XX IMEKO World Congress
Metrology for Green Growth
September 9–14, 2012, Busan, Republic of Korea

STATE-SPACE FORMULATION FOR THE NODAL LOAD OBSERVER FOR SMART ELECTRICAL GRIDS WITH IMPERFECT MEASUREMENT INFRASTRUCTURE

W. Heins [1,2], *G. Gewiss* [3], *C. Bohn* [2] *and H.-P. Beck* [4]

[1] Physikalisch-Technische Bundesanstalt, Berlin, Germany, wiebke.heins@ptb.de
[2] Institut für Elektrische Informationstechnik, TU Clausthal, Clausthal-Zellerfeld, Germany, {heins,bohn}@iei.tu-clausthal.de
[3] N/A, Goslar, Germany, gunner.gewiss@gmx.de
[4] Energie-Forschungszentrum Niedersachsen, Goslar, Germany, vorsitzender@efzn.de

Abstract: This paper presents a discrete-time state-space formulation of the nodal load observer for middle-voltage distribution grids, developed at Clausthal University of Technology ([1], [2]). The nodal load observer is a procedure for the on-line reconstruction of unknown load and generation in a middle-voltage distribution grid based on measurements of complex nodal voltage or line current. Conceptually it is inspired by the idea of applying the state observer concept, as known from control system theory for general dynamic systems, to a set of algebraic equations. In [1], a first draft of the method was developed. In the present paper, an alternative formulation is presented, which leads to a discrete-time version of the method and reveals that in fact the nodal load observer can be interpreted as a disturbance observer structure and is closely related to dynamic state estimation, which allows for system-theory-based general analysis and extensions.

Keywords: Distribution grid, dynamic state estimation, state observation, nodal load estimation, nodal load observer, disturbance observer, Kalman filter.

1. INTRODUCTION

The increasing number of decentralized generation units on the distribution level of power grids causes new dynamics in the electric grid which need to be observed through measurement or estimation in order to ensure safe operating conditions. State estimation methods as known from the transmission grid level [3] cannot be used for the distribution grid level without significant modifications. On the one hand, this is due to usually unbalanced conditions on distribution grid level which often demand the use of three-phase-models [4]. On the other hand, on distribution grid level, a high number of nodes has to be observed through very few measurements, while on the transmission grid level a high number of measurements for few nodes even allows for robust state estimation including bad data detection [3].

In approaches based on static state estimation, missing critical measurements are often replaced by pseudo-measurements such as load and generation forecasts, which differ considerably from the actual real-time values and therefore affect the accuracy of the estimated quantities. Special attention is therefore paid to the improvement of forecasts and procedures which are able to estimate active and reactive power on-line ([5], [6]).

In [1], a procedure is derived that is able to reconstruct missing active and reactive power for nodal load and generation from the available measurements of complex nodal voltages. Load and generation forecasts are used to improve convergence speed. Estimated active and reactive nodal power can then in turn be used to reconstruct missing values of complex nodal voltage. Simulations for a small middle-voltage grid show promising results also for some cases in which not all complex nodal voltages are measured.

The remainder of this paper is organized as follows. In Section 2, the disturbance observer as known from control system theory is recapped briefly. The grid model on which the nodal load observer is based is described in Section 3. In Section 4, the design of a disturbance observer for the grid model is explained and its relation to known state estimation approaches is pointed out. A Kalman filter is applied to the resulting state-space representation in Section 5. First simulation results are shown in Section 6. Summary and conclusions are given in Section 7.

2. DISTURBANCE OBSERVER

For the disturbance observer approach in this paper, a linear time-varying discrete-time dynamic system

$$\begin{aligned} \boldsymbol{x}_{\text{sys}}(k+1) &= \boldsymbol{A}_{\text{sys}}(k)\boldsymbol{x}_{\text{sys}}(k) + \boldsymbol{B}_{\text{sys}}(k)(\boldsymbol{u}(k)+\boldsymbol{z}(k)), \\ \boldsymbol{y}(k) &= \boldsymbol{C}_{\text{sys}}(k)\boldsymbol{x}_{\text{sys}}(k) + \boldsymbol{D}_{\text{sys}}(k)(\boldsymbol{u}(k)+\boldsymbol{z}(k)) \end{aligned} \tag{1}$$

in state-space representation is considered. Here, $\boldsymbol{x}_{\text{sys}} \in \mathbb{R}^{n_{\text{sys}}}$ denotes the state, $\boldsymbol{u} \in \mathbb{R}^m$ the input and $\boldsymbol{y} \in \mathbb{R}^r$ the output vector, and $\boldsymbol{A}_{\text{sys}}, \boldsymbol{B}_{\text{sys}}, \boldsymbol{C}_{\text{sys}}, \boldsymbol{D}_{\text{sys}}$ time-dependent matrices of appropriate dimensions. A deterministic disturbance $\boldsymbol{z} \in \mathbb{R}^m$ affects the input of the system.

For a non-decaying disturbance, an identity observer for this system that does not take into account the dynamics of the disturbance will not be able to reconstruct the system states properly. Therefore, an appropriate disturbance model is added to the system dynamics by state augmentation and an identity observer for the resulting overall system is designed, which will then be able to reconstruct both the system states and the disturbance states. It is assumed that the disturbance can be modelled through a possibly time-varying, but unforced linear system of the form

$$\begin{aligned} \boldsymbol{x}_\mathrm{s}(k+1) &= \boldsymbol{A}_\mathrm{s}(k)\boldsymbol{x}_\mathrm{s}(k), \\ \boldsymbol{z}(k) &= \boldsymbol{C}_\mathrm{s}(k)\boldsymbol{x}_\mathrm{s}(k). \end{aligned} \tag{2}$$

Then, through definition of

$$\boldsymbol{x}(k) := \begin{bmatrix} \boldsymbol{x}_\mathrm{sys}(k) \\ \boldsymbol{x}_\mathrm{s}(k) \end{bmatrix}, \tag{3}$$

$$\boldsymbol{A}(k) := \begin{bmatrix} \boldsymbol{A}_\mathrm{sys}(k) & \boldsymbol{B}_\mathrm{sys}(k)\boldsymbol{C}_\mathrm{s}(k) \\ \boldsymbol{0} & \boldsymbol{A}_\mathrm{s}(k) \end{bmatrix}, \ \boldsymbol{B}(k) := \begin{bmatrix} \boldsymbol{B}_\mathrm{sys}(k) \\ \boldsymbol{0} \end{bmatrix}, \tag{4}$$

and

$$\boldsymbol{C}(k) := \begin{bmatrix} \boldsymbol{C}_\mathrm{sys}(k) & \boldsymbol{D}_\mathrm{sys}(k)\boldsymbol{C}_\mathrm{s}(k) \end{bmatrix}, \ \boldsymbol{D}(k) := \boldsymbol{D}_\mathrm{sys}(k), \tag{5}$$

the overall system is described by

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}(k)\boldsymbol{x}(k) + \boldsymbol{B}(k)\boldsymbol{u}(k), \\ \boldsymbol{y}(k) &= \boldsymbol{C}(k)\boldsymbol{x}(k) + \boldsymbol{D}(k)\boldsymbol{u}(k). \end{aligned} \tag{6}$$

An identity observer for (6) is then given by

$$\begin{aligned} \hat{\boldsymbol{x}}(k+1) &= \boldsymbol{A}(k)\hat{\boldsymbol{x}}(k) + \boldsymbol{B}(k)\boldsymbol{u}(k) + \boldsymbol{L}(k)(\boldsymbol{y}(k) - \hat{\boldsymbol{y}}(k)), \\ \hat{\boldsymbol{y}}(k) &= \boldsymbol{C}(k)\hat{\boldsymbol{x}}(k) + \boldsymbol{D}(k)\boldsymbol{u}(k), \end{aligned} \tag{7}$$

where $\hat{\boldsymbol{x}}$ denotes the estimated overall state vector and $\hat{\boldsymbol{y}}$ the estimated output vector, with time-varying observer gain $\boldsymbol{L}(k)$ such that the estimation error $\boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k)$ decays for $k \to \infty$, i.e. the system

$$\boldsymbol{x}(k+1) - \hat{\boldsymbol{x}}(k+1) = (\boldsymbol{A}(k) - \boldsymbol{L}(k)\boldsymbol{C}(k))(\boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k)) \tag{8}$$

is asymptotically stable. If such an $\boldsymbol{L}(k)$ exists depends on the observability of (6). Definitions and a detailed discussion for the case of general linear discrete-time time-varying systems can be found, for example, in [7]. The observer gain $\boldsymbol{L}(k)$ is not unique and can be designed according to several desired performance objectives. The system from (7) is called disturbance observer since not only the system states, but also the states of the disturbance model are reconstructed.

3. GRID MODEL BASED ON POWER FLOW EQUATIONS

The nodal load observer in [1] uses a simple grid model which is based on a nodal formulation of the power flow equations. Assuming stationary or quasi-stationary conditions, grid quantities are represented as complex phasors. A single-phase representation of the grid is used for modelling under the assumption of sufficiently balanced conditions. This leads to a set of algebraic equations relating the vectors of injected nodal active and reactive power $\boldsymbol{P}$ and $\boldsymbol{Q}$ to complex nodal voltages $\underline{\boldsymbol{V}}$ via the bus admittance matrix $\underline{\boldsymbol{Y}}$. It is furthermore assumed that $\underline{\boldsymbol{Y}}$ is constant, so admittances as well as grid topology do not change over time, shunt conductances are not neglected and, apart from the slack node, every node can be considered or approximated as P-Q. Rectangular coordinates are chosen for the real calculation of the complex values. With the, now real, quantities

$$\boldsymbol{S} := \begin{bmatrix} \boldsymbol{P} \\ \boldsymbol{Q} \end{bmatrix}, \ \boldsymbol{V} := \begin{bmatrix} \mathrm{Re}(\underline{\boldsymbol{V}}) \\ \mathrm{Im}(\underline{\boldsymbol{V}}) \end{bmatrix}, \ \boldsymbol{Y} := \begin{bmatrix} \mathrm{diag}(\mathrm{Re}(\underline{\boldsymbol{Y}})) & \text{-}\mathrm{diag}(\mathrm{Im}(\underline{\boldsymbol{Y}})) \\ \mathrm{diag}(\mathrm{Im}(\underline{\boldsymbol{Y}})) & \mathrm{diag}(\mathrm{Re}(\underline{\boldsymbol{Y}})) \end{bmatrix} \tag{9}$$

and

$$\boldsymbol{M}(\boldsymbol{V}) := 3\begin{bmatrix} \mathrm{diag}(\mathrm{Re}(\underline{\boldsymbol{V}})) & \mathrm{diag}(\mathrm{Im}(\underline{\boldsymbol{V}})) \\ \mathrm{diag}(\mathrm{Im}(\underline{\boldsymbol{V}})) & -\mathrm{diag}(\mathrm{Re}(\underline{\boldsymbol{V}})) \end{bmatrix}, \tag{10}$$

this gives the nonlinear relation

$$\boldsymbol{S} = \boldsymbol{M}(\boldsymbol{V})\,\boldsymbol{Y}\,\boldsymbol{V}. \tag{11}$$

One node, without loss of generality labeled with 0, is chosen as slack node. Its phase angle is chosen as reference angle and set to zero, while its magnitude is assumed to be known. Calculation of nodal active and reactive power at the slack node are omitted. This leads to

$$\boldsymbol{S}^{(0)} = \boldsymbol{M}(\boldsymbol{V}^{(0)})\left(\boldsymbol{Y}^{(0,0)}\,\boldsymbol{V}^{(0)} + \boldsymbol{Y}_\mathrm{slack}\begin{bmatrix} \mathrm{Re}(\underline{V}_0) \\ \mathrm{Im}(\underline{V}_0) \end{bmatrix}\right), \tag{12}$$

where superscripts (0) and $(0,0)$ mean that rows resp. columns and rows corresponding to the slack node have been removed from the original vector or matrix, $\underline{V}_0$ denotes the complex nodal voltage at the slack node, and $\boldsymbol{Y}_\mathrm{slack}$ is a matrix containing the columns which have been deleted from $\boldsymbol{Y}$ without the former diagonal element.

Under the described assumptions, in every measurement instant k the relation between nodal active and reactive power $\boldsymbol{S}(k)$ and complex nodal voltage, represented by $\boldsymbol{V}(k)$, is given through

$$\boldsymbol{S}^{(0)}(k) = \boldsymbol{M}(\boldsymbol{V}^{(0)}(k))\left(\boldsymbol{Y}^{(0,0)}\,\boldsymbol{V}^{(0)}(k) + \boldsymbol{Y}_\mathrm{slack}\begin{bmatrix} \mathrm{Re}(\underline{V}_0(k)) \\ \mathrm{Im}(\underline{V}_0(k)) \end{bmatrix}\right). \tag{13}$$

For the nodal load observer, it is necessary to find an expression for $\boldsymbol{V}^{(0)}$ in terms of $\boldsymbol{S}^{(0)}$. Since an explicit representation does not exist, Equation (13) is usually solved with iterative methods. In [1], it is assumed that nodal voltages do not change significantly between two measurement instants. Therefore, if an estimate $\hat{\boldsymbol{V}}^{(0)}(k-1)$ is known, (13) can be approximated by

$$\boldsymbol{S}^{(0)}(k) \approx \boldsymbol{M}(\hat{\boldsymbol{V}}^{(0)}(k-1))\left(\boldsymbol{Y}^{(0,0)}\,\boldsymbol{V}^{(0)}(k) + \boldsymbol{Y}_\mathrm{slack}\begin{bmatrix} \mathrm{Re}(\underline{V}_0(k)) \\ \mathrm{Im}(\underline{V}_0(k)) \end{bmatrix}\right), \tag{14}$$

where $\boldsymbol{M}(\hat{\boldsymbol{V}}^{(0)}(k-1))$ now denotes a known quantity. For $k=1$, a flat start (voltage magnitudes are set to the nominal value and phase angles equal to zero) is applied. Regularity of $\boldsymbol{M}(\hat{\boldsymbol{V}}^{(0)}(k-1))$ and $\boldsymbol{Y}^{(0,0)}$ is usually verified easily, such that

$$\boldsymbol{V}^{(0)}(k) \approx \left(\boldsymbol{Y}^{(0,0)}\right)^{-1}\left(\left(\boldsymbol{M}(\hat{\boldsymbol{V}}^{(0)}(k-1))\right)^{-1}\boldsymbol{S}^{(0)}(k) - \boldsymbol{Y}_\mathrm{slack}\begin{bmatrix} \mathrm{Re}(\underline{V}_0(k)) \\ \mathrm{Im}(\underline{V}_0(k)) \end{bmatrix}\right), \tag{15}$$

assuming that the complex nodal voltage at the slack node is known or measured.

4. DISTURBANCE OBSERVER FOR MIDDLE-VOLTAGE DISTRIBUTION GRIDS

Starting from Equation (15), a disturbance observer is derived in order to reconstruct unknown nodal active and reactive power from available measurements of complex nodal voltages. In [1], also an alternative formulation is given which uses line currents instead of nodal voltages. Since the argumentation is basically the same, only nodal voltages are considered here.

It is assumed that nodal active and reactive power is either measured or forecasted. Denoting as $\boldsymbol{S}_{\mathrm{m}}^{(0)}$ the vector of measured active and reactive power, as $\boldsymbol{S}_{\mathrm{nm}}^{(0)}$ the vector of unknown exact values for the remaining nodes and as $\tilde{\boldsymbol{S}}_{\mathrm{nm}}^{(0)}$ the vector of corresponding forecasts, $\boldsymbol{S}^{(0)}$ can be expressed as

$$\boldsymbol{S}^{(0)} = \boldsymbol{D}_{\mathrm{m}}\boldsymbol{S}_{\mathrm{m}}^{(0)} + \boldsymbol{D}_{\mathrm{nm}}\boldsymbol{S}_{\mathrm{nm}}^{(0)}, \tag{16}$$

where $\boldsymbol{D}_{\mathrm{m}}$ and $\boldsymbol{D}_{\mathrm{nm}}$ are possibly non-square matrices with at most one entry equal to 1 in every row, and zeros else. If for some node, neither a measurement nor a forecast is available, the corresponding entry in $\tilde{\boldsymbol{S}}_{\mathrm{nm}}^{(0)}$ is set to zero.

The difference between the actual real-time values $\boldsymbol{S}_{\mathrm{nm}}^{(0)}$ and the forecasts $\tilde{\boldsymbol{S}}_{\mathrm{nm}}^{(0)}$ will be denoted as

$$\Delta\boldsymbol{S}_{\mathrm{nm}}^{(0)} = \boldsymbol{S}_{\mathrm{nm}}^{(0)} - \tilde{\boldsymbol{S}}_{\mathrm{nm}}^{(0)}. \tag{17}$$

Assuming noise-free measurement, it then holds that

$$\boldsymbol{S}^{(0)} = \boldsymbol{D}_{\mathrm{m}}\boldsymbol{S}_{\mathrm{m}}^{(0)} + \boldsymbol{D}_{\mathrm{nm}}\left(\Delta\boldsymbol{S}_{\mathrm{nm}}^{(0)} + \tilde{\boldsymbol{S}}_{\mathrm{nm}}^{(0)}\right). \tag{18}$$

Let $\boldsymbol{V}_{\mathrm{m}}^{(0)}$ denote the vector of measured complex nodal voltages in rectangular coordinates, and $\boldsymbol{C}_{\mathrm{m}}$ the corresponding matrix with exactly one entry equal to 1 in every row, and zeros else, such that

$$\boldsymbol{V}_{\mathrm{m}}^{(0)} = \boldsymbol{C}_{\mathrm{m}}\boldsymbol{V}^{(0)}. \tag{19}$$

Then, with (18) and (19), from Equation (15) follows

$$\begin{aligned}
&\boldsymbol{V}_{\mathrm{m}}^{(0)}(k) \approx \\
&\quad \boldsymbol{C}_{\mathrm{m}}\left(\boldsymbol{Y}^{(0,0)}\right)^{-1}\left(\boldsymbol{M}(\hat{\boldsymbol{V}}^{(0)}(k-1))\right)^{-1}\boldsymbol{D}_{\mathrm{nm}}\Delta\boldsymbol{S}_{\mathrm{nm}}^{(0)}(k) \\
&\quad + \boldsymbol{C}_{\mathrm{m}}\left(\boldsymbol{Y}^{(0,0)}\right)^{-1}\left(\boldsymbol{M}(\hat{\boldsymbol{V}}^{(0)}(k-1))\right)^{-1}\left(\boldsymbol{D}_{\mathrm{m}}\boldsymbol{S}_{\mathrm{m}}^{(0)}(k) + \boldsymbol{D}_{\mathrm{nm}}\tilde{\boldsymbol{S}}_{\mathrm{nm}}^{(0)}(k)\right) \\
&\quad - \boldsymbol{C}_{\mathrm{m}}\left(\boldsymbol{Y}^{(0,0)}\right)^{-1}\boldsymbol{Y}_{\mathrm{slack}}\begin{bmatrix}\mathrm{Re}(\underline{V}_0(k)) \\ \mathrm{Im}(\underline{V}_0(k))\end{bmatrix}.
\end{aligned} \tag{20}$$

Now define

$$\boldsymbol{y}(k) := \boldsymbol{V}_{\mathrm{m}}^{(0)}(k) + \boldsymbol{C}_{\mathrm{m}}\left(\boldsymbol{Y}^{(0,0)}\right)^{-1}\boldsymbol{Y}_{\mathrm{slack}}\begin{bmatrix}\mathrm{Re}(\underline{V}_0(k)) \\ \mathrm{Im}(\underline{V}_0(k))\end{bmatrix} \tag{21}$$

as output vector (measured quantities),

$$\boldsymbol{u}(k) := \boldsymbol{D}_{\mathrm{m}}\boldsymbol{S}_{\mathrm{m}}^{(0)}(k) + \boldsymbol{D}_{\mathrm{nm}}\tilde{\boldsymbol{S}}_{\mathrm{nm}}^{(0)}(k) \tag{22}$$

as input vector (known quantities), and

$$\boldsymbol{z}(k) := \boldsymbol{D}_{\mathrm{nm}}\Delta\boldsymbol{S}_{\mathrm{nm}}^{(0)}(k) \tag{23}$$

as unknown, yet deterministic disturbance. With

$$\boldsymbol{D}(k) := \boldsymbol{C}_{\mathrm{m}}\left(\boldsymbol{Y}^{(0,0)}\right)^{-1}\left(\boldsymbol{M}(\hat{\boldsymbol{V}}^{(0)}(k-1))\right)^{-1}, \tag{24}$$

Equation (20) becomes

$$\boldsymbol{y}(k) = \boldsymbol{D}(k)(\boldsymbol{u}(k) + \boldsymbol{z}(k)). \tag{25}$$

A simple dynamic behavior is assumed for $\boldsymbol{z}(k)$ via

$$\begin{aligned}\boldsymbol{x}_{\mathrm{s}}(k+1) &= \boldsymbol{x}_{\mathrm{s}}(k), \\ \boldsymbol{z}(k) &= \boldsymbol{D}_{\mathrm{nm}}\boldsymbol{x}_{\mathrm{s}}(k).\end{aligned} \tag{26}$$

This is justified if the used forecasts reflect the qualitative behavior of nodal active and reactive power (for example, periodic oscillations during the day), such that the error $\Delta\boldsymbol{S}_{\mathrm{nm}}^{(0)}(k)$ is approximately constant. Other disturbance models can be useful for other cases, but are not considered in this paper. Since no plant states have to be considered, the augmented state vector is given directly through $\boldsymbol{x}(k) = \boldsymbol{x}_{\mathrm{s}}(k)$. Building the overall model from (25) and (26) and defining $\boldsymbol{C}(k) := \boldsymbol{D}(k)\boldsymbol{D}_{\mathrm{nm}}$ gives

$$\begin{aligned}\boldsymbol{x}(k+1) &= \boldsymbol{x}(k), \\ \boldsymbol{y}(k) &= \boldsymbol{C}(k)\boldsymbol{x}(k) + \boldsymbol{D}(k)\boldsymbol{u}(k).\end{aligned} \tag{27}$$

Designing an observer for (27) with observer gain $\boldsymbol{L}(k)$ as described in Section 2 now gives a procedure for the reconstruction of $\boldsymbol{S}_{\mathrm{nm}}^{(0)}$. Once an estimate $\hat{\boldsymbol{S}}_{\mathrm{nm}}^{(0)}(k)$ for $\boldsymbol{S}_{\mathrm{nm}}^{(0)}(k)$ is known, an estimate $\hat{\boldsymbol{V}}^{(0)}(k)$ for $\boldsymbol{V}^{(0)}(k)$ is given through (15). Since the complex voltage at the slack node is measured, based on these estimates all relevant quantities in the grid can be reconstructed.

Accuracy of estimates depends on convergence speed and robustness of the applied observer and accuracy of the approximation introduced in Equation (14). Regarding the latter, it can be shown that the applied approximation can be interpreted as a fixed-point iteration which is carried out for only one step. So, if the iteration converges fast enough for every k, a reasonable accuracy of the estimates $\hat{\boldsymbol{V}}^{(0)}(k)$ as well as boundedness of $\boldsymbol{C}(k)$ and $\boldsymbol{D}(k)$ is given.

In order to find an $\boldsymbol{L}(k)$ such that asymptotic stability of the observer error dynamics (see Equation (8)) is given, observability of (27) is crucial. Note however, that observability is meant here in terms of system theory, which might differ from the understanding of observability in the context of static state estimation. While in system theory even a single-output system with a state vector of dimension greater than one might be observable due to convenient system dynamics or time-variance of several parameters, in static state estimation usually a crucial condition for observability is that at least as many measurements as unknowns are available ([3], [8]). A detailed analysis of conditions based on grid topology for observability in terms of system theory will be part of future research.

From Equation (27) it can be seen that the resulting grid model is conceptionally similar to models used in dynamic state estimation (for a review of the latter see [9]). Both dynamic state estimation and the here proposed disturbance observer lead to models of the general form

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{x}(k), \\ \boldsymbol{y}(k) &= \boldsymbol{f}(\boldsymbol{x}(k)), \end{aligned} \tag{28}$$

or, considering general process and measurement noise,

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{x}(k) + \boldsymbol{w}_1(k), \\ \boldsymbol{y}(k) &= \boldsymbol{f}(\boldsymbol{x}(k)) + \boldsymbol{w}_2(k). \end{aligned} \tag{29}$$

The main difference lies in the choice of the quantities that are to be estimated. In state estimation, usually the so called “state of the electric power system”, which is typically the vector of complex nodal voltages either represented by voltage magnitudes and phase angles (polar coordinates) or in rectangular coordinates, is to be reconstructed [9]. In the disturbance observer approach considered here, basically nodal active and reactive power take the place of the state vector.

Choosing nodal active and reactive power as state vectors is not a new idea. In [6], for example, static state estimation techniques are used to estimate and correct nodal loads. The here proposed disturbance observer therefore can be interpreted as a corresponding dynamic estimation technique for load and generation data.

5. KALMAN FILTER DESIGN

So far, process and measurement noise have not been considered in the model. A direct approach for a convenient estimation procedure in this case is for example a Kalman predictor [10]. If the inputs and measurements at instant $k+1$ are available for calculation of the state estimate $\hat{\boldsymbol{x}}_s(k+1)$, the more accurate Kalman filter can be applied. For clarification, it should be noted that the Kalman filter is not an observer in the classical sense, nonetheless conditions for convergence of the estimation error and estimation error covariance in terms of observability remain the same [11].

Both approaches assume an underlying state-space model of the form

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}(k)\boldsymbol{x}(k) + \boldsymbol{B}(k)\boldsymbol{u}(k) + \boldsymbol{w}_1(k), \\ \boldsymbol{y}(k) &= \boldsymbol{C}(k)\boldsymbol{x}(k) + \boldsymbol{D}(k)\boldsymbol{u}(k) + \boldsymbol{w}_2(k), \end{aligned} \tag{30}$$

with measurement and process noise $\boldsymbol{w}_1(k)$ and $\boldsymbol{w}_2(k)$ with zero mean and covariance matrices $\boldsymbol{Q}(k)$ and $\boldsymbol{R}(k)$. It is furthermore assumed that

$$\mathrm{E}(\boldsymbol{w}_1(k)\boldsymbol{w}_1(j)^{\mathrm{T}}) = \mathrm{E}(\boldsymbol{w}_2(k)\boldsymbol{w}_2(j)^{\mathrm{T}}) = 0 \tag{31}$$

for $k \neq j$ and

$$\mathrm{E}(\boldsymbol{w}_1(k)\boldsymbol{w}_2(j)^{\mathrm{T}}) = \mathrm{E}(\boldsymbol{w}_2(k)\boldsymbol{w}_1(j)^{\mathrm{T}}) = 0 \tag{32}$$

for all k and j. In the Kalman filter approach, estimated states and outputs are then calculated via

$$\begin{aligned} &\hat{\boldsymbol{x}}(k+1) = \\ &\quad \boldsymbol{A}(k)\hat{\boldsymbol{x}}(k) + \boldsymbol{B}(k)\boldsymbol{u}(k) + \boldsymbol{K}(k+1)(\boldsymbol{y}(k+1) \\ &\quad - \boldsymbol{C}(k+1)(\boldsymbol{A}(k)\hat{\boldsymbol{x}}(k) + \boldsymbol{B}(k)\boldsymbol{u}(k)) - \boldsymbol{D}(k+1)\boldsymbol{u}(k+1)), \\ &\hat{\boldsymbol{y}}(k+1) = \boldsymbol{C}(k+1)\hat{\boldsymbol{x}}(k+1) + \boldsymbol{D}(k+1)\boldsymbol{u}(k+1), \end{aligned} \tag{33}$$

where the Kalman gain $\boldsymbol{K}(k+1)$ and the covariance matrix of the estimation error $\boldsymbol{P}(k+1)$ at instant $k+1$ are given as

$$\begin{aligned} &\boldsymbol{K}(k+1) = \\ &\quad (\boldsymbol{A}(k)\boldsymbol{P}(k)\boldsymbol{A}(k)^{\mathrm{T}} + \boldsymbol{Q}(k))\boldsymbol{C}(k+1)^{\mathrm{T}} \\ &\quad [\boldsymbol{C}(k+1)(\boldsymbol{A}(k)\boldsymbol{P}(k)\boldsymbol{A}(k)^{\mathrm{T}} + \boldsymbol{Q}(k))\boldsymbol{C}(k+1)^{\mathrm{T}} + \boldsymbol{R}(k+1)]^{-1}, \end{aligned} \tag{34}$$

$$\boldsymbol{P}(k+1) = (\mathbf{J} - \boldsymbol{K}(k+1)\boldsymbol{C}(k+1))(\boldsymbol{A}(k)\boldsymbol{P}(k)\boldsymbol{A}(k)^{\mathrm{T}} + \boldsymbol{Q}(k)), \tag{35}$$

where $\mathbf{J}$ denotes the identity matrix of appropriate dimension and it holds $\boldsymbol{P}(0) = \mathrm{E}((\boldsymbol{x}(0) - \hat{\boldsymbol{x}}(0))(\boldsymbol{x}(0) - \hat{\boldsymbol{x}}(0))^{\mathrm{T}})$.

Assuming that all assumptions are correct, the Kalman filter is optimal regarding the covariance of the estimation error $\boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k)$. Nevertheless, since the matrices $\boldsymbol{P}(0)$, $\boldsymbol{Q}(k)$ and $\boldsymbol{R}(k)$ are usually not known exactly, they are rather used as setting parameters and only a suboptimal performance of the filter can be expected. For a short review and direct comparison of the Kalman predictor and Kalman filter equations see [12].

In the case considered here, the filter equations are applied using the definitions given in Section 4, which yields $\boldsymbol{A}(k) = \mathbf{J}$ and $\boldsymbol{B}(k) = \mathbf{0}$ for all k. If other disturbance models are engaged, these matrices have to be chosen accordingly.

6. COMPUTATION EXAMPLE

In order to validate the derived model from (27) and the proposed algorithm in principle, a simple example is calculated for a small middle-voltage (20 kV) distribution grid consisting of nine nodes and ten lines, which already has been used for simulation studies in [1]. The network graph is shown in Figure 1. At node no. 0, the grid is connected to the superior grid level. This node is chosen as slack node. At node no. 2, a generation unit is applied. The remaining nodes are provided with loads. A Kalman filter for linear time-varying systems has been applied in a straightforward fashion. A complete set of data for nodal active and reactive power and complex nodal voltage has been generated in [1] via power flow calculations in Matlab based on the Newton-Raphson method. Values are given every 15 minutes for the course of one day. Validation of this simulation algorithm has been carried out against measurements at a physical grid simulator of the University of Applied Science of Oldenburg/Ostfriesland/Wilhelmshaven, Germany, such that the data set can be considered realistic. Line admittances are given through the parameters in Table 1.

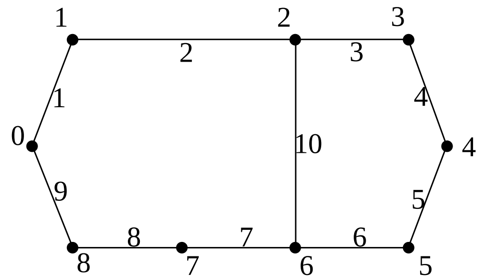

Figure 1: Network graph

Figures 2 shows results for node no. 3 for measurement of complex nodal voltages at nodes no. 2, 4, 6 and 8. Very simple forecasts are used for all nodes with a load. Active and reactive power injection at node no. 2 are given (through measurement, for example). Values are shown in Figure 3. The results show that in this case the proposed procedure is (in principle) able to reconstruct unknown active and reactive power even though no measurements are taken at the node and characteristics of active and reactive power at the neighboring node no. 2 are significantly different. Nodal voltages are reconstructed with good accuracy, as Figure 4 shows. Nonetheless there are some significant deviations in load estimations between minute 70 and 90. There might be several reasons for this, one of which is the inaccurate disturbance model. It forces the filter to correct the estimation, but due to poor observability properties, convergence of the filter is very slow. In comparison, in regions where the disturbance model is more accurate, convergence is significantly better.

	Lines 1,2,6-9	**Lines 3-5,10**
Length in km	4	8
X **in** Ω	3	1.5
R **in** Ω	2.6	1.3
C **in** nF	28	19.25

Table 1: Line parameters

7. SUMMARY AND CONCLUSION

A discrete-time state-space representation of the nodal load observer presented in [1] has been derived. This reveals the relation of the procedure to state estimation techniques and offers possibilities for further extensions of the method. A computation example shows the general ability of the method to reconstruct unknown values at grid nodes where no measurements are available.

Nonetheless, many points still have to be investigated. Performance of the method could be improved through application of more intrigued filtering techniques. Measurements at the slack node could be better exploited by reformulation of the grid model. More complex disturbance models could improve performance and observability. The effect of noise on the method has to be investigated in detail. This as well as a thorough analysis of observability with respect to network topology and an extension of the method to a three-phase formulation are part of future research.

8. ACKNOWLEDGMENT

The research leading to the results described in this paper is part of the European Metrology Research Program (EMRP), which is jointly funded by the EMRP participating countries within EURAMET and the European Union.

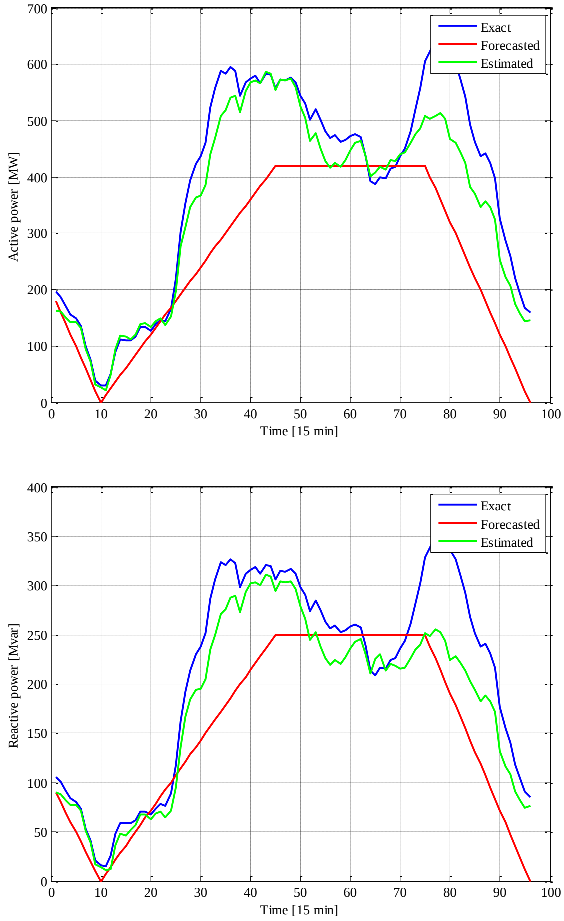

Figure 2: Results for node no. 3 with measurement of complex nodal voltages only at nodes no. 2, 4, 6 and 8, simple forecasts for all loads and given active and reactive power injection at node no. 2.

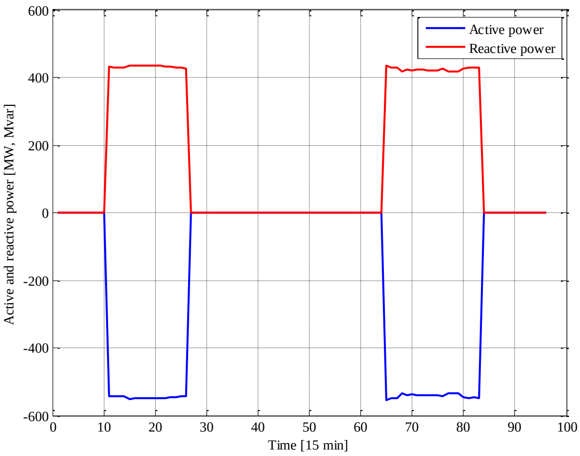

Figure 3: Active and reactive power at node no. 2 (generation).

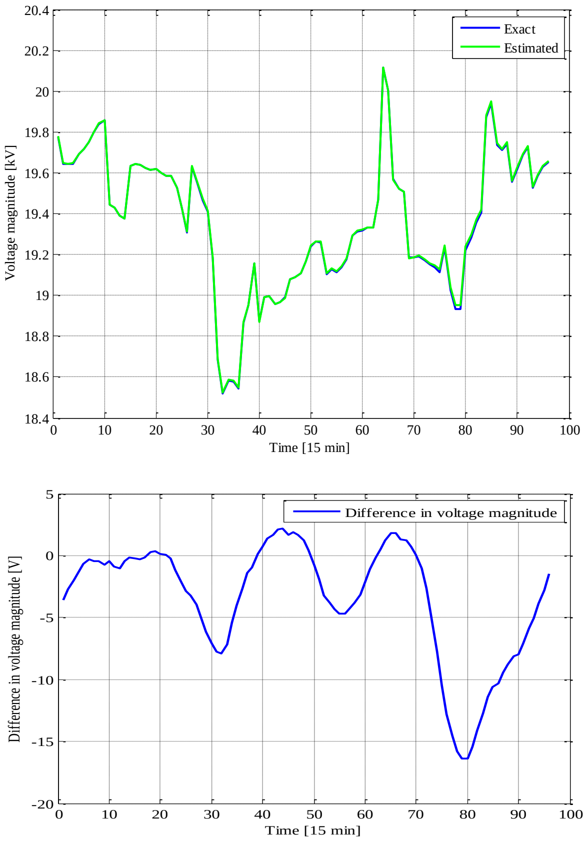

Figure 4: Actual and estimated voltage magnitude and difference between actual and estimated voltage magnitude at node no. 3.

9. REFERENCES

[1] J. z. Hingst, Prozessbegleitende Simulation elektrischer Mittelspannungsnetze bei dezentraler Einspeisung und unvollständiger Messinfrastruktur. Doctoral Dissertation, Clausthal University of Technology. Clausthal-Zellerfeld, Germany: Papierflieger Verlag, 2009. (In German, Online simulation of electrical mid-voltage grids with distributed generation and incomplete instrumentation.)

[2] G. Gewiss, H.-P. Beck, J. zum Hingst, "Nodal load observer with imperfect measurement infrastructure for (smart) electrical grids". Advanced Mathematical and Computational Tools in Metrology and Testing, vol.9, Series on Advances in Mathematics for Applied Sciences vol. 84, Singapore: World Scientific, 2012.

[3] A. Abur, A. G. Gómez Expósito, Power System State Estimation: Theory and Implementation. New York: Marcel Dekker, 2004.

[4] I. Dzafic, S. Henselmeyer, H.-T. Neisius, "High performance state estimation for smart grid distribution network operation". IEEE-PES Conference on Smart Grid Technologies, Anaheim, California, January 2011.

[5] A. Seppala, M. Lehtonen, "Distribution load estimation – a function for the electric utility in the deregulated electricity market". International Conference on Energy Management and Power Delivery. Singapur, November 1995, pp. 240-243.

[6] J. Wan, K. N. Miu, "Weighted least squares methods for load estimation in distribution networks". IEEE Transactions on Power Systems, vol. 18, pp. 1338-1345, 2003.

[7] G. Ludyk, Time-variant discrete-time systems. Braunschweig, Germany: Vieweg, 1981.

[8] R. Gelagaev, P. Vermeyen, J. Vandewalle, J. Driessen, "Numerical observability analysis of distribution systems". 14th International Conference on Harmonics and Quality of Power, Milano, September 2010.

[9] N. R. Shivakumar, A. Jain, "A review of power system dynamic state estimation techniques". Joint International Conference on Power System Technology and IEEE Power India Conference, New Delhi, October 2008.

[10] R. E. Kalman, "A new approach to linear filtering and prediction problems". Transactions of the ASME-Journal of Basic Engineering, vol. 82, pp. 35-45, 1960.

[11] J. B. Moore, B. D. O. Anderson, "Coping with singular transition matrices in estimation and control stability theory". International Journal of Control, vol. 31, pp. 571-586, 1980.

[12] B. Teixeira, „Kalman filters". IEEE Control Systems Magazine, vol. 28, pp. 16-18, 2008.