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# LIKE AND UNLIKE IMPEDANCE COMPARISONS WITH THE SAME INSTRUMENT

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**Abstract** – IEN system for unlike impedance comparison (resistance vs. inductance, or resistance vs. capacitance) is based on an implementation of the *three-voltmeter* method, which configures both standards in comparison as four terminal-pair, by using automatic electronic compensations to achieve the purpose. Basic relative accuracy of the instrument is between a few parts in  $10^{-6}$  for comparison of impedances in the range  $10 \Omega$  to  $10 \text{ k}\Omega$ , and at frequencies in the range 100 Hz - 10 kHz.

The same system, without any changes in the instrumentation employed or its wiring, can be effectively used also as a four terminal-pair ratio bridge for comparison of *like* impedances (resistance vs. resistance, or capacitance vs. capacitance), in the same impedance and frequency range. This capability has been checked by conducting measurements of resistance ratios of four terminal-pair ac resistors, having nominal ratios of 1:1 and 10:1. Results show that, for like impedances, relative accuracies at the level of a few  $10^{-7}$  can be within reach.

**Keywords**: impedance metrology; ac ratio bridges; four terminal-pair.

#### 1. INTRODUCTION

The comparison of *unlike* impedances, like a resistor with a capacitor, is performed at the highest levels of relative accuracy, a few parts in  $10^{-8}$ , with quadrature transformer bridges [1]. Such devices are complex, expensive and difficult to operate correctly. If the relative accuracy target can be limited to a few parts  $10^{-6}$ , the *three-voltmeter method* [2] permits to compare unlike impedances of any kind (even those with big stray parameters, like air-core inductors) with a much simpler circuit, also suitable of automation.

At the Istituto Elettrotecnico Nazionale Galileo Ferraris (IEN) the method is employed to mantain the Italian national standard of electrical inductance in the audio frequency range, by tracing inductance to ac resistance [3]; more recently, the method has been extended to calibration of high-valued capacitors (10 nF  $- 1 \mu$ F).

When dealing with impedances ranging below 100 k $\Omega$ , any accurate comparison of like or unlike impedances must deal with coaxial *four terminal-pair* definition of the standards [4]. Transformer ac bridges [1] accomplish this definition by employing a number of passive networks of variable inductive voltage dividers and auxiliary impedances, which must be operated manually until a number of secondary balances is reached.

We developed at IEN an implementation of the three-voltmeter method which define both impedances under comparison as four terminal-pair standards, by employing automatic synchronous electronic compensators [5]. This permitted the complete automation of the implementation, which is now suitable for unlike impedance comparisons in the range 10  $\Omega$  - 10 k $\Omega$ , and frequencies in the range 100 Hz - 10 kHz; typical relative accuracies lie in the 10<sup>-6</sup> range (for example, in the calibration of L = 100 mH with R = 1 k $\Omega$ , at f = 1 kHz, we reach  $u_{\rm R}(L) = 13 \cdot 10^{-6}$ , k = 2 [3]).

The subsystem defining the standards as four terminal-pair is completely independent from the measurement subsystem. Thus, the latter can be replaced with alternative measurement subsystems, retaining the four terminal-pair definition anyway.

Here we investigate the possibility of employing the setup as a four terminal-pair transformer ratio bridge, for the comparison of *like* impedances (e.g., resistor with resistor). It will be shown that this is possible without any change in the electrical wiring of the three-voltmeter setup. Like impedances are defined as four terminal-pair, and compared in a semi-automatic way; first experiment show that a realistic accuracy target is in the order of  $10^{-7}$  for impedances between 10  $\Omega$  and 10 k $\Omega$ .

## 2. EXPERIMENTAL

The implementation of the three-voltmeter method is described in detail in [3]: a schematics is reported in Fig. 1, and a photo of the system in Fig. 2. The impedances to be compared  $Z_s$  and  $Z_x$  are fed by a two-phase sinewave generator G. Voltages  $U_s$  and  $U_x$  develop on  $Z_s$  and  $Z_x$ ;  $U_s$  and  $U_x$  form a combination voltage  $U_m$ , constructed with an inductive voltage divider IVD, usually set to a fixed voltage ratio of 0.5.

The rms values  $U_s$ ,  $U_x$  and  $U_m$  are measured with an voltmeter V and a switching unit S. The complex ratio  $Z_x/Z_s$  can then be computed from the real ratios  $U_x/U_s$  and  $U_m/U_s$ , thus only the V linearity rather than its absolute accuracy actually enters the uncertainty evaluation.

Four-terminal pair standard definition is obtained with two automatic synchronous compensations,  $C_S$  and  $C_X$ , fed

by a slave generator  $G_c$ . The compensation  $C_S$  ( $C_X$ ) measures the voltage  $U_{sl}$  ( $U_{xl}$ ) and synthesizes a compensation voltage, injected through a feedthrough transformer, continuously readjusted to minimize  $U_{sl}$  ( $U_{xl}$ ).

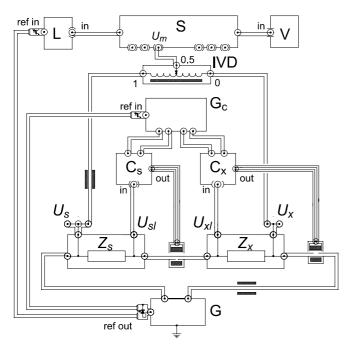


Fig. 1. Coaxial schematics of the three-voltmeter method.

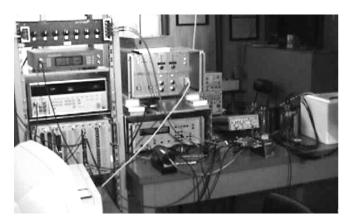


Fig. 2. A photograph of the system.

The effectiveness of the compensation is verified with a remotely controlled lock-in amplifier L, switched by S on  $U_{sl}$  or  $U_{xl}$ . All reference signals are distributed by optical fibers to minimize interferences and ground loops. Overall system accuracy has been estimated to be about  $10^{-5}$  [3].

Although in principle the three-voltmeter method could be used also to compare *like* impedances, in practice it behaves poorly. The vector diagram of measured voltages collapse from a triangle to a straight line, and the sensitivity of the measurement to voltmeter noise increases.

However, for like impedances, it is possible to operate the implementation of Fig. 1 as a four terminal-pair *ratio bridge*. The voltage vectors  $U_s$  and  $U_x$  have now similar phases and can be directly compared by IVD (a 7-decade divider Electro Scientific Industries ESI DT72 has been employed), by connecting the lock-in amplifier L (by S) to read  $U_m$  and adjust manually IVD to a ratio k which minimize it. If  $U_m \equiv 0$  is perfectly reached, then one can write the usual bridge expression  $p(Z_x) = p(Z_s) k / (1-k)$ , where  $p(Z_s)$  and  $p(Z_x)$  are the principal components of the impedances compared (e.g., the resistances  $R_s$  and  $R_x$  when comparing resistors). Small in-quadrature residuals permit to compute the ratio of the auxiliary parameter (e.g., time constants of resistors).

## 3. RESULTS

Tests on this capability of the three-voltmeter system have been performed by measuring ac resistance ratios.

We measured Tinsley resistance standards mod. 5685A/B, encased in an electrostatic screen and configured as four terminal-pair standards (with British Post Office MUSA connections). Fig. 3 shows a photo of the modified standards.



Fig. 3. Four terminal-pair ac resistance standards compared.

#### 3.1 Measurement of 1:1 nominal ratio

We measured two nominally equal 10 k $\Omega$  resistors. The standards have been first compared with an automatic setup for 1:1 calibration by substitution, based on a pneumatic coaxial switch and an automatic four terminal-pair RLC bridge used as comparator [6]; Fig. 4 displays the ratio  $M_{\rm sw} = [R_x / R_{\rm s}]_{\rm switch}$  for each measurement, and also the the corresponding average  $\langle M_{\rm sw} \rangle = 0.99996014$ ; the experimental standard deviation of the mean is  $u(M_{\rm sw}) = 4.6 \cdot 10^{-7}$ .

The comparison with the three-voltmeter setup gives the stable ratio value  $M_{\rm B} = 0.99996000$ .

The ratio difference between the two techniques is

$$\Delta M = M_{\rm B} - M_{\rm sw} = 1.4 \cdot 10^{-7};$$

thus the two ratio mesurements are thus fully compatible within  $u(M_{sw})$ . This measurement assess mainly the symmetry of the system, but because of such symmetry several possible systematic errors can be cancelled.

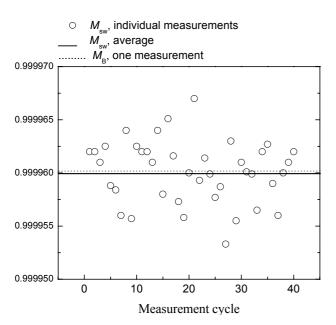


Fig.4. Resistance ratio for 1:1 comparison between two 10 k $\Omega$  resistors, see text for details.

## 3.2 Measurement of 1:10 nominal ratio

A test having more practical significance has been made by comparing two resistors,  $R_{10}$  and  $R_1$ , with nominal values of 10 k $\Omega$  and 1 k $\Omega$  respectively. The reference values have been obtained with dc calibration (by a Guildline 6675 automatic dc current comparator, against Italian national dc resistance scale), and ac-dc corrections (established during a BCR [7] intercomparison of ac resistance):

$$R_{10} = 10001.177 \ \Omega, \ u_R(R_{10}) = 2.5 \times 10^{-6} \text{ (k=1)};$$
  
 $R_1 = 1000.0710 \ \Omega, \ u_R(R_1) = 2.5 \times 10^{-6} \text{ (k=1)};$ 

The reference ratio is thus

$$M_{\mu} = R_{10} / R_1 = 10.000467, \ u(M_{\mu}) = 3.5 \times 10^{-5} \ (\text{k}=1).$$

We performed two measurements with the setup, "direct" ( $R_{10}$  as standard S) and "inverse" ( $R_1$  as standard S). Results are as follows:

 $k_d = 0.9090950 \Rightarrow M_d = 10.000495$  $k_i = 0.0909050 \Rightarrow M_i = 10.000495$ 

Again, , we find that the deviation of  $\{M_d, M_i\}$  from the reference value  $M_r$  is within  $u(M_r)$ .

## 3. REMARKS ON UNCERTAINTY

A complete estimation of the uncertainty of the method is beyond the aim of the paper. However, we can state some major limiting factors of the system:

- Ratio divider calibration  $u_r(k) \approx 5 \times 10^{-8}$ ;
- Accuracy of the realization of four terminal-pair definition:  $\approx 0.1 \,\mu\text{V}$  (with 2V excitation);
- Voltage transmission error to IVD  $\approx 1 \times 10^{-7}$  (but can be corrected by extrapolation);
- Lock-in alignment;
- Short-term G instabilities (at frequencies above the bandwidth of the automatic synchronous compensators);

Considering such limiting factors, we have confidence that, with the present setup, a relative uncertainty level around  $2\div 3\times 10^{-7}$  for 10:1 like impedance comparisons is within reach. The same system mantains the uncertainty level of a few  $10^{-6}$  for unlike impedance comparisons [2].

## 4. CONCLUSIONS

We have shown that the three-voltmeter implementation at IEN can be employed for comparisons of both like and unlike four terminal-pair impedances; for like impedance, a relative uncertainty level in the order of  $10^{-7}$  appear within reach. The employment of automatic synchronous compensations for four terminal-pair standard definition, in high-precision transformer ratio bridges, is also demonstrated.

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