[6].

(ii) Relative standard uncertainty of voltage ratio traceability

The calibration of voltage measurement accuracy is performed on periodic basis. Voltages with different traceable amplitudes and frequencies were acquired by voltage ratio acquisition on both voltage measurement channels AI0 and AI1, one is for the reference sensor voltage, $V_r$, measurement, and the other is for the check standard output voltage, $V_o$. The standard uncertainty of the voltage ratio is listed below:

$$u(V_{Vol,trace}) = \sqrt{u_{AI0}(V_R)^2 + u_{AI1}(V_R)^2}$$

where $u_{AI0}(V_R)$ and $u_{AI1}(V_R)$ are standard uncertainties from the calibration report for each channel under difference voltage amplitudes and frequencies. Here, for every calibrated channel $x$,

$$u_{AIx}(V_R) = V_{mean,x} \times u_{Vol,trace} \times K$$

where $V_{mean,x}$ is the voltage input amplitude for each channel calibrated

$U_{Vol,trace}$ is the relative expanded uncertainty from calibration report [7], for frequency range from 10 Hz to 10 kHz and the voltage range from 10 mV to 1 V, the relative expanded uncertainty is 0.38 %.[7]

$K$ is the coverage factor, and $K=2$

(iii) Relative standard uncertainty of shaker transverse and rocking motion

The transverse and rocking motion, which is the acceleration deviating from the intended exciter moving direction, will vary with the mounting orientation of the reference and test sensor, the exciting amplitude, and the loading on the shaker with different frequency. This contributor is not considered as random because the influence of the transverse motion remains mostly consistent for check standard is mounted on the excitor.

The transverse and rocking motion from shaker is vector addition of the two lateral axes (i.e. x-axis and y-axis in this case) normalized by the primary direction(z-axis). It can be determined using a tri-axial sensor mounted on the reference sensor surface as shown in Fig. 2.

Firstly, the frequency response function of acceleration level was performed on both normal axes and the cover factor, $K$, which is the expected exciting (calibration) direction. The rocking and transverse acceleration level on two normal direction, $a_x$ and $a_y$, were determined individually by following expressions:

<table>
<thead>
<tr>
<th>Table 3 Uncertainties from reference standard calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
</tr>
<tr>
<td>--------------</td>
</tr>
<tr>
<td>Relative Expand Uncertainty, $U$ [%]</td>
</tr>
</tbody>
</table>
of the reference standard is estimated to be 0.05 % per year. For two years calibration interval the worst drift is 0.1 %, and here we used 0.2 % for the sensitivity shift, $S_{\text{Diff}}$.

3.4 Covariance estimation

The assumption in this system validation is based on that each of measurement input is not correlated. Thus covariance between different measurement inputs is negligible.

3.5 Combined standard uncertainty

The combined standard uncertainty of accelerometer sensitivity $S_a$ by comparison method is concluded as:

$$u_c(S_a) = \sqrt{(u_a(S_a))^2 + (u_b(S_a))^2 + (u_d(S_a))^2}$$

After substituting the appropriate evaluated values into the above uncertainty equation, then calculated results of $U_r(S_a)$ are listed in Table 4 and Table 5.

3.6 Expanded uncertainty

The expanded uncertainty is obtained from multiplying the combined standard uncertainty by the associated coverage factor $k$ at a required confidence level (presently 95 % adopted in the system). The value of coverage factor $k$ at the 95 % confidence level is then found from the Student-t distribution with a known effective degrees of freedom, $v_{eff}$. The effective degrees of freedom is approximated by the Welch-Satterthwaite equation shown as follows:

$$v_{eff} = \frac{\sum_{i=1}^{n} \frac{u_i^2(S_a)}{v_i}}{\sum_{i=1}^{n} \frac{u_i^2(S_a)}{v_i}}$$

where $u_c(S_a)$ is combined standard uncertainty of $S_a$

$$u(X) = \text{the standard uncertainty of each input measurand}$$

$v_i$ is the degrees of freedom associated with each uncertainty component which can be estimated shown in the following way

The degrees of freedom of type A uncertainty is $v_i=n-1$ with $n$ repetitions of measurement. The degrees of freedom of type B uncertainty may be approximated as $v_i = (1/2)(R)^2$, in which $R$ is reliability of the uncertainty and expressed as (R = 1 – confidence level) in this document. For 95 % confidence level, $R = 5 \%$. Thus, $v_i = 200$ for each uncertainty source of related instrument, and $v_{eff}$ will be obtained by substituting each $u(X)$ and associated $v_i$ into the Welch-Satterthwaite equation.

4. CONCLUSION

The system configuration and the uncertainty estimation method through type A and Type B uncertainties are introduced for calculating the measurement uncertainty of accelerometers' voltage sensitivity and effective degrees of freedom with a chosen coverage factor. The calibration station is for conducting the accelerometer sensitivity with frequency range from 50 Hz to 7 kHz. The assessment on
the best measurement capability of the calibration station was perform using the uncertainties evaluation for calibration performed using a check standard. The results of the relative expanded uncertainty for the voltage sensitivities calibration for accelerometer are less than 1.4 %, 1.9 % and 2.9 % within frequency range from 50 Hz to 1.5 kHz, from 1.5 kHz to 5 kHz, and from 5 kHz to 7 kHz, respectively.

REFERENCES

Table 4 Uncertainty components for accelerometer’s voltage sensitivity magnitude determination

<table>
<thead>
<tr>
<th>Uncertainty Component, ( x_i )</th>
<th>Probability distribution</th>
<th>Type</th>
<th>Relative Standard uncertainty ( u(x_i) )</th>
<th>Sensitivity coefficients ( c_i )</th>
<th>Degree of freedom ( v_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage Error, ( V_{Vol_Trace} )</td>
<td>Rectangular ( 1/\sqrt{3} )</td>
<td>B</td>
<td>( \sqrt{u_{AB}(V_R)^2 + u_{All}(V_R)^2} )</td>
<td>( S_r )</td>
<td>200</td>
</tr>
<tr>
<td>Reference standard Calibration, ( S_{Trace} )</td>
<td>Normal ( (k=2) )</td>
<td>1/2</td>
<td>( S_r \times (U_{sen.trace}/2) )</td>
<td>( S_r )</td>
<td>200</td>
</tr>
<tr>
<td>Reference standard sensitivity drift, ( S_{Drift} )</td>
<td>Rectangular ( 1/\sqrt{3} )</td>
<td>B</td>
<td>( 0.2/\sqrt{3} )</td>
<td>( S_r )</td>
<td>50</td>
</tr>
<tr>
<td>Shaker transverse and rocking motion, ( V_{R-Rocking} )</td>
<td>Special ( 1/\sqrt{18} )</td>
<td>B</td>
<td>( \sqrt{(S_{r,1}^2 + S_{r,2}^2) \times a_k/\sqrt{18}} )</td>
<td>( S_r )</td>
<td>12.5</td>
</tr>
<tr>
<td>The long term repetition, ( V_{SDV_Rep,1} )</td>
<td>( t )</td>
<td>( 1/\sqrt{3} )</td>
<td>A</td>
<td>( V_{SDV_Rep,1}/\sqrt{3} )</td>
<td>( V_R )</td>
</tr>
<tr>
<td>The short term repetition, ( V_{SDV_Rep,2} )</td>
<td>( t )</td>
<td>( 1/\sqrt{3} )</td>
<td>A</td>
<td>( V_{SDV_Rep,2}/\sqrt{3} )</td>
<td>( V_R )</td>
</tr>
</tbody>
</table>

Table 5 Uncertainty Budgets for accelerometer sensitivity calibration from 50 Hz to 7 kHz

| Relative uncertainty Component, \( x_i \) | Frequency Range [Hz] |
|---------------------------------|-----------------|-----------------|-----------------|
| Voltage Error                   | 0.30            | 0.30            | 0.30            |
| Reference standard Calibration  | 0.58            | 0.86            | 1.13            |
| Reference standard sensitivity drift | 0.07        | 0.07            | 0.07            |
| Shaker transverse and rocking motion | 0.06        | 0.08            | 0.11            |
| The long term repetition         | 0.04            | 0.12            | 0.24            |
| The short term repetition        | 0.01            | 0.01            | 0.01            |
| Relative combined standard uncertainty, \( u_c(S_r) \) | 0.66            | 0.92            | 1.20            |
| Effective degree of freedom, \( V_{eff} \) | 306.3           | 258.7           | 248.3           |
| Coverage factor, \( k \)        | 1.97            | 1.97            | 1.97            |
| Relative expanded uncertainty, \( U \) [%] | 1.4             | 1.9             | 2.4             |