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THE PARAMETERS OF MOTION MECHANICAL EQUATIONS AS A SOURCE OF UNCERTAINTY FOR TRACTION SYSTEMS SIMULATION

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Abstract: Electromechanical analysis and simulation of traction systems are required to estimate the power consumption and the best optimization for energy saving. The number of variables and parameters (mechanical and electrical) is huge and they are deemed by a high degree of uncertainty. The sensitivity of the mechanical equations to the track and train parameters is considered, as well as the spread of the output variables.

Keywords: Davis equation, Electromechanical simulation, Guideway transportation systems, Model validation.

1. INTRODUCTION

Electromechanical simulation of electric transportation systems has several applications: during design, the sizing and rating of electric substations, of traction supply conductors and supply feeders and the optimization of the feeding points, with respect to the maximum absorbed power and the tolerated supply voltage drops along the line; moreover, under major system revamping and modernization, the calculations may be repeated to push optimization even further, reducing power absorption peaks, resizing some feeders and conductors, adjusting the time table and train scheduling. In general, also with the aim of optimizing the power consumption and demonstrating overall system efficiency, electromechanical simulation is a precious tool to support direct measurements.

The validation of an electromechanical simulator is based at last on the direct comparison of simulator outputs with experimental data. One of the goals of the simulation is the evaluation of the overall system efficiency and a fraction of % on the evaluated system efficiency can really bring to tangible differences in terms of money, if the involved power consumption terms of a whole railway or metro system are taken in the due consideration. This translates into a demanding requisite concerning the overall simulator accuracy and analogously, the same accuracy is required when measurements are designed and performed on the system. A big effort is necessary to limit and to cover comprehensively the system under measurement. Second, the used sensors, instruments and measurement techniques must ensure an adequate accuracy, that is barely achievable for electrical variables (voltages and currents at substations and on trains), but represents a major difficulty for the evaluation of mechanical variables.

In this first work the attention is focused on the mechanical equations, on the relationships between variables and coefficients and on how their values are determined with reference to experimental results and published data. The goal is the identification of the intrinsic uncertainties, related to the quantification of equations coefficients and to the determination of the most influencing parameters, with the aim of designing the correct experimental activities. For this reason the expression of the mechanical train resistance and its terms are analyzed, defining the parameters and the input data and the uncertainties related to their determination. It is quite common in reality that several data are known only in tabular form and that the adopted values are average values for a similar type of train consist or track. For this reason the present analysis focuses on the identification of the most relevant parameters and data in terms of sensitivity and spread.

2. TRAIN RESISTANCE FORCE AND GENERAL FORMULATION

The train resistance force R is approximated by a quadratic function that is variously known as the “von Borries Formel”, the “Leitzmann Formel”, the “fonction de Barbier” and, in the Anglo-Saxon world, the “Davis equation” [1]:

$$R = A + Bv + Cv^2 \tag{1}$$

The coefficients A and B include the mechanical resistances and depend on the train mass, so that at lower speed (□30 m/s, that is about 100 km/h) the resistance force R is mainly dependent on the train mass. At higher speed, the Cv^2 term related to the aerodynamic resistance becomes dominant. The values of the coefficients in (1) are usually set for open air conditions and require modification for the tunnel environment, where the C term, in particular, is larger.

Armstrong and Swift [2] proposed a set of empirical expressions to determine the coefficients A, B and C of the Davis equation for the electric multiple units (EMU) in service at that time on the former British Rail lines. The coefficients A, B, C in (1) are put in relationship with the following constants:

$$A = a_1 m_{TC} + a_2 m_{PC} \tag{2}$$

$$B = b_1 m + b_2 n_{TC} + b_3 n_{PC} P \tag{3}$$

$$C = c_1 C_x S + c_2 dl + c_3 dI_g (n_{TC} + n_{PC} - 1) + + c_4 C_x^B n_B + c_5 n_P \quad (4)$$

where:

m_{TC} in tons is the total mass of trailer cars;
m_{PC} in tons is the total mass of power cars;
m in tons is the train mass;
n_{TC} is the number of trailer cars;
n_{PT} is the number of power cars;
P in kW is the total power;
C_x is the head/tail drag coefficient;
S in m^2 is the cross-sectional area;
d in m is the perimeter;
l in m is the train length;
I_g in m is the inter-vehicle gap;
C_x^B is the bogie drag coefficient;
n_B is the number of bogies;
n_P is the number of pantographs.

The units of the empirical coefficients a_1 to c_5 are chosen to results in the correct units for A, B and C, e.g. c_3 is in [N s^2/m^3]. Armstrong and Swift provide values for a_1 to c_5 that lead to the following expression:

$A = 6.4\, m_{TC} + 8.0\, m_{PC}$
$B = 0.18\, m + 1\, n_{TC} + 0.005\, n_{PC}\, P$
$C = 0.6125\, C_x\, S + 0.00197\, d\, l +$
$+ 0.0021\, d\, I_g\, (n_{TC} + n_{PC} - 1) +$
$+ 0.2061\, C_x^B\, n_B + 0.2566\, n_P$

Rochard and Schmid provide in [1] a validation of the above equations (1) to (4), using data obtained by SNCF as a result of run-down tests and reported by M. de la Broise [1], for the Class 373 Eurostar train.

Different authors [1-4] in recent years have described the approaches of various national railway undertakings to the calculation of train resistance. Most of them are empirically modified versions of the Davis equation and include coefficient related to particular types of rolling stock, putting coefficient A, B and C in relationship with different figures not considered in Armstrong-Swift equations (such as the number of axles and the axle load).

3. ANALISYS OF THE DAVIS COEFFICIENTS

The terms of (1) are further analyzed in the following, with particular reference to their relationship with train and track characteristic, non idealities, dependency of speed itself and possibility of experimental determination and related accuracy.

3.1 Term A of Davis equation

The first term, A, of the Davis equation (1) is purely mass dependent; from the tests reported in [3][4] the term A is approximately linear with respect to the number of axles and the running resistance increases approximately linearly and only slightly with the increasing axle load. It is reasonable to expect that the term A, as well as the other mass-related coefficient B below, should depend upon track construction and maintenance standards. However, it is very difficult to determine how the track type influences the coefficient A, because of the number of variables involved in the numerical quantification of the track.

3.2 Term B of Davis equation

The influence of several system parameters on the value of the term B is reviewed here below:

- Influence of train mass and train length: the coefficient B is normally expressed as a function of train mass [3]. However, from tests reported in [4], where the axle load was varied for the same train configurations, no systematic variation in coefficient B, due to axle load, was observed. This may indicate that the main part of this coefficient is not due to the mechanical resistance, but rather originates from portions of air drag not covered by the term Cv^2. Coefficient B then may be expressed as a function of the total train length rather than the train mass.
- Influence of track type: from test covered in [4] it is not possible to conclude that changes in B originate from the difference in track type; however according to Davis [5] concussion and swaying of the vehicles contribute to B.
- Influence of air momentum drag: a train set ingests air for cooling and ventilation and this causes additional air momentum drag [6]. According to Gawthorpe [3], the contribution to the resistance from the air momentum drag of a locomotive is calculated in Newton as:

$$F_{D,in} = \rho \frac{dV_{in}}{dt} \cong 20v \quad (5)$$

  where:
  ρ in kg/m^3 is the density of air;
  V_{int} in m is the volume of air intake by the cooling and ventilation system.
- The air density ρ is about 1.3 kg/m^3 at 0 ºC and 1013 hPa, therefore – especially for non high-speed and freight trains – the variation of coefficient B due to air momentum drag is only some percent of the total running resistance (e.g. about 600 N for train moving at 30 m/s) and is hard to distinguish.

3.3 Term C of Davis equation

The aerodynamic drag (the part which is dependent upon speed squared) is usually expressed in Newton for no wind conditions as:

$$F_D = \tfrac{1}{2} \rho A_f (C_p + C_s l) v^2 = Cv^2 \quad (6)$$

where:

A_f in m^2 is the projected cross-section area;
C_p is the total mean front pressure and rear suction drag coefficient;
C_s is the total mean pressure and friction drag coefficient along the train;
l in m is the train length.

It is convenient to express the coefficient C as air drag area $C_D A_f$ in m^2 [7]:

$$C_D A_f = (C_p + C_s l) A_f = 2C/\rho \tag{7}$$

The results reported in [4] reveal that the aerodynamic drag for passengers and freight trains increases approximately linearly with length (this is also supported by Hammit [7]). By means of the method of least squares, a line is fitted to the wind average results of $C_D A_f$, giving:

$C_D A_f \approx 8.3 + 0.057\, l$ for conventional passengers trains;
$C_D A_f \approx 4.7 + 0.050\, l$ for high speed trains;
$C_D A_f \approx 8.2 + 0.133\, l$ for freight trains of mixed consist.

The first constant term expresses the contribution to the pressure and suction drag, acting on the front and rear of a train [7]; the drag contribution from pantograph and roof equipment of the loco is also included. The linear term originates from the skin friction and pressure effects along the train.

3.4 Other resistance terms

In addition to the three friction forces originated by the mass, viscous component of mass and aerodynamic characteristic described by (1), the traction equipment of trains also has to overcome the resistance to acceleration, gradient force and curving resistance.

Acceleration is simply a function of the masses with the necessary allowance for the rotating components (also known as rotary allowance), while curving is highly dependent on wheel and rail profiles, track cant and the geometry of the vehicle concerned.

3.4.1 Gradient resistance

The gradient force is mass-related and can be added as an equivalent linear force:

$$R_{slope} = mgi \tag{8}$$

where:
m in kg is the mass of the train;
g in m/s^2 is the gravity acceleration;
i in % is the track grade (slope).

The grade i can be expressed as:

$$i = 100 \tan(\alpha) = 100\ \Delta y/\Delta x \tag{9}$$

where:
α in rad is the slope angle;
Δy in m is the rise;
Δx in m is the horizontal run.

3.4.2 Resistance due to track curvature

The curve resistance is an estimate of the added resistance a vehicle has to overcome when operating through a horizontal curve. The exact details of the mechanics contributing to curve resistance are not easy to define. The effects of resistance due to track curvature are small for curves with radius larger than 250 m.

A common formula for calculating the resistance due to track curvature, provided by Profillidis [8], is

$$r_c = 0.01\left(\frac{k}{R_c}\right) \tag{10}$$

where:
r_c in kN/t is the specific resistance force, assuming that the gravity acceleration is 10 m/s^2;
k is a dimensionless parameter, depending on the train design and varying in general from 500 to 1200;
R_c in m is the curve radius in a horizontal plane.

However it is generally accepted in the railway industry that the curve resistance is approximately the same as a 0.04% up grade per degree of curvature for standard gauge tracks [9]. At very slow speed (up to 5 km/h) the curve resistance is closer to 0.05% up grade per degree of curve.

4. VARIABILITY OF MECHANICAL EQUATIONS

Using Armstrong-Swift's equations (2) to (4) to calculate the coefficients A, B and C of the Davis equation, the effects of varying train parameters are investigated. The sensitivity of the mechanical equations to the train parameters is considered, as well as the spread of the output variables. Then the same equations are also compared to those proposed in the past and still used in France, Germany and Japan, with slightly different assumptions and identification of the input parameters.

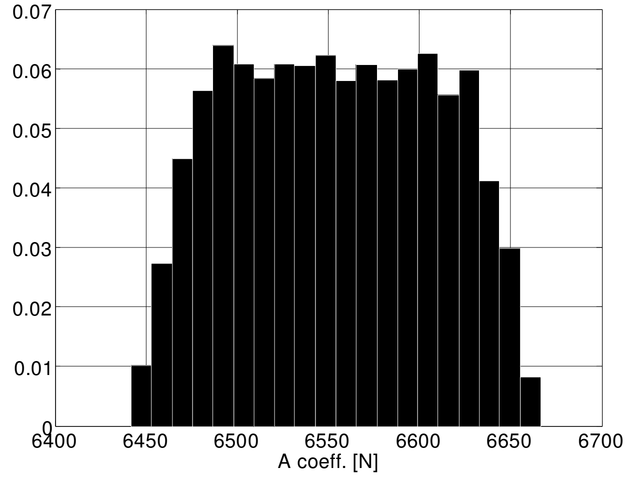
4.1 Sensitivity analysis

A Series 100 Shinkansen and a Class 373 Eurostar are used as the reference systems; the sensitivity analysis is performed around their nominal values reported below in Table 1, over a speed range from 0 to 300 km/h. The sensitivity is evaluated by a Monte Carlo approach with the variations applied by a random fractional change around the nominal value with a uniform distribution. The extremes of the distribution are fixed so to have two normalized dispersion values, 1% (for an analysis around a fixed operating point) and 20% (for an analysis that includes also the effects of the non-linear terms of (2) to (4)).

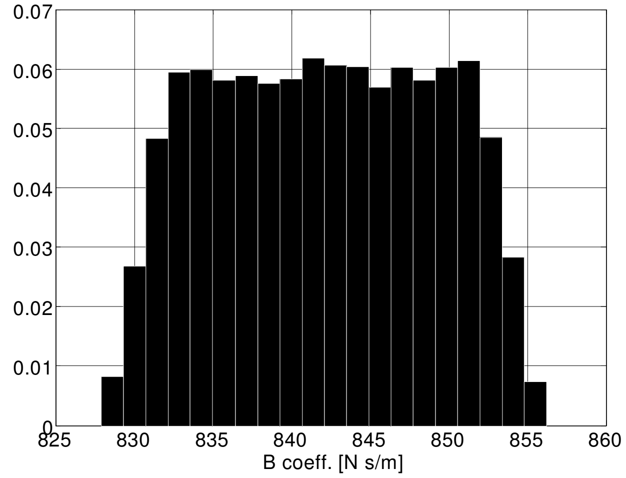
Table 1. Comparison of Armstrong-Swift equation input data for Series 100 Shinkansen and Class 373 Eurostar.

Parameter	Series 100	Class 373
Length l [m]	402	394
Mass of power cars m_{PC} [t]	672	137
Mass of trailer cars m_{TC} [t]	184	730
Total mass m [t]	856	867
Power P [MW]	11.04	12
Number of power cars n_{PC}	12	2
Number of trailer cars n_{TC}	4	18
Cross-sectional area S [m^2]	12.6	8.9
Perimeter d [m]	14.24	11.24
Number of pantographs n_P	3	2
Head drag coefficient C_x	0.075	0.0702
Tail drag coefficient C_x^B	0.075	0.0743

Fig. 1 shows the histograms of A, B and C coefficients for a uniform distribution with 1% dispersion of the coefficients after (4), except integer coefficients n_{TC}, n_{PT}, n_B, n_P held constant (a 1% dispersion on an integer variable in the range of up to some tens is not relevant and doesn't change the value of the coefficients themselves).



(a)



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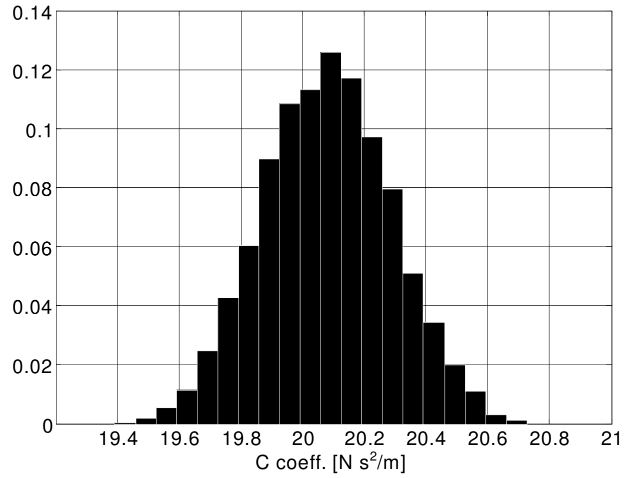
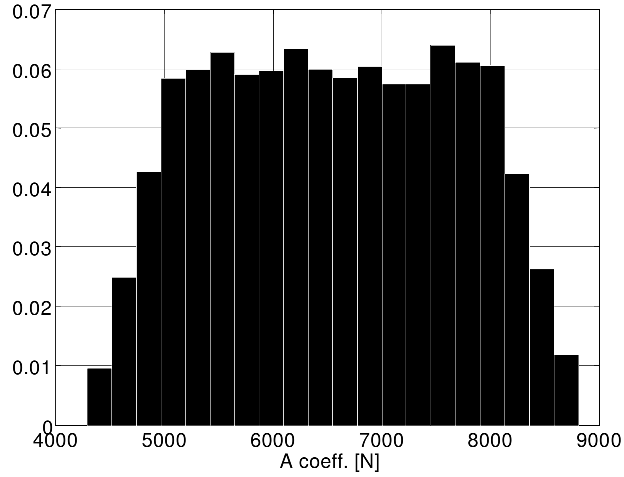


(c)

Figure 1. Histograms of *A*, *B* and *C* coefficients for a normalized 1% dispersion of all parameters except n_{TC}, n_{PT}, n_B, n_P

The probability density functions (pdfs) of the *A* and *B* coefficients are trapezoidal distributions, as evident by observing (2) and (3), where the resulting coefficient is the sum of two terms with a uniform distribution. The third pdf of the *C* coefficient resembles a Normal distribution, but it will be shown in the next figure that it is a skewed distribution similar to a Weibull distribution.

In Fig. 2 all the assumed uniform distributions are characterized by a twenty times larger dispersion, including the integer parameters, neglected in the first analysis.



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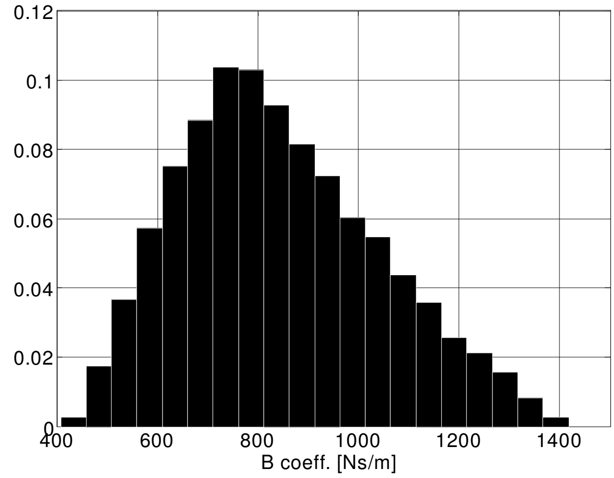

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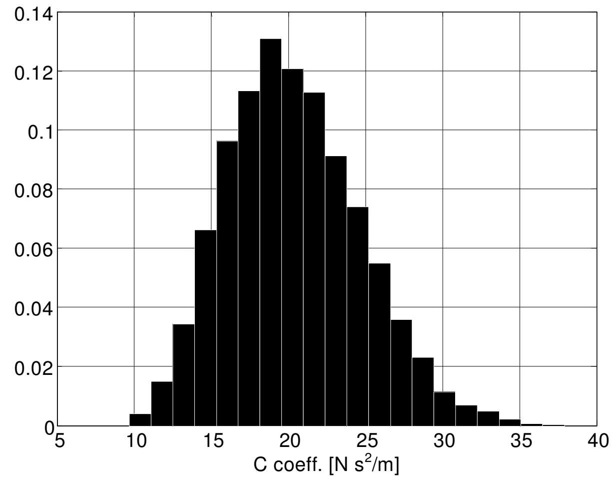


(c)

Figure 2. Histograms of *A*, *B* and *C* coefficients for a normalized 20% dispersion of all parameters including n_{TC}, n_{PT}, n_B, n_P

Fig. 2(c) is showing a skewed distribution, already visible in Fig. 1(c) for a 1% only dispersion. What is interesting to see is the effect of the two integer variables n_{TC} and n_{PC}, modelled as random variables only in the case of 20% dispersion of Fig. 2(c): for the smaller 1% dispersion the two integer variables are considered constant, being the applied random changes masked by the round-off.

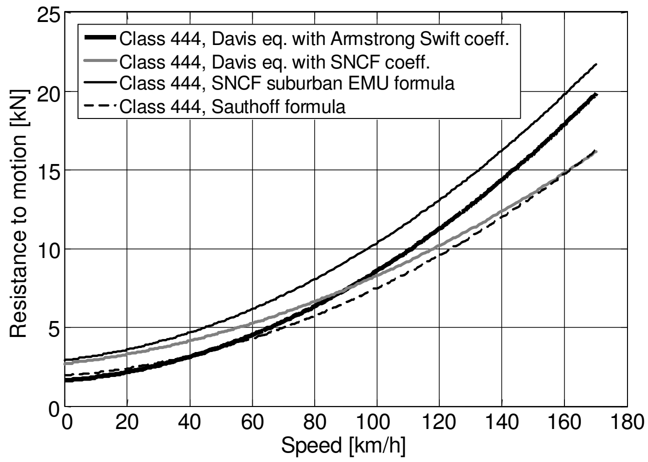
The pdf of the coefficient B shown in Fig. 2(b) takes a triangular shape, while in Fig. 1(b) the pdf was very narrow around the average value, because the influence of the 1% dispersion of the only random variable in B expression was weak.

4.2 Comparison of equivalent formulations

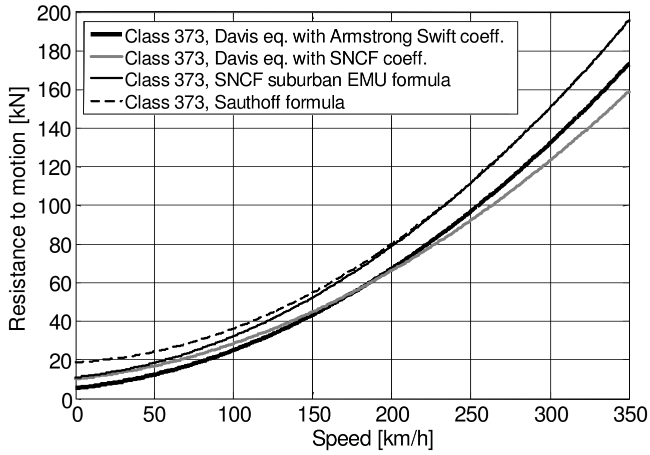
The results of the Armstrong-Swift approach applied to the Davis equation are further compared with the results of other empirically or semi-empirically formulae commonly used in France and Germany to calculate the resistance to motion of trains [8]. This part of the analysis gives a direct estimation of the dispersion of results due to different assumptions and interpretations of available data and parameters. We could identify the resulting dispersion as an "operative" spread of results, while the Monte Carlo simulations lead to a "theoretical" spread (and to the evaluation of the sensitivity).

Two trains are considered in Fig. 3 with different characteristics: a British Class 444 suburban train of the EMU type and a European Class 373 Eurostar high speed train. The curves of the resistance to motion in kN are calculated and plotted versus speed from standstill to the maximum commercial speed. The curves are characterized by slightly different values and slopes, so that there is in general no definitely overestimating and underestimating curve at all speed values.

The spread of the curves at the end of the respective speed intervals for the largest speed values is practically due to the fact that the parameters for a train class are normally derived from approximations and interpolation of experimental data, mostly valid and accurate at the center of the speed intervals.



(a)



(b)

Figure 3. Curves of resistance to motion calculated for the same train with different formulations: (a) suburban train Class 444, (b) high speed train Class 373

Fig. 3 shows that the spread of the results is 20-30% on average. A more accurate estimate is shown in Table 2 and 3, where the difference in percentage with respect to the Armstrong-Swift equation (taken as reference) is reported for the two trains.

Table 2. Comparison of Armstrong-Swift equation and other formulations for Class 444 suburban train.

Used formulation	**Speed [km/h]**							
	30	**50**	**70**	**80**	**100**	**120**	**150**	**180**
Armstrong-Swift [kN]	2.56	3.78	5.38	6.35	8.60	11.27	16.09	19.84
SNCF formula %	42.5	24.1	10.2	4.9	-3.5	-9.5	-15.7	-18.7
SNCF formula suburban %	57.6	42.3	31.3	27.1	20.7	16.2	11.6	9.4
Sauthoff formula %	7.7	-0.8	-6.8	-9.0	-12.3	-14.6	-17.0	-18.0

Table 3. Comparison of Armstrong-Swift equation and other formulations for Class 373 Eurostar.

Used formulation	**Speed [km/h]**											
	30	**50**	**70**	**80**	**100**	**120**	**150**	**180**	**200**	**250**	**300**	**330**
Armstrong-Swift [kN]	9.24	12.69	17.04	19.56	25.28	31.91	43.55	57.23	67.48	97.09	132.37	156.26
SNCF formula %	50.2	35.2	24.4	20.2	13.7	8.8	3.7	0.1	-1.6	-4.8	-6.8	-7.7
SNCF formula suburban %	62.4	47.8	37.9	34.3	28.7	24.8	20.8	18.3	17.0	15.0	13.8	13.3
Sauthoff formula %	135.3	94.6	68.4	58.9	45.1	35.8	26.9	21.6	19.2	15.5	13.7	13.1

By observing the relative difference with respect to Armstrong-Swift formulation shown in Table 2 and 3 above, it may be concluded that the other formulations show a spread of the average difference of +80% and –9%, indicating that the Armstrong-Swift formulation in the Davis equation represents a lower bound of the resistance to motion estimation. This statement does not mean that this formulation is not enough conservative, since the present study has been performed in terms of sensitivity and dispersion, without posing the question of the determination of the "real" resistance to motion.

5. CONCLUSIONS

In the present work an overview of the phenomena and equations related to the mechanical motion of a train are presented. The electromechanical analysis and simulation of traction systems of various kinds (heavy and light railways, metros, etc.) are required to estimate the power consumption and where optimization can be applied more profitably for energy savings. Energy efficiency has been the target of a great research effort all over the world; large traction systems are subject to a huge number of variables (mechanical and electrical), an intrinsic difficulty in the identification of system boundaries and related power flows, and a high degree of uncertainty, in particular on mechanical variables and parameters.

Here the sensitivity of the mechanical equations to the set of parameters describing the track and the trains was considered, together with the spread of the output variables for typical variations of the input parameters. Two different dispersions were applied (1% and 20%) to the assumed uniform distributions, the latter case including also integer parameters. Some considerations are derived in this case on the shape of the resulting probability density functions of the coefficients of the commonly used resistance to motion expression by Armstrong and Swift [2]. Moreover, alternative formulations and the dispersion of results are then considered for two trainsets used as the reference cases: the resulting dispersion is an indication of the expected variability in real cases [10].

It is easy to see that mechanical phenomena play a major role in terms of uncertainty due to the dispersion of the relevant parameters (e.g. non constant values or scarcely know values). So, to the aim of the electromechanical simulation and estimation of energy efficiency, the modeling of the electrical supply system and the related uncertainties are of less significance, in particular if the so called "hot path" (the catenary and the overall return circuit) is considered [11][12]. In this case the system is adequately modeled even if a simplifying approach by means of reduced number of conductors is followed [13][14].

6. REFERENCES

[1] B. P. Rochard and F. Schmid, "A review of methods to measure and calculate train resistances", *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, vol. 214, no. 4, pp. 185–199, 2000.

[2] D. S. Armstrong and P. H. Swift, "Lower energy technology. Part A, identification of energy use in multiple units. Report MR VS 077, British Rail Research, Derby, 20 July 1990.

[3] R. G. Gawthorpe, "Train drag reduction from simple design changes", *International Journal of Vehicle Design*, vol. 3, 1983.

[4] P. Lukaszewicz, "Running resistance - results and analysis of full-scale tests with passenger and freight trains in Sweden", *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, vol. 221, no. 2, pp. 183–193, 2007.

[5] W. J. Davis, Jr., "The tractive resistance of electric locomotives and cars", *General Electric Review*, 1926, vol. 29, pp. 2–24.

[6] A. J. Scibor-Rylsky, *Road vehicle aerodynamics*, London: Pentech Press, 1984.

[7] A. Hammit, *Aerodynamic forces on freight trains*, Springfield: National Technical Information Services, 1976, vol. I.

[8] V. Profillidis, "Railway Engineering", Avebury Technical: Ashgate Publishing Limited, Aldershot, 1995, pp. 213-224.

[9] AREMA Committee 24, Educational and Training, "Practical Guide To Railway Engineering", AREMA, 2003, pp. 56-62.

[10] G. Boschetti and A. Mariscotti, "Integrated Electromechanical Simulation of Traction Systems: Relevant Factors for the Analysis and Estimation of Energy Efficiency", Proc. of the 2nd Intern. Conf. on Electrical Systems for Aircraft, Railway and Ship Propulsion, Bologna, Italy, Oct. 12-14, 2012.

[11] A. Mariscotti, "Distribution of the traction return current in AC and DC electric railway systems", *IEEE Transactions on Power Delivery*, vol. 18 n. 4, Oct. 2003, pp. 1422-1432.

[12] A. Mariscotti, P. Pozzobon, "Determination of the Electrical Parameters of Railway Traction Lines: Calculation, Measurement and Reference Data", *IEEE Transactions on Power Delivery*, vol. 19 n. 4, Oct. 2004 pp. 1538-1546.

[13] A. Mariscotti, P. Pozzobon, M. Vanti, "Simplified modelling of 2x25 kV AT Railway System for the solution of low frequency and large scale problems", *IEEE Transactions on Power Delivery*, vol. 22 n. 1, Jan. 2007, pp. 296-301.

[14] H. Lee, C. Lee, G. Jang and S. Kwon, "Harmonic Analysis of the Korean High-Speed Railway Using the Eight-Port Representation Model", *IEEE Transactions on Power Delivery*, Vol. 21, no. 2, Apr. 2006, pp. 979-986.