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SIMULATION OF VARIOUS ARRANGEMENTS FOR THE MULTI LASER TRACKER SYSTEM

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Abstract: This paper presents a Monte Carlo simulation of the self-calibration for the multi laser tracker system (MLTS) which can track a retro-reflector mounted on the kinematical system (e.g. positioning stage, robot manipulator etc.). Four laser trackers build up the MLTS. In the first part of the study the required algorithms enabling the MLTS to measure the position of the retro-reflector are presented. The algorithms include the localization of the retro-reflector, the communication between the laser trackers, the tracking controller and the calculation of the Tool Centre Point (TCP) position. In the second part of this study a deeper analysis of the self-calibration algorithm is carried out. A Monte Carlo simulation shows that the quality of the parameter estimation highly depends on the optimal arrangement of the MTLS.

Keywords: optical measurement, multi laser tracker system, Monte Carlo simulation, self-calibration

1. INTRODUCTION

In the industrial environment at present day there is an increasing demand for more accurate automation and control systems. Capacitive as well as inductive position sensors achieve a limited measurement resolution and/or a limited working range. In contrast, optical sensors offer a high precision as well as a large working range. Furthermore, optical sensors measure the TCP position directly and do not need to be integrated in the process machine. An example for the mentioned optical sensors is a laser tracker.

A laser tracker is an instrument which opens the possibility of realizing a noncontact 3D measurement. It consists of a very precise laser interferometer and a beam deflection system. The development of a laser trackers began in the 1980's when Lau *et al.* [5] [6] used the laser tracking technique to determine the performance of a robot. This tracking system was implemented in a real-time system to identify the three-dimensional static and dynamic positioning accuracy of a robot end effector. The measuring volume of the described tracking system was approximately 3x3x3 meters and a maximum speed of 300 millimeters per second was reached. Parker and Mayer were the first to develop an optical laser tracking system using a 2-axis rotational galvanometer scanner [12]. Their system was used to measure the absolute position of a moving optical target which was mounted on the robot's manipulator. Takatsuji *et al.* presented a laser tracking interferometer system based on trilateration called distance-only-measurement (DOM). Using four laser interferometers detecting one target offers the advantage of a redundant measurement. Thus, the position of the interferometers and the initial position of the target can be calculated out of the multiple position measurements. The achieved measurement error was about 40 micrometers on a distance of one meter [7].

Due to high resolution of the interferometer the DOM method for accurate measurement is state of the art. Furthermore, the three-dimensional position of the reflector is detected directly and hence the Abbe's principle is not violated. To determine the absolute coordinate of the reflector the initial measurement lengths as well as the position of each laser tracker have to be known. These parameters are determined by self-calibration methods without a reference calibration machine. Using the self-calibration method the number of the free measurement points must be greater than the number of the unknown parameters, where the resulting overconstrained system allows the parameter identification using numerical techniques.

The quality of the self-calibration depends on various metrological arrangements of the system e.g. the number of measurements, the working range, the distance of the laser tracker among each other, the distance between the laser tracker and the measurement point etc. In [14] an optimal arrangement of four laser trackers is already clarified. Inherently there is no mathematical description for the various arrangements of measurement points and the laser trackers. Hence an analytical optimal configuration for the self-calibration is not given. A possible solution for this problem is using numerical simulation e.g. the Monte Carlo simulation technique.

Within the framework of the "Kompetenzdreieck Optische Mikrosystem (OPTIMI)", which is supported by the Ministry of Education and Research (BMBF), the Systems Analysis Group of Ilmenau University of the Technology has developed a MLTS for tracking a kinematical system shown in Figure 1 [1][2][3][4]. In section 2 the experimental set-up of the laser tracker system and the hard and software is presented. Section 3 describes the required tracking algorithms including the calculation of the TCP position using the trilateration method. Finally, some results of the Monte Carlo simulation to increase the accuracy of the calibration are presented.

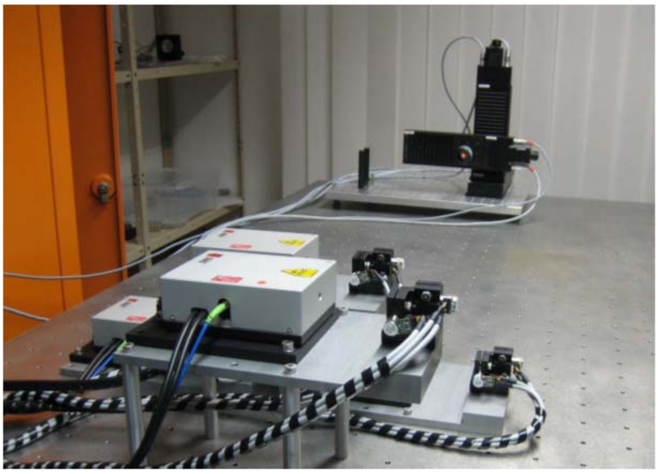

Figure 1: The developed MLTS for tracking kinematic

2. EXPERIMENTAL SET-UP

The laser tracker setup can be divided into the opto-electronic components, the supply electronic unit and the opto-electronic detection unit. The opto-electronic components include a Michelson interferometer as well as a galvanometer scanner. Both components are fixed on a base panel. The interferometer uses a stabilized *He-Ne* laser as laser beam source and exhibits a position resolution less than 0.1 nanometers [8] [9]. Beside the Michelson interferometer a position sensitive detection (PSD) unit is integrated to detect the motion of the retro-reflector. In this work a corner cube mirror is used as reflector [10]. The two opto-electronic components are arranged in the way that the emitted interferometer beam hits the galvanometer scanner. The galvanometer scanner consists of two beam deflection units and the laser beam is reflected by both of them [1]. Each deflection unit includes a mirror, a torque transducer, a high precision position sensor as well as an analogue servo closed loop control. The servo is used to control the angles and provide the needed current for the motors. The developed algorithms (localization, communication, tracking control and determination of the TCP position) are implemented in Matlab/Simulink® using C-code-s-functions. The Real-Time Workshop (RTW) is utilized to automatically generate C code, which is carried out by a modular Rapid Control Prototyping System of dSpace®. The algorithms work with a sample rate of 10 kHz.

3. TRACKING ALGORITHM

A. Tracking control and Localization

A closed loop tracking controller of the galvanometer scanner is designed. This allows the laser beam to follow the retro-reflector. The model-based control approach consists of a digital PID controller in combination with disturbance compensation. The controller has a high bandwidth. Moreover, the controller is stable and robust against external disturbances. For deeper information about the controller design the reader is referred to [1]. In case the retro-reflector is not found, the galvanometer scanner is controlled by a localization algorithm. This algorithm searches for the retro-reflector in the complete working range of the laser tracker. The proposed localization method is based on an Archimedean spiral which is derived in polar coordinates. The spiral is defined by three parameters. The radius r_0 is used to describe the maximum rotational angle of the galvanometer scanner in polar coordinates. The duration t_0 describes the rotation of the laser beam from the origin of the spiral to the predefined radius r_0. The number of rotations is given by the parameter ω_0. Due to the fact that the rotation of the galvanometer scanner is specified in angle coordinates, the designed Archimedean spiral in polar coordinates has to be transformed after its calculation [3].

The proposed localization algorithm is used for a single laser tracker. Due to the fact that four laser trackers are used in an MLTS, the developed algorithm is expanded by a communication channel between the four laser trackers. The single laser tracker is able to share the angle information and the retro-reflector position with all connected laser trackers and this accelerates the localization of the whole MLTS significantly. To realize the communication between the trackers, the position of each tracker is required in a global coordinate system. If at least two laser trackers hit the retro-reflector, they will become the transceiver and share their angle information with all connected laser trackers. Using this angle information, the radial distance between laser tracker and retro-reflector can be calculated. After the determination of this distance, the retro-reflector's position is computed in spherical coordinates, based on the angle information and the radial distance. As the calculated position of the retro-reflector is in the local coordinate system of the considered laser tracker, the position of the retro-reflector has to be transformed into the global coordinate system of MLTS. In the last step, the global position of the retro-reflector is provided to the other laser trackers of the MLTS [2].

B. Calculation of the TCP Position

The TCP position can be calculated by the relative length measurement of all interferometers. Figure 2 shows the four laser trackers (T_1, T_2, T_3 and T_4) and the position of the retro-reflector (X, Y, Z) in the Cartesian coordinate system. Every laser tracker has three position parameters $T_i = [x_i, y_i, z_i]^T$ in the Euclidian space and hence the number of parameters, which describe the whole multi-laser tracking system, is twelve.

To reduce the unknown parameters from twelve to six and simplify the calculation of the TCP position, we choose the configuration depicted in figure 2. The tracker T_1 is located in the origin of the coordinate system. Therefore, the position of the tracker T_1 is known with $[0,0,0]^T$. The tracker T_2 is located on the x-axis, the tracker T_3 is located on the x-y-plane and the position of the tracker T_4 is freely selectable. The parameter ℓ_i describes the radial distance of the four laser trackers and is defined as follows:

$$\ell_i = \ell_{0i} + \Delta\ell_i, i = 1 \ldots 4 \tag{1}$$

The parameter ℓ_{0i} represents the absolute distance after the initialization phase and the interferometer detects the relative radial distance $\Delta\ell_i$. If the positions of all trackers as well as all distances ℓ_{0i} are known, the TCP position can be determined by an approach called multi-lateration.

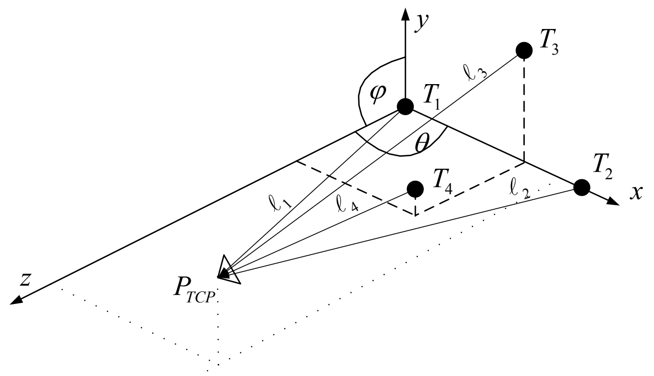

Figure 2: The position configuration of the MLTS

The four spherical equations in the Euclidian space are given by:

$$X^2 + Y^2 + Z^2 = \left(\ell_{01} + \Delta\ell_1\right)^2 \tag{2}$$

$$\left(X - x_2\right)^2 + Y^2 + Z^2 = \left(\ell_{02} + \Delta\ell_2\right)^2 \tag{3}$$

$$\left(X - x_3\right)^2 + \left(Y - y_3\right)^2 + Z^2 = \left(\ell_{03} + \Delta\ell_3\right)^2 \tag{4}$$

$$\left(X - x_4\right)^2 + \left(Y - y_4\right)^2 + \left(Z - z_4\right)^2 = \left(\ell_{04} + \Delta\ell_4\right)^2 \tag{5}$$

After the insertion of Eqn. (2) in Eqn. (3), Eqn. (4), Eqn. (5) and under the utilization of the relation $\ell_i = \ell_{0i} + \Delta\ell_i$, the following linear system of equations can be defined [13]:

$$2\underbrace{\begin{bmatrix} x_2 & 0 & 0 \\ x_3 & y_3 & 0 \\ x_4 & y_4 & z_4 \end{bmatrix}}_{M}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \ell_1^2 - \ell_2^2 - \ell_2^2 \\ \ell_1^2 - \ell_3^2 + x_3^2 + y_3^2 \\ \ell_1^2 - \ell_4^2 + x_4^2 + y_4^2 + z_4^2 \end{bmatrix} \tag{6}$$

It is possible to calculate the TCP position if the matrix M can be inverted:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{2} M^{-1} \begin{bmatrix} \ell_1^2 - \ell_2^2 - x_2^2 \\ \ell_1^2 - \ell_3^2 + x_3^2 + y_3^2 \\ \ell_1^2 - \ell_4^2 + x_4^2 + y_4^2 + z_4^2 \end{bmatrix} \tag{7}$$

The inverted matrix M is given by:

$$M^{-1} = \begin{bmatrix} \frac{1}{x_2} & 0 & 0 \\ -\frac{x_3}{x_2 y_3} & \frac{1}{y_3} & 0 \\ -\frac{x_4}{x_2 z_4} + \frac{x_3 y_4}{x_2 y_3 z_4} & -\frac{y_4}{y_3 z_4} & \frac{1}{z_4} \end{bmatrix} \tag{8}$$

The proposed model of the TCP position includes in total nine system parameters (ℓ_1, ℓ_2, ℓ_3, x_2, x_3, y_3, x_4, y_4 and z_4) because ℓ_4 can be derived from the other three initial distances. Due to the fact that the system parameters cannot be identified by experimental data with the needed precision, a calibration is indispensable. By using at least four laser trackers, the system parameters can be self-calibrated and thus a reference kinematic is not necessary [7], [11]. The self-calibration only requires N static measurements of the position of the retro-reflector. The absolute distance can be calculated as follows:

$$\ell_{ij} = \sqrt{\left(X_j - x_i\right)^2 + \left(Y_j - y_i\right)^2 + \left(Z_j - z_i\right)^2}, \quad with\ i = 1 \ldots 4;\ j = 1 \ldots N \tag{9}$$

Furthermore the parameter ℓ_{ij} is defined as follows:

$$\ell_{ij} = \ell_{0i} + \Delta\ell_{ij} \tag{10}$$

Eqn. 9 shows that for every point-measurement there are three unknown parameters (X_j, Y_j, Z_j) and four measured distances (ℓ_{1j}, ℓ_{2j}, ℓ_{3j}, ℓ_{4j}). Hence, the nonlinear system of equations is over-determined and the unknown parameters can be calculated by using a nonlinear numerical optimization method. The objective function of the optimization can be defined as follows:

$$R = min \sum_{j=1}^{N} \sum_{i=1}^{4} \left\{ f\left(\ell\right) \right\}^2 \tag{11}$$

Where the function $f(\ell)$ is the difference between Eqn. (9) and Eqn. (10) :

$$f\left(\ell\right) = \ell_{ij} - \left(\ell_{0i} + \Delta\ell_{ij}\right) \tag{12}$$

4. MONTE CARLO SIMULATION

After the self-calibration, an estimation of the nine system parameters is available. Furthermore, the real values are, within the simulation, well known. To evaluate the quality of the estimation, the maximum difference between the known and the estimated parameters is calculated:

$$Q = max\,|\,\vec{p}_{real} - \vec{p}^*\,|$$

First tests have shown that the optimization algorithm is able to converge to the designated minimum. To analyse the influence of noise and local minima, the starting point for optimization is set into an area of ±50 millimeters around the real parameters. The measurement noise of the interferometers has a Gaussian distribution with standard deviation of 1 micrometer. These assumptions match with the experimental setup.

The choice of the boundary values for calibration has influence on the accuracy of the self-calibration algorithm. Then, the number of measurement points (*aG*), the distance between the trackers among each other (*aTT*), the size of the working range for the reference points (*abTCP*), the distance between the laser trackers and measurement area (*aTTCP*) are investigated. In the Monte Carlo simulation these configuration values are varied within the following intervals:

- *aG*: [27, 64, 125, 256]
- *aTT* in [m]: [0.2, 0.3, 0.5, 0.8]
- *abTCP* in [m]: [0.3, 0.4, 0.5, 0.6, 0.7, 0.8]
- *aTTCP* in [m]: [0.5, 0.7, 0.9, 1.1, 1.3, 1.5, 1.7, 1.9]

For *aG* and *aTT* there exist four variations of boundary values, for *abTCP* six, and for *aTTCP* eight, respectively. The total number of the calibration set-ups N_C is the product of the configuration variations, which provide the different maximum combination options. N_C is calculated as follows:

$$N_C = aG \cdot aTT \cdot abTCP \cdot aTTCP = 4 \cdot 4 \cdot 6 \cdot 8 = 768 \tag{13}$$

For every combination the optimization is realized and the quality Q is determined. The current boundary and the optimization results are stored. For the evaluation the maximum aberration between the estimated parameters and the real parameters is compared with a threshold value, which is given as 50 micrometers. The combination is valid, if the maximum aberration is below the threshold value. In the case that all combinations are valid, a maximum number of the valid calibration N_{MAX} can be defined as the quotient between the total number of the calibration N_C and the number of the used intervals for every boundary:

aG $\rightarrow N_{MAX} = 192$
aTT $\rightarrow N_{MAX} = 192$
abTCP $\rightarrow N_{MAX} = 128$
aTTCP $\rightarrow N_{MAX} = 96$

The first test simulation is presented in figure 3 without measurement noise. As can be seen the number of valid calibrations is minimal below the maximum valid calibrations N_{MAX}. It can be stated that the utilized estimation algorithm appropriate for the selected boundary values.

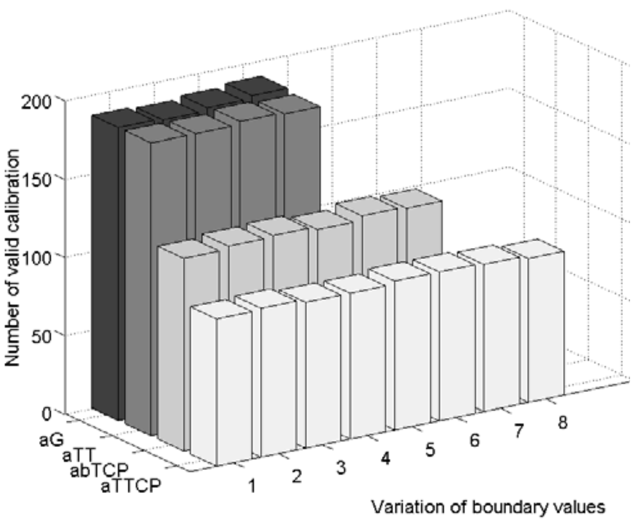

Figure 3: Variation of four boundary values and number of valid calibration without measurement noise, threshold value = *50µm*

The figure 4 depicts the simulation results with measurement noise. It can clearly be seen that the choice of the boundaries influences the quality of the parameter estimation. Only 508 of 768 calibrations are below the threshold value. The boundary *aTT* shows the strongest sensitivity regarding a valid calibration. Furthermore, a large distance between the trackers among each other increases the number of the valid calibrations significantly. A step-by-step rising of the *aTTCP* decreases the number of the valid calibrations. When, in contrast, increasing *aG* as well as *abTCP,* the number of the valid calibrations rises slightly.
To show the effect of the boundary values, we change the threshold value in the simulation from 50 micrometers to 10 micrometers. In figure 5 the variation of the boundary values and the related valid calibrations is depicted. Only 240 of 768 calibrations are below the threshold value**.** It can be seen that the number of successful optimizations varies extremely with the choice of the boundaries. The number of valid calibrations increases with the number of measurement points *aG*, as does the distance between the laser trackers *aTT*. The results improve with a wider area of reference points. Best performance for *aTTCP* is reached for a minimal distance between trackers and measurement interval. Further simulations show that non-equally spaced reference points have no negative impact on the estimation error, as long as every direction in space is used for measurement. If points only lie in one or two spatial directions, one cannot act on the assumption that all effects of the parameters are gathered.

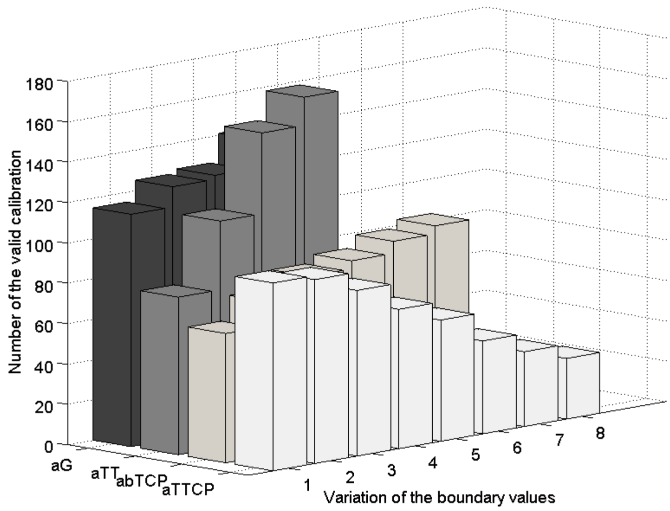

Figure 4: Variation of four boundary values and number of valid calibration with measurement noise, threshold value = *50µm*

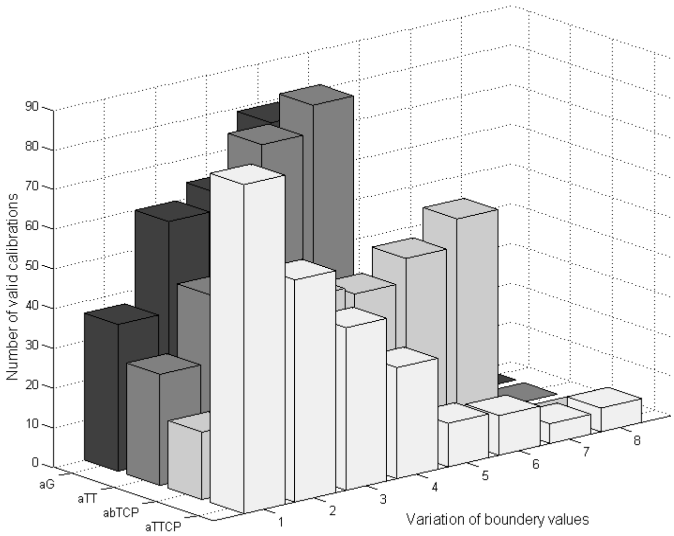

Figure 5: Variation of four boundary values and number of valid calibration with measurement noise, threshold value = *10µm*

In table 1 the estimated parameter values for non-uniform distribution is shown, following the rules mentioned above.

Table 1: Estimated Parameters [*mm*]

Parameter	ℓ_1	ℓ_2	ℓ_3	x_2	x_3
Real	786.8714	692.2187	706.3521	300.0000	150.0000
Diff.	0.0022	0.0032	0.0029	0.0005	0.0001
Parameter	y_3	x_4	y_4	z_4	
Real	160.0000	150.0000	50.0000	150.0000	
Diff.	0.0002	0.0005	0.0000	0.0008	

5. CONCLUSION

In the first part of this contribution, we present the developed multi laser tracker system and the required algorithms which open the possibility to track the TCP. The required algorithms include the tracking control, the localization of the retro reflector as well as communication between the four laser trackers. In the second part of this contribution a calculation method of the TCP is shown, which is used by the multi-lateration measurement. To calculate the TCP, nine system parameters have to be estimated. The estimation of these parameters is depending on four boundary values. To evaluate the sensitivity of the parameters, a Monte Carlo simulation is presented. The simulation shows that the choice of the boundary values affects the estimation significantly. To achieve good estimation results, the distance between the laser trackers among each other as well as the size of the working range of the reference points should be set sufficiently large. Furthermore, a minimal distance between laser tracker and the measurement area is recommended and a big number of measurement points ensure the quality of the parameter estimation.

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