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MODEL OF PROCEDURE FOR MEASUREMENT RESULT ERROR CORRECTION

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Abstract: An extension of the formal PSIQ-model of measurement procedure is proposed in order to have possibility to describe algorithms of measurement result correction in the ratio scale. An example of application of the model proposed to synthesis of a speedy correction method is given.
Keywords: measurement science, measurement procedure, error correction

1 INTRODUCTION

In [1] a formal model of measurement procedure has been proposed that ensures a determination of the elements of a numerical system with the same relations as in an empirical system. In accordance to the model, the measurement procedure is considered to be consisted of the following four operations:

- **p**artitioning of the initial solution set S of the Measurement Problem into subsets T_i;
- **s**electing one of the subsets T_i and an element $y_i \in S$ in this subset as its representative;
- **i**nverse mapping, possibly by use of a reference set A_r, the image y_i of the element x to the empirical system, i.e. $x_i = f^{-1}(y_i)$;
- **q**uestioning whether the result of the inverse mapping x_i is equivalent to the element x, fixed in the empirical system, in some measure of similarity.

Let each of these operations be implemented by some abstract apparatus, i.e. P-element, S-element, I-element, and Q-element. Then the model of measurement procedure can be represented by a four-unit formal scheme shown in Figure 1. In [1] it was called the PSIQ-model. Measurement procedures described by the PSIQ-model are distinguished from each other by methods of partitioning, selecting, inverse mapping, and questioning. A combination of these methods defines the particular procedure. In [1] such the particular models were constructed for three basic scale types, and their analysis was carried out; also a connection to well-known methods of AD-converting and to combinatorial search methods was considered. Structural properties of the model were investigated in [2]. In [3] a place of the PSIQ-model was defined in the common structure of the Measurement Science which corresponds to the representational approach (see, for example, [4]).

Offered in [1-3] approach to the measurement procedure model construction allows to have a compact formalized representation for a rather large number of the different tasks at various stages of measurement. In particular, it can be used for the description of correction methods of measurement results. This paper is devoted to an extension of the formal PSIQ-model of the measurement proce-

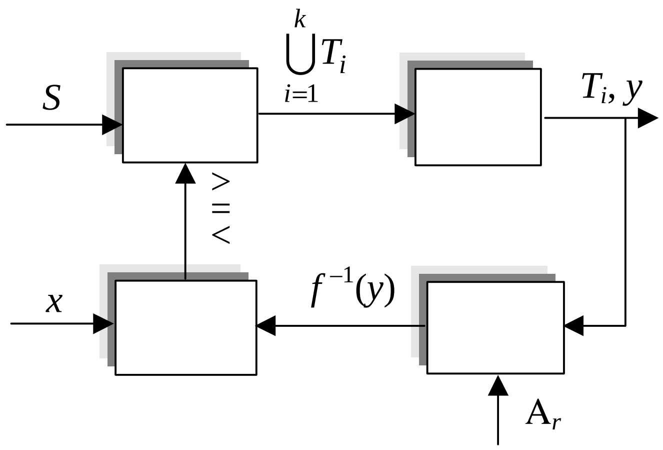

Figure 1. PSIQ-model of measurement procedure

dure in order to have a possibility to describe formally algorithms of error measurement correction in the ratio scale. An example of application of the model to synthesis of a new fast correction algorithm will be considered.

2 MEASUREMENT PROCEDURE MODEL OF SECOND ORDER

One of ways for a gain of measurement results accuracy is the use of so-called structural-algorithmic methods (see, for example, [5,6]). In this case the measurement errors are diminished with the help of auxiliary means, such as measurement standards, digital-to-analog converters and switching elements.

For the formalized description of these methods a measurement procedure model of second order, SCI-model (see Figure 2), can be introduced where

- the S-element realizes a choice between the measurand and some reference value;
- the PSIQ-element, which is described by PSIQ-model, forms the measurement result;
- the C-element executes calculation and memorization functions;
- the I-element, possibly by using a reference value a_r, fulfils inverse mapping of the measurement result into the empirical set.

Thus, the main measurement channel in the second order model is represented by the PSIQ-model, and the auxiliary tools by the S-, C- and I-elements. In the real measurement practice, the S-element of the SCI-model may be corresponded to the gate, the PSIQ-element to the measuring instrument, for example, digital voltmeter, the ∇-element to the computer, and the I-element to the digital-to-analog converter. In Figure 2 x denotes the measurand and y is the measurement result.

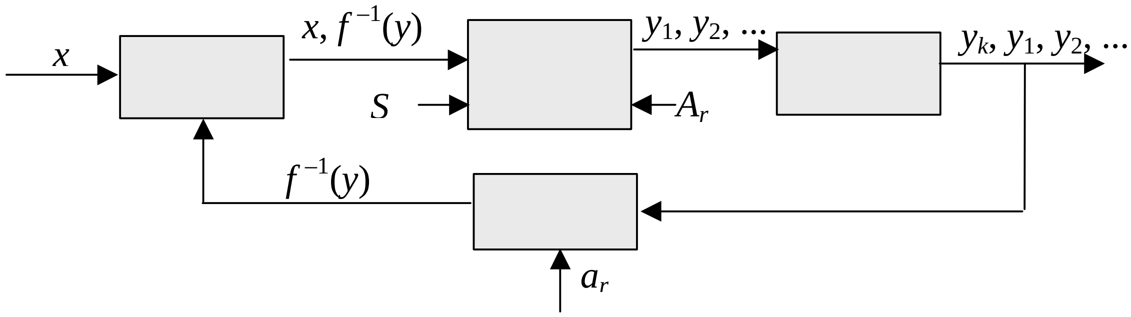

Figure 2. SCI-model of measurement result correction procedure

3 SYNTHESIS OF ALGORITHM OF MEASUREMENT RESULT ERROR

Let us show as the introduced model can be used for the analysis and synthesis of algorithms for a correction of measurement results obtained in the ratio scale. In what follows, the question will be about the minimization of a so-called correlated (low-frequency) component of the error including systematic, progressing (stipulated by a drift of parameters or a deterioration of elements) and rather slowly varying random errors.

For description of the general measuring channel transfer function of the basic we will use following formula:

$$ó = x(1+\gamma) = x(1+\frac{\Delta_0}{x} + \gamma_2) = x(1+\gamma_1 + \gamma_2), \tag{1}$$

where $õ$ is the measurand, $ó$ is the measurement result, γ is the initial relative error, Δ_0 is the absolute zero error, $\gamma_1 = \frac{\Delta_0}{x}$ is the relative zero error, and γ_2 is the relative sensitivity error.

For a simplicity, in further the S-, C- and I-elements are considered to be ideal, that is, they have no own error.

One of most widely used structural-algorithmic methods of measurement accuracy raise is the *iterative method* known also as the inverse transformation method [5,6]. Let us consider, at first, how this method can be described with use of the offered model, and then it will be shown how to improve its performance.

3.1 The iteration method

This method is useful, for example, for the correction of frequency errors under measurement of voltage of alternating current in a wide frequency range. The essence of the algorithm consists in the following. In the first step, the measurand x is connected to the input of the measuring channel. The numerical value $\acute{o}_1$ is stored by the C-element and is converted by the I-element into the empirical object $\tilde{o}_1 = f^{-1}(y_1)$. In the second step, the object $\tilde{o}_1$ is connected to the measuring channel input. In the C-element the difference $\Delta y = y_1 - y_2$ is computed and stored, where y_2 is the result of the second measurement. In the third step, the measurand x is connected to the input of the measuring channel again. The C-element computes the difference $y_k = y_3 - \Delta y$ where y_3 is the third measurement result. The value y_k is supposed to be the first corrected measurement result. Then y_k is subjected to the inverse converting, and the second and third steps are repeated. The iteration process can continue till a moment of reach a necessary measuring accuracy. The formal presentation of the algorithm in the SCI-model terms is shown in Figure 3. Here and in further a is the value of the S-element output.

	Step 1.
S:	$a := x$
PSIQ:	$\acute{o}_1 := f(a)$
	while $\gamma \geq \gamma^*$ **do**
	$\{$*Step 2.*
S:I:	$a := f^{-1}(\acute{o}_1)$
PSIQ:	$y_2 := f(a)$
C:	$\Delta y := y_1 - y_2$
	Step 3.
S:	$a := x$
PSIQ:	$y_3 := f(a)$
C:	$y_k := y_3 - \Delta y$ $\}$

Figure 3. Algorithm 1 of the iteration error correction

Let us determine the corrected error after the first iteration. Accounting the formula (1), the expression for the difference Δy is

$$\Delta y = x(1 + \gamma_1 + \gamma_2)\left(1 + \frac{g_1}{1 + g_1 + g_2} + g_2\right) - x(1 + \gamma_1 + \gamma_2), \qquad (2)$$

which, after simple transformations, becomes of the form:

$$\Delta y = x(1 + \gamma_1 + \gamma_2 + \gamma_1\gamma_2 + \gamma_2^2). \qquad (3)$$

For the first corrected result we have:

$$y_k = x(1 + \gamma_1 + \gamma_2) - x(\gamma_1 + \gamma_2 + \gamma_1\gamma_2 + \gamma_2^2) = x(1 - \gamma_2(\gamma_1 + \gamma_2)). \qquad (4)$$

Thus, after the first iteration the initial error γ has been multiplied by the amount of $(-\gamma_2)$. It is not complicate to show that after N iterations the error γ_{cor1} of the corrected measurement result will have the form:

$$\gamma_{\mathrm{cor1}} = \gamma\,(-\gamma_2)^N. \qquad (5)$$

As always $|\gamma_2| < 1$, the iterative process of the accuracy raise converges.

3.2 The modified method

As it is evident from the equation (5), the iterative process of the accuracy raise converges rather slowly, and is the slower the less $|\gamma_2|$ is. Moreover, the random error is increased at this case. There are various ways to do faster this method, for example, the element-wise correction or space separation of cycles of iterations [5]. However, all of them requires to use additional S- and I-elements. At the same time Algorithm 1 can be improved without a modification of the SCI-model structure, and by changing only functions of the C-element.

One of possible realizations of such the way is offered below. The SCI-model structure for the offered algorithm does not differ from shown in Fig. 2. The algorithm's essence is the following. In the first step, the measurand x is connected to the input of the measuring channel. The measurement result y_1 is stored by C-element and converted into the empirical object $x_1 = f^{-1}(y_1)$ by the I-element. In the second step, the value $\tilde{o}_1$ is connected to the input of the measuring channel. In the

	Step 1.
S:	$a := x$
PSIQ:	$\acute{o}_1 := f(a)$
	Step 2.
S: I:	$a := f^{-1}(\acute{o}_1)$
PSIQ:	$y_2 := f(a)$
C:	$y_k := \dfrac{y_1^2}{y_2}$

Figure 4. Algorithm 2 of the modified error correction

C-element the expression $y_k = \dfrac{y_1^2}{y_2}$ is computed where y_2 is the result of the second measurement. The value of y_k is supposed to be the final corrected result. Thus, instead of the operation of subtraction, the C-element will fulfil the square and the division. The formal presentation of the algorithm in the SCI-model terms is shown in Figure 4.

Let us determine the corrected result error that is obtained by means of Algorithm 2.

Using formula (1), the first measurement result has the form:

$$y_1 = x(1 + \gamma_1 + \gamma_2). \tag{6}$$

The second measurement result is

$$y_2 = x\,(1 + \gamma_1 + \gamma_2)\left(1+\frac{\gamma_1}{1+\gamma_1+\gamma_2}+\gamma_2\right). \tag{7}$$

The final corrected result is defined by the formula:

$$y_k = \frac{x(1+\gamma_1+\gamma_2)^2}{1+2\gamma_1+2\gamma_2+\gamma_1\gamma_2+\gamma_2^2}\ . \tag{8}$$

Reducing the expression (8) to the form (1), we have:

$$y_k = x\left(1+\frac{\gamma_1(\gamma_1+\gamma_2)}{(1+\gamma_1+\gamma_2)^2-\gamma_1(\gamma_1+\gamma_2)}\right) = x\left(1+\frac{\gamma_1\gamma}{(1+\gamma)^2-\gamma_1\gamma}\right). \tag{9}$$

Finally, the corrected result error has the form:

$$\gamma_{\mathrm{cor2}} = \frac{\gamma_1}{(1+\gamma)^2-\gamma_1\gamma}\gamma\,. \tag{10}$$

3.3 Comparative analysis of the correction algorithms

It is interesting to evaluate a number N of iterations, which are required to be carried out by Algorithm 1 in order to reach the accuracy of Algorithm 2, which is defined by the expression (10).

For this aim, on the base of expressions (5) and (10) compose the transcendental equation:

$$\gamma_2^N = \frac{\gamma_1}{(1+\gamma_1+\gamma_2)^2-\gamma_1(\gamma_1+\gamma_2)}\ , \tag{11}$$

solving whish yields the necessary number of iterations:

$$N = \lceil \log_{\gamma_2}\gamma_1 - \log_{\gamma_2}((1+\gamma_1+\gamma_2)^2-\gamma_1^2-\gamma_1\gamma_2)\ \rceil.$$

Using the modulus $1/\lg\gamma_2$ in order to go over to decimal logarithms, we finally have

$$N = \lceil \frac{1}{\lg\gamma_2}(\lg\gamma_1 - \lg((1+\gamma_1+\gamma_2)^2-\gamma_1^2-\gamma_1\gamma_2))\ \rceil. \tag{12}$$

In Figure 5 by the expression (12) the collection of curves $N(\gamma_1)$ is plotted with various fixed values of the error γ_2. It is clear from Figure 5 that the preeminent by execution speed for Algorithm 2 exists if $\gamma_2 > \gamma_1$, that is, the sensitivity error predominates. The gain increases additionally if $\gamma_2 >> \gamma_1$ and sharply increases, if γ_2 approaches to 1. For example, at γ_1 = 0,1 % and γ_2 = 0,9 % four iterations (eight slopes) is required to carry out by Algorithm 1 in order to reach the accuracy of Algorithm 2 which is obtained for only one cycle (two slopes).

The initial variant of Algorithm 2 was firstly published in [7].

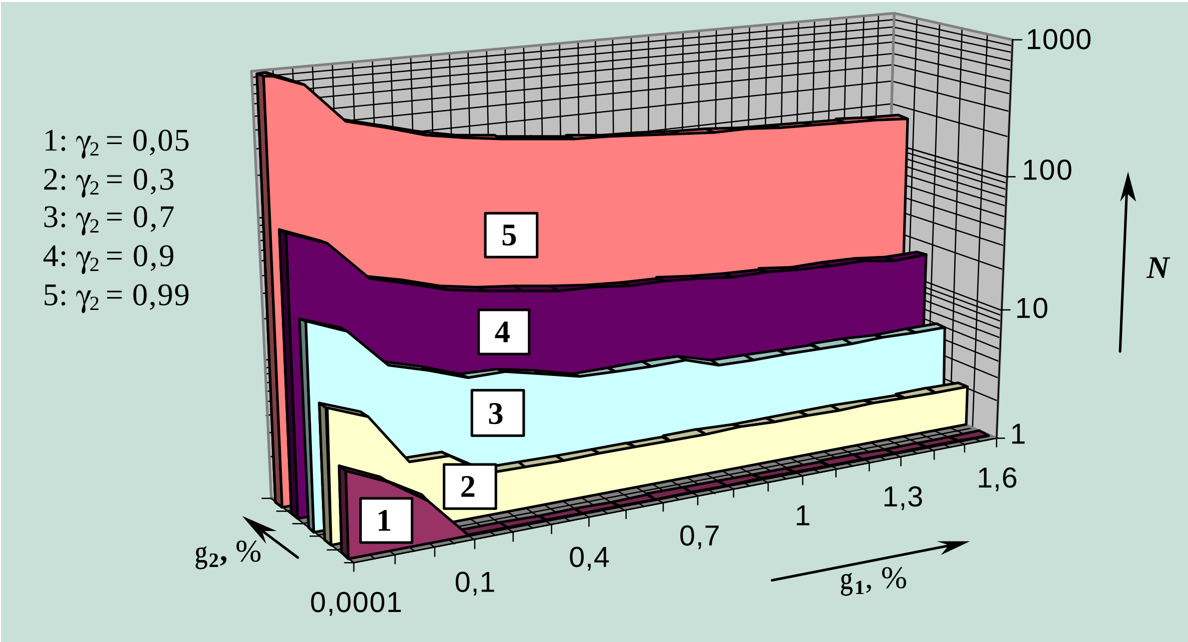

Figure 5. Dependence of the iteration number N of the error γ_1 with various fixed values of the error γ_2

4 CONCLUSION

In this paper the possibilities of the formal PSIQ-models for the analysis and synthesis of measurement procedures in the ratio scale have been shown. Particularly, as one of possible applications of the formal model to solving the problem of raise of measurement accuracy the algorithm of the error correction of measuring channel has been proposed which allows to increase the measuring channel accuracy only for two steps.

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