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A "BLIND" CORRECTION OF DYNAMIC ERROR OF A NONSTATIONARY FIRST ORDER TRANSDUCER FOR THE PERIODIC CASE – SIMULATION INVESTIGATION

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Abstract: The paper presents an extension of the "blind correction" method with respect to nonstationary transducers of the first order. An analytical description of the measuring channels' dynamics is presented for the case where both: the measured signal and the measuring channels' parameters are varying with same fundamental frequency. The influence of the measuring system parameters on the correction accuracy was investigated using the simulation methods.

Keywords: dynamic error, blind correction, nonstationary transducer.

1. INTRODUCTION

A measurement of time-varying quantities always contains a dynamic error caused by the analogue signal transducers. The instantaneous values of the measured signal can be reconstructed if the models of these transducers' dynamics, and the model coefficients are known. The results of such correction, however, will not be satisfactory when the model coefficients vary with time, depending of the measuring system operating condition. The system structure and algorithms that allow self-identification of the measuring channel analogue model's coefficients at the system operating site, using solely the measured signal, are presented in [1]. This method of correction allowed for a substantial reduction of dynamic error, but only under the assumption that the identified coefficients are varying considerably slower than the measured signal. One of the possible variants of the self-identification and correction method was introduced in [2] for a more general case, where the coefficients of a measuring channel's dynamics model vary with time at the same rate as the measured signal. The paper presents simulation results that illustrate the extended method. They were carried out for the case in which the coefficients of an analogue channel are varying with same fundamental frequency as the measured signal does.

An example of application of the presented correction method may be the measurement of an instantaneous temperature of exhaust gases from a reciprocating piston engine, operated at a constant rotational speed. Gas of variable temperature, flowing with pulsating velocity around the transducer, changes the instantaneous value of the heat transfer coefficient. These two time-variable quantities: the temperature and the heat transfer coefficient, have the same fundamental frequency and they contain harmonics. They are differentiable over the whole period, and hence satisfy the Dirichlet conditions. They therefore can be subjected to harmonic analysis and, using advanced methods of digital signal processing, the measured signal can be reconstructed. Consequently, there is no need to identify the non-linear relations, depending on the instantaneous gas velocity, which determine the dynamic process of heat transfer between the flowing gas and the transducer. So there is no need to determine the instantaneous value of the gas velocity and define its physical properties, as well as the geometry of transducers.

2. THE SYSTEM STRUCTURE

The measuring system which performs "blind correction" consists of two independent analogue channels [1], synchronously measuring the same input quantity $u(t)$ with the fundamental harmonic period $\Theta=1$, $\omega=2\pi/\Theta$. The symbols $\Phi 1(t,t_0)$ and $\Phi 2(t,t_0)$ in Fig.1 are the fundamental matrices of differential equations solutions describing the dynamics of the first and the second channel, respectively. A unique analytical solution of this task requires the dynamic properties of these analogue channels to be different. The sensitivity of both channels was assumed unity $k=1$, and it is not subject to identification. It was also assumed that the signals $u(t)$, $x(t)$ and $p(t)$ are expressed as relative dimensionless quantities, related to the conversion range of measuring channels. The procedures of identification of the coefficients for both measuring channels' models, and the correction of dynamic error are subsequently performed after converting the transducers' output signals $x(t)$, $p(t)$ to their digital representation $x_i=x(t_i)$, $p_i=p(t_i)$ at time instants t_i.

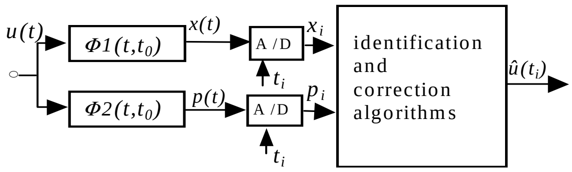

Fig. 1. The system structure

3. MATHEMATICAL MODEL OF MEASURING CHANNELS

Each of the two channels is described by a differential equation of the 1st order, and each of them is measuring the same input quantity $u(t)$, at the same instants of time, hence:

$$k\,u(t)=x(t)+f(t)\,x'(t)=p(t)+g(t)\,p'(t)\,. \tag{1}$$

The solution of this equation are time-variable functions $x(t)$ and $p(t)$. Functions $f(t)$ and $g(t)$ play the roles of time-variable coefficients in the differential equation (1). In the classic stationary case the coefficients of derivatives in equation (1) would play the role of time-constants. In the discussed case they are a periodic functions of time $f(t)$ and $g(t)$ with unity fundamental harmonic frequency, containing subsequent harmonics (2). Only the components F_0 and G_0 could be interpreted as time-constants in the commonly used meaning.

$$\begin{aligned} f(t) &= F_0 + \sum_{m=1}^{M} F_m \sin(m\omega t + \varphi_m) \\ g(t) &= G_0 + \sum_{l=1}^{L} G_l \sin(l\omega t + \gamma_l) \end{aligned} \tag{2}$$

4. THE IDENTIFICATION PROCEDURE

As a result of harmonic analysis of signals $x(t)$ and $p(t)$ we obtain their approximate analytical form. Thus, the analytical approximation of their derivatives $x'(t)$ and $p'(t)$ can also be obtained. Substituting obtained instantaneous values of the recorded signals and their derivatives into the central and right sides of equation (1) we transform it to algebraic form. The functions $f(t)$ and $g(t)$ that describe the channels' dynamics take on the role of unknowns. This operation, executed for each sampling instant t_i, yields an overdetermined system of algebraic equations. Then, applying QR decomposition of rectangular matrix of this equation, we obtain approximate sought values of the coefficients of measuring channels' models. For instance, for the case $L=M=2$ the differential equation (1) for the time instant t_i takes the algebraic form (3). There is no need to express the amplitudes and phase angles of individual harmonics in an explicit form, for the determined values of expressions containing these quantities are directly used in the procedure of correction.

$$F_0 x'(t_i) + F_1\cos(\varphi_1)x'(t_i)\sin(\omega t_i) + F_1\sin(\varphi_1)\,x'(t_i)\cos(\omega t_i) + F_2\cos(\varphi_2)\,x'(t_i)\,\sin(2\omega\,t_i) + F_2\sin(\varphi_2)\,x'(t_i)\,\cos(2\omega\,t_i) - G_0 p'(t_i) - G_1\cos(\gamma_1)p'(t_i)\sin(\omega t_i) - G_1\sin(\gamma_1)p'(t_i)\cos(\omega t_i) - G_2\cos(\gamma_2)p'(t_i)\sin(2\omega t_i) - G_2\sin(\gamma_2)p'(t_i)\cos(2\omega t_i) = p(t_i) - x(t_i) \tag{3}$$

5. THE CORRECTION PROCEDURE

The reconstruction of instantaneous values of the measured signal is obtained by means of series correctors using the identified $f(t)$ and $g(t)$, operating as numerical algorithms, independently in each of the two signal-processing channels $\hat{u}_x(t_i)$, $\hat{u}_p(t_i)$ according to equations (4). They are slightly different due to measurement errors. So, the final result of correction $\hat{u}(t_i)$ is determined as (5).

$$\hat{u}_x(t_i)=x(t_i)+f(t_i)\,x'(t_i) \qquad \hat{u}_p(t_i)= p(t_i)+g(t_i)\,p'(t_i) \tag{4}$$

$$\hat{u}(t_i)= (\hat{u}_x(t_i),+\hat{u}_p(t_i))/2 \tag{5}$$

6. SIMULATION RESULTS

The investigation was carried out for the signal $u(t)$, modelled as a trapezoid waveform with unity period $\Theta=1$ and adjustable coefficients which determine its shape. Duration of the signal high value $Uhi=0.9$ is 0.4, and duration of the low value $Ulo=0.4$ is 0.2. The rise time is 0.2, and fall time is 0.2. The following coefficients were used for the channel associated with the signal $x(t)$: $F_0=1$, $F_1=0.7$, $\varphi_1=\pi/6$, $F_2=0.2$, $\varphi_2=\pi/9$, and for the channel associated with the signal $p(t)$: $G_0=0.95$, $G_1=0.65$, $\gamma_1=\pi/5$, $G_2=0.15$, $\gamma_2=\pi/7$. Solutions of differential equations were found using ODE45 procedure of MATLAB package; they are presented in Fig. 2.

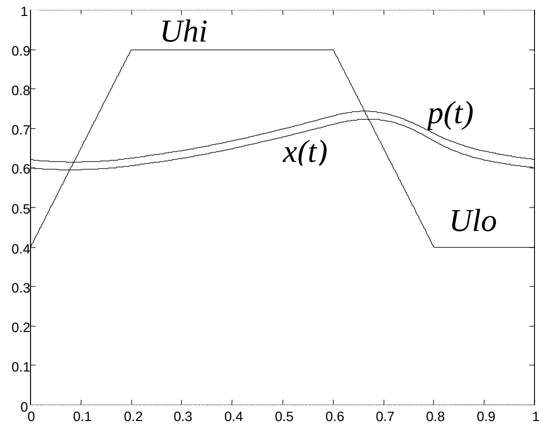

Fig. 2. Example waveforms of signals' *u(t)*, *x(t)* and *p(t)*

The simulation results of correction obtained for A/D converters with the number of bits denoted Lb, and sampling frequency f_s, are presented in Table 1. The accuracy of correction index E is determined as (6). It is assumed that the correction and identification algorithm uses all the available harmonics of the measured signals $x(t)$ and $p(t)$, according to the Nyquist principle. It is also assumed that the identification algorithm seeks the coefficients $f(t)$ and $g(t)$ that determine the channels' dynamics, in the form of periodic functions containing components up to two harmonics, according to equation (3).

$$e(t_i) = \hat{u}(t_i) - u(t_i) \qquad E = \max_i \left|e(t_i)\right| \tag{6}$$

Table 1. Dynamic error *E* after the correction, depending on the number of bits A/D converter *Lb* and the sampling frequency f_s.

$Lb\backslash f_s$	16	64	256	1024
8	0.21420	0.28305	0.28292	0.29713
12	0.04632	0.16158	0.31136	0.27471
16	0.07158	0.00455	0.01428	0.15723
20	0.07300	0.00448	0.00122	0.00137
24	0.07322	0.00447	0.00124	0.00030

Example of the dynamic error $e(t_i)$ waveform for Lb=24 and fs=1024 is shown in Fig. 3. The highest instantaneous values of the error occur at time instances corresponding with the instances of switching the measured signal, and reach values of the order of 0.0003 in the sense of the H_∞ norm, whereas the measurement without correction contains error of maximum value 0.2872. In the investigated case the presented method allows therefore for 1000 times reduction of dynamic error.

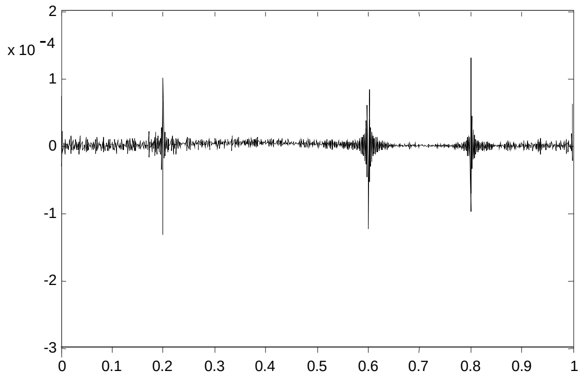

Fig.3. The dynamic error *e(t_i)* after correction

In order to compare the quality of correction in each channel of the pair, the waveform $e_{xp}(t_i)$ (7) of difference between the signals reconstructed independently in each channel associated with signals *x(t)* and *p(t)*, is shown in Fig 4. The instantaneous values of this difference are of one order of magnitude smaller than those of the signal *e(t)*. That confirms the good quality of correction in each channel.

$$e_{xp}(t_i) = \hat{u}_x(t_i) - \hat{u}_p(t_i) \qquad (7)$$

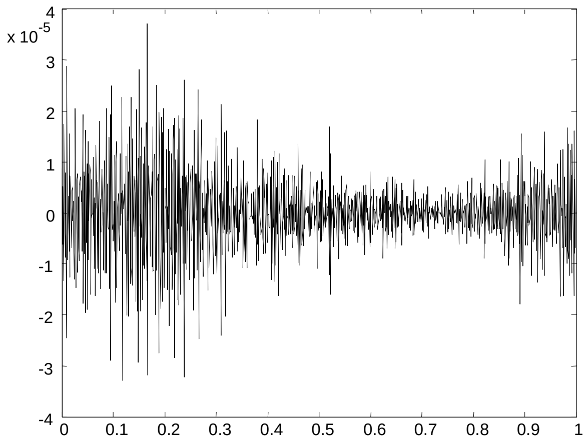

Fig.4. The difference of output signals $e_{xp}(t_i)$ of the pair of channels, after the correction

For the purpose of further verification of the identification procedure properties, it has been assumed that coefficients of differential equation (1) are determined for a greater number of harmonics. It has also been assumed that *f(t)* contains up to *M=7*, and *g(t)* contains up to *L=6* harmonics, whereas only 2 harmonics were applied in both coefficients. The values of index *E* for different numbers of harmonics being sought in the coefficients *f(t)* and *g(t)* of both channels, are tabulated in Table 2. These results should be analysed with regard to Table 3 which shows the matching index *Exp*, determined in the same way as *E*, but for the signal $e_{xp}(t_i)$. In an actual measurement there is no possibility of determining the value of index *E* because the signal *u(t)* remains unknown. It is only possible to determine the *Exp* from the correction results in both channels. If the identification procedure is seeking at least two harmonics in coefficients *f(t)* and *g(t)*, then the *Exp* values are of several orders of magnitude smaller than those when only the fundamental component of these coefficients is sought. This preliminary analysis of the *Exp* values allows to specify more accurately the number harmonics of the final identification procedure. Seeking a redundant number of harmonics unnecessarily increases the task size, what results in decreasing the accuracy of calculations, as can be seen in Table 2.

Table 2. The correction accuracy index *E* versus the number of harmonics assumed for the channels dynamics model identification.

M\L	1	2	3	4	5	6
1	194.95	212.9	258.7	266.3	271.0	256.6
2	157.22	1.245	1.243	1.238	1.236	1.223
3	201.53	1.243	1.239	1.245	1.247	1.230
4	202.37	1.239	1.248	1.244	1.233	1.229
5	200.67	1.236	1.247	1.233	1.214	1.204
6	144.19	1.219	1.229	1.224	1.209	3.908
7	150.28	1.211	1.204	1.763	6.997	7.722
						$x10^{-3}$

Table 3. The matching index *Exp* versus the number of harmonics used for the channels dynamics model identification.

M\L	1	2	3	4	5	6
1	14065	1496	1072	555.4	586.5	497.9
2	1042	9.138	9.159	9.202	9.201	9.178
3	812.3	9.161	9.184	9.223	9.202	9.190
4	407.6	9.198	9.218	9.222	9.140	9.149
5	389	9.200	9.204	9.114	9.119	9.059
6	254.8	9.188	9.188	9.133	9.069	9.268
7	153.0	9.161	9.169	9.06	9.282	9.212
						$x10^{-6}$

Results of identification for *M=7* and *L=6* harmonics sought for the coefficients *f(t)* and *g(t)*, respectively, are presented in Table 4. Rows in the Table show both: the set and the identified values of moduli and arguments of subsequent harmonics.

Table 4. The set values and the identified values of the models' dynamics parameters, for 6 and 7 harmonics being identified.

	f_set	f_id	g_set	g_id
Mod0	1	0.99452	0.95	0.94455
Mod1	0.7	0.69762	0.65	0.64862
Arg1	0.5236	0.53769	0.62832	0.64361
Mod2	0.2	0.20647	0.15	0.15614
Arg2	0.34907	0.33669	0.4488	0.4286
Mod3	0	0.0038797	0	0.003929
Arg3	0	-2.4774	0	-2.4716
Mod4	0	0.0020175	0	0.0020573
Arg4	0	1.4416	0	1.454
Mod5	0	0.00061562	0	0.00063263
Arg5	0	-0.65997	0	-0.66742
Mod6	0	7.2288 e-5	0	7.4558 e-5
Arg6	0	-1.8641	0	-2.2536
Mod7	0	1.4258 e-5	0	0
Arg7	0	-2.8822	0	0

Reduction of the number of sought harmonics to 2, results in the increase in the accuracy of identification, as shown in Table 5.

Table5. The set values and the identified values of parameters of the models' dynamics, for 2 harmonics identified of the models.

	f_set	f_id	g_set	g_id
Mod0	1	0.99997	0.95	0.94997
Mod1	0.7	0.69998	0.65	0.64998
Arg1	0.5236	0.52369	0.62832	0.62842
Mod2	0.2	0.20001	0.15	0.15001
Arg2	0.34907	0.34914	0.4488	0.44889

Another parameter of the correction procedure is the number of harmonics in the recorded signals *x(t)* and *p(t)*, used in the procedure. These harmonics are first of all used to determine the *x(t)* and *p(t)* signals derivatives. Moreover, limiting the number of harmonics allows to filter out disturbances from the recorded signals. The values of index *E* for various numbers of harmonics *Nd* used in determining the derivative are presented in columns of Table 6, the rows refer to the number of harmonics *Nf* of filtered signals *x, p*.

Table 6. The correction accuracy index *E* versus the number of harmonics in the recorded signals used for determinaton of derrivatives, and filtering.

Nf\Nd	3	6	9	12	15	18
3	0.3318	0.1964	0.0825	0.0637	0.0647	0.0622
9	0.3415	0.1646	0.0258	0.0101	0.0090	0.0080
12	0.3415	0.1646	0.0259	0.0100	0.0090	0.0079
18	0.3415	0.1646	0.0259	0.0101	0.0090	0.0079

The dynamic properties of both: the measured signal and sensors employed, have significant influence on the correction quality. Rows in Table 7 show values of the index *E* obtained in the example experiment. In this experiment has been altered time t_3, which determines the initial instant of signal *u(t)* falling from the value *Uhi* to *Ulo*. The columns refer to changes in the argument φ_1 around $\pi/6$ (0.5236). The investigation was carried out for *fs*=256, *Lb*=16 and *M*=*L*=2.

Table7. The quality of correction index *E* versus the rate of change of the input signal and phase of the channel dynamics model.

$t_3\backslash\varphi_1$	0.36652	0.5236	0.68068	0.83776
0.56	0.026552	0.01292	0.030155	0.0019116
0.58	0.025029	0.019541	0.024989	0.0018792
0.60	0.030187	0.01428	0.033775	0.0012792
0.62	0.024603	0.021764	0.029327	0.0021026
0.64	0.025756	0.018429	0.040505	0.0017061

All previous investigations were carried out under the assumption that static parameters of both measuring channels are precisely known. (sensitivity and offset *k=kp=kx=1*, *x0=p0=0*). Investigation of a simpler, stationary variant (*f(t), g(t)* –const) has revealed a significant influence of disturbances in these parameters on the correction quality. This is why the consequences of disregarding the lack of knowledge of these parameters have also been investigated for a non-stationary case. Table 8 presents the values of index *E* for various gain *kp* and offset *p0* values, introduced to the channel described by the variable *p*, with *Lb*=24, *fs*=1024 and *M*=*L*=2. Such a disturbance has not been taken into account in the correction procedure. These results demonstrate the importance and influence of the static accuracy (for DC component) of a measuring channel on the quality of dynamic correction. When DC parameters of the two channels are different, but known, then adequate corrections to the recorded signals *x(t)* and *p(t)* can be introduced. If they, however, remain unknown, the self-identification procedure can be extended to determining static parameters of one channel with respect to the other one. The absolute accuracy of both channels is not significant. The sensitivity and offset of any channel can be determined with respect to the other one. This modification, however, requires a considerable modification of equations (1) and (3). For this reason this problem is presented in a separate publication [3].

Table 8. The change in correction accuracy index *E*, resulting from disregarding the DC parameters of one channel.

p0\kp	0.96	0.98	1	1.02	1.04
-0.00441	0.44812	0.25468	0.06515	0.13244	0.32594
-0.00147	0.40735	0.21392	0.02178	0.17323	0.36672
0	0.38697	0.19353	0.00030	0.19363	0.38711
0 .00147	0.36659	0.17315	0.02169	0.21402	0.40751
0 .00441	0.32582	0.13238	0.06501	0.2548	0.44829

7. CONCLUSIONS

The presented method of correction, confirmed with the simulation example, shows that the responsibility for the correctness of a complex measurement can be allocated to the algorithms processing the measurement data. It is not necessary the input circuits should meet stringent requirements, only a static calibration of measuring channels is required for measuring time-varying signals.

ACKNOWLEDGMENTS

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