Ultrasonic Transit-time Discharge Determination in Rectangular Open Channel

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Abstract

Ultrasonic Transit-time method is a good choice to measure the open channel discharge. To investigate its performance in the rectangular open channels, a 26m long, 2m wide, 1.2m deep open channel facility was built with an over-fall head tank providing a very stable flowrate of 1.5m³/s in maximum and 39 typical cases were tested. The tested results verified that ultrasonic transit-time method can be well used to measure the discharge in rectangular open channels. However, due to unavoidably disturbing the flow patterns and introducing some errors, thus the transducers should be made as small as possible to improve the measuring accuracy. In addition, more paths should be used to produce smaller errors. And at least two paths are suggested to be mounted in the range of h/H>0.7 to better reflect the influence of the free surface. Comparing sub-discharges in all zones, it can be found that substantial differences between mean-section method and Giordano Law exist below the lowest path. Larger Kᵣ, for example 0.9, makes mean section method produce closer measurements to Giordano Law.

Key words: open channel, discharge determination, ultrasonic transit-time method, transducers’ arrangement

1 Introduction

Discharge is a most important hydraulic parameter for open channels. Lots of different devices, including rated sections, ramp flumes, Parshall flumes, cutthroat flumes, weirs, are used to determine the flow discharges in open channels. Especially in the recent years, electromagnetic and ultrasonic Doppler flowmeters (Lynnworth and Liu, 2006), which are based on direct velocity measurements, have been rapidly developed and successfully applied in the open-channel's flow measurements. However, the discharge measuring accuracies are still not good enough. Heiner et al. (2011) evaluated the accuracies of multiple flow meters which were installed in various open channels, and the results showed two-thirds of the instruments exceeded the allowed errors with the maximum values being up to 40%. As high as 71% of the devices were unacceptable by visual inspection alone, and the reasons are mainly the improper site conditions couldn't fully meet the strict requirements. Therefore, in order to improve the accuracy of flowrate measurements in open channels, newly developed ultrasonic transit-time flowmeters maybe a very good alternative, due to their good stability and applicability (Hu et al. 2015).

As is well known, when travelling in the flow, the ultrasonic wave will take some time. And it will take some more time when travelling in the upstream direction than in the downstream direction due to the flow velocity. Based on this time difference, the flow velocity can easily be determined. Then, according to the velocities at different path heights, the flow rate can be obtained through numerical integration. This is the basic procedure of discharge determination with velocity measurements using ultrasonic transit-time method. Clearly, the discharge measuring accuracy is influenced by many factors, and the transducers’ influence is one of them. In the actual fact, lots of works have been done in the discharge determination in pipes. Lüscher et al. (2007) analyzed the performance of ultrasonic transit-time method in pipes, and concluded that a higher number of acoustic paths could decrease the integration error, however they couldn’t reduce the influence of the swirl or rotation. Hu et al. (2015) analyzed the influence of the swirl or transverse velocity to the line velocity measurements in a theoretical aspect and presented a mechanism analysis and estimation.
In this paper, the design and construction of a testing system is presented which allows to study the velocity determination and the improvement of the precision of ultrasonic transit-time flowmeters used for rectangular open channel measurements. Based on it, groups of experiments are conducted and analyzed. Especially both the transducers’ influences and how to improve the arrangements are discussed. In addition, the integration methods are also studied, with the velocity distribution taken into consideration.

2 Principle of discharge determination

2.1 Line velocity measurement

An ultrasonic pulse travels in a downstream direction faster than a similar pulse travels upstream. The speed of a pulse travelling diagonally across the flow in a downstream direction will be increased by the velocity component of the water. Conversely, the speed of a pulse moving in the opposite direction will be decreased. If two transducers in pairs are mounted at the same elevation but on different sides and locations of the channel, as in Fig.1, the line velocity can be determined using the difference in the transit time in the two directions as Eq.1.

\[ v_a = \frac{L}{2 \cos \phi} \left( \frac{1}{t_2} - \frac{1}{t_1} \right) \]  

(1)

in which, \( v_a \) is the line velocity; \( L \) is the path length (distance between transducer A and transducer B); \( \phi \) is the angle between the path and direction of flow; \( t_1 \) is the transit time from transducer A to B; \( t_2 \) is the transit time from transducer B to A.

Fig.1 Schematic illustrating the line velocity determination

2.2 Discharge determination

Based on the line velocities at different elevations, the channel discharge can be determined with the help of an integration algorithm. Up till now, the integration algorithm mostly used in open channels is the mean-section method.
In this method, the whole cross section beneath the surface is divided into three types of sub-layers: top layer, middle layers and bottom layer, as in Fig. 2.

In the top layer, the mean velocity is calculated from a limited extrapolation of the line velocities of the top two paths. In the middle layers, the mean velocities are calculated from the two line velocities measured by the paths which bound the layer. In the bottom layer, the mean velocity is calculated according to the velocity measured by the lowest path and the near-bed velocity. The near-bed velocity is commonly determined empirically. Then the sub-discharges of each layer can be determined with the layer area evaluated as the mean values of the corresponding layer, as are shown in Eq. 2–Eq. 5.

\[ Q = \sum_{i=1}^{\infty} Q_i \]  
For \( i = n+1 \)

\[ Q_{i+1} = 0.5(W_i + W_j)(h_i - h_s)\left(v_{n} + K_i v_i\right) / 1 + K_i \]  
For \( i = 2,3,\ldots,n \)

\[ Q_i = 0.25(W_{i-1} + W_j)(h_i - h_{i-1})\left(v_{i-1} + v_i\right) \]  
For \( i = 1 \)

\[ Q_1 = 0.25h_1\left(W_1 + W_0\right)\left(v_1 + K_1 v_i\right) \]  

in which, \( Q \) is the total discharge; \( Q_i \) is the sub-discharge of \( i \)-th layer; \( W_i \) is the distance wall to wall of \( i \)-th path; \( W_0 \) is the width at channel bed; \( W_1 \) is the width at flow surface; \( h_i \) is the height of \( i \)-th path from the channel bed; \( h_s \) is the height of the flow surface; \( v_i \) is the line velocity of \( i \)-th path; \( v_1 \) is the line velocity at flow surface which can be obtained by extrapolation as Eq. 6 and Eq. 7; \( K_i \) is surface factor chosen commonly between 0–1, sometimes negative values are also allowed; \( K_b \) is bottom factor normally between 0.4 and 0.8; Subscript \( i = 1,2,\ldots,n \) represents the order of path from bottom.

If \( h_i - h_s < h_s - h_{i-1} \)

\[ v_i = v_n + (v_n - v_{i-1}) \times \frac{h_i - h_s}{h_s - h_{i-1}} \]  
(6)

If \( h_i - h_s \geq h_s - h_{i-1} \)

\[ v_i = v_n + (v_n - v_{i-1}) \]  
(7)

If we know the velocity profile, the discharge can also be calculated by an analytical integration method. Giordano Law (Giordano, 1995) provides an analytical expression for the line velocity profile for open channels (see Eq. 8). It can be determined only if the parameter \( a, b \) and \( c \) are all calibrated using the measured line velocities. Then combining with the width \( W(z) \) at different elevation \( z \), the total discharge \( Q \) can be got using Eq. 9.

\[ v(z) = a \left( \frac{z}{h_i} \right)^c e^{-\frac{z}{h_i}} \]  
(8)

\[ Q = \int_{z=0}^{h} W(z) v(z) \, dz \]  
(9)

3 Testing system

3.1 General design

The testing system was built in Daxing Experimental Base of China Institute of Water Resources and Hydropower Research (IWHR). It consisted of a water tank, inflow pipes, a stilling pool, a rectangular flume, a tail gate and a tail water collecting device, as is shown in Fig. 3. The water tank has a volume of \( 7.0 \text{m} \times 6.0 \text{m} \times 13.0 \text{m} \) (length \( \times \) width \( \times \) height) and can provide a high water head and a large stable discharge for the tests. Inflow pipes, made up of two iron pipes of diameter 500mm, then supply the flow to the stilling pool. Both the master ultrasonic flow meters for pipes and flow adjusting valves are installed in the inflow pipes. The master ultrasonic flow meters were both pre-calibrated using weighing methods in the laboratory as is shown in Table 1. Indication errors were for both meters well below 0.5%, and the repeatability was better than 0.1%, which indicated that the precision and stability could both fulfill requirements.
The stilling pool is 10.0 m \times 4.8 m \text{(length \times width)}, and was designed to dissipate energy and provide stable flow for the flume. The flume was designed with a size of 26.0 m \times 2.0 m \text{(length \times width)}, and its bottom was horizontal. The test sections were set 15m~20m downstream of the beginning of the flume. At the end of the flume, a tail gate was installed to adjust the water level in the flume. In addition, a drop of 0.4 m was designed to avoid the tail water’s back-propagation’s influence.

![Diagram of the testing system](image)

**Fig. 3 General design of the testing system**

**Table 1 Calibration of master flow meter**

<table>
<thead>
<tr>
<th>Flow rate (m³/h)</th>
<th>UF1(L) errors (%)</th>
<th>Repeatability (%)</th>
<th>UF2 (R) errors (%)</th>
<th>Repeatability (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1414</td>
<td>/</td>
<td>0.31</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>1060</td>
<td>-0.18</td>
<td>0.04</td>
<td>0.36</td>
<td>0.05</td>
</tr>
<tr>
<td>707</td>
<td>-0.09</td>
<td>0.08</td>
<td>0.38</td>
<td>0.07</td>
</tr>
<tr>
<td>353</td>
<td>-0.11</td>
<td>0.08</td>
<td>0.46</td>
<td>0.05</td>
</tr>
<tr>
<td>Average errors (%)</td>
<td>-0.13</td>
<td>0.38</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>Regulating factor</td>
<td>1.0013</td>
<td>0.9962</td>
<td>/</td>
<td></td>
</tr>
</tbody>
</table>

**3.2 Ultrasonic flow meters in the open channel**

In this testing system, two sets of ultrasonic transit-time flow meters were installed, and both were designed as crossed-path systems (see Fig.4). The upstream flow meter was labelled as No.1. Its mounting seats were made of aluminum, and were embedded in the pre-drilled troughs. Their outer surfaces were flush with the corresponding sidewalls, as is shown in Fig.5, to avoid disturbing flows. For flow meter No.1, at most eight pairs of transducers were planned to be installed in the seats, which is shown in Fig.5a. They are all small cylinders with diameters of 15 mm. In addition, the path angles were designed as 60°. The downstream flow meter is labelled as No.2. Its mounting seats were installed similar to flow meter No.1. However, the transducers of flow meter No.2 are much bigger. They are semispherical with diameters of 30 mm. Only 5 pairs were fixed in the mounting seats (see Fig.5b). Additionally, the transducers can be placed in a protruded or recessed position as can be seen in Figure 5b.

![Diagram of flow meters](image)

**Fig. 4 Open channel flow meters under test**

(a) Flowmeter position

<table>
<thead>
<tr>
<th>Flowmeter</th>
<th>path height</th>
</tr>
</thead>
<tbody>
<tr>
<td>P8</td>
<td>822</td>
</tr>
<tr>
<td>P7</td>
<td>722</td>
</tr>
<tr>
<td>P6</td>
<td>622</td>
</tr>
<tr>
<td>P5</td>
<td>522</td>
</tr>
<tr>
<td>P4</td>
<td>422</td>
</tr>
<tr>
<td>P3</td>
<td>322</td>
</tr>
<tr>
<td>P2</td>
<td>222</td>
</tr>
<tr>
<td>P1</td>
<td>122</td>
</tr>
</tbody>
</table>

(b) path height (mm)

<table>
<thead>
<tr>
<th>layer no</th>
<th>path height</th>
</tr>
</thead>
<tbody>
<tr>
<td>P8</td>
<td>/</td>
</tr>
<tr>
<td>P7</td>
<td>/</td>
</tr>
<tr>
<td>P6</td>
<td>/</td>
</tr>
<tr>
<td>P5</td>
<td>845</td>
</tr>
<tr>
<td>P4</td>
<td>655</td>
</tr>
<tr>
<td>P3</td>
<td>465</td>
</tr>
<tr>
<td>P2</td>
<td>275</td>
</tr>
<tr>
<td>P1</td>
<td>85</td>
</tr>
</tbody>
</table>

(a) Small transducer (No.1)
3.3 Parameter determination

The geometrical parameters, such as flume width $W$, transducer installing height $h_i$, acoustic path length $L_i$, and water level are all of great importance in the open channel discharge determination using ultrasonic flow meter. The water level was determined by an ultrasonic water level meter. The other parameters were all measured using a FARO arm and analyzed using POLYWORKS. In details, the side walls and the bottom are fitted by points measured using the FARO arm while the distance between the side walls is calculated by POLYWORKS. If the transducer positions are adjusted, the geometrical parameters will be refreshed.

In addition, both sets of open channel ultrasonic flow meters were used to indicate the discharges in the flume. For flow meter No.1, all the transducers were specially designed. In case 1 and case 2, the transducers were protruding into the flume, while in other cases, the transducers were retracted in the seats. The integration algorithm used in all cases was the mean-section method which is described in detail in ISO6416 (2005) and section 2.2 above. $K_t$ was evaluated as 0.1 while $K_b$ took the value of 0.4. The discharge readings $Q_r$ are shown in column 4, and their corresponding measurement error $\varepsilon$, defined as in Eq.10, are shown in column 5. For flow meter No.2, the transducers were made like a semi-sphere, and occupied more space than those in flow meter No.1. The integration algorithm used is the same as that of flow meter No.1. The detailed results of flow meter No.2 are shown in column 7 and column 8.

$$\varepsilon = \frac{Q_r - Q}{Q} \times 100\% \quad (10)$$

$Q_r$ represents the discharge measurement of the tested flow meter.

### Table 2 Tested cases and measurement errors of flow meter No.1 & No.2
According to Table 2, it can be concluded that the transducers’ sizes play an important part in the discharge measurements. Clearly the bigger the transducers are protruding into the flume, the more their influences are. For flow meter No. 1, although the transducers are made very small, their influences are still apparent depending on their way of mounting. When the transducers are retracted in the seats, they only produce an average measurement error of 0.35%. However, when the transducers are protruding into the flume, the average measurement error increases more than double to 0.82%. If the transducer size is much larger, protruding into the flume produces much more measurement errors. For flow meter No.2, the transducers are much larger than those of flow meter No.1 with the diameter of 15mm, and produce as large measurement errors as 3.50~4.01%. In order to analyze the mechanism, a numerical simulation using Ansys Fluent is conducted and the result is shown as Fig.7. It can be seen that larger transducers will occupy more space, and will disturb the flows around the transducers more heavily. Clearly it will influence the time measurements and thus the discharge measurement. Besides, transducers can also cause the overestimation of the path length. In these physical model tests, a protrusion of 1mm can make an error of about 0.05% in path length. Thus to minimize the measurement errors of discharges, the transducers should be made small, and should be retracted in the mounting seats if possible. In addition, in order to give an accurate analysis as possible, flow meter No.1 was chosen to conduct further tests, and small transducers were retracted in the mounting seats.

Table 3 Tested cases and line velocities of each path

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Q (m³/h)</th>
<th>ε₁ (%)</th>
<th>h₁ (m)</th>
<th>v₁ (m/s)</th>
<th>v₂ (m/s)</th>
<th>v₃ (m/s)</th>
<th>v₄ (m/s)</th>
<th>v₅ (m/s)</th>
<th>v₆ (m/s)</th>
<th>v₇ (m/s)</th>
<th>v₈ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1500.819</td>
<td>0.50%</td>
<td>0.851</td>
<td>0.233</td>
<td>0.238</td>
<td>0.245</td>
<td>0.247</td>
<td>0.251</td>
<td>0.257</td>
<td>0.257</td>
<td>0.257</td>
</tr>
<tr>
<td>9</td>
<td>2426.366</td>
<td>0.75%</td>
<td>0.930</td>
<td>0.337</td>
<td>0.345</td>
<td>0.356</td>
<td>0.362</td>
<td>0.368</td>
<td>0.379</td>
<td>0.381</td>
<td>0.389</td>
</tr>
<tr>
<td>10</td>
<td>2699.328</td>
<td>0.49%</td>
<td>0.897</td>
<td>0.391</td>
<td>0.400</td>
<td>0.412</td>
<td>0.417</td>
<td>0.426</td>
<td>0.438</td>
<td>0.441</td>
<td>0.451</td>
</tr>
<tr>
<td>11</td>
<td>3363.608</td>
<td>0.33%</td>
<td>0.949</td>
<td>0.454</td>
<td>0.465</td>
<td>0.479</td>
<td>0.487</td>
<td>0.498</td>
<td>0.514</td>
<td>0.521</td>
<td>0.537</td>
</tr>
<tr>
<td>12</td>
<td>3505.288</td>
<td>0.53%</td>
<td>0.930</td>
<td>0.487</td>
<td>0.500</td>
<td>0.513</td>
<td>0.520</td>
<td>0.530</td>
<td>0.545</td>
<td>0.553</td>
<td>0.568</td>
</tr>
<tr>
<td>13</td>
<td>1374.462</td>
<td>-0.35%</td>
<td>0.789</td>
<td>0.229</td>
<td>0.234</td>
<td>0.243</td>
<td>0.246</td>
<td>0.250</td>
<td>0.257</td>
<td>0.255</td>
<td>--</td>
</tr>
<tr>
<td>14</td>
<td>1381.112</td>
<td>-0.31%</td>
<td>0.653</td>
<td>0.280</td>
<td>0.286</td>
<td>0.296</td>
<td>0.303</td>
<td>0.310</td>
<td>0.319</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>15</td>
<td>2413.577</td>
<td>1.00%</td>
<td>0.568</td>
<td>0.567</td>
<td>0.585</td>
<td>0.606</td>
<td>0.618</td>
<td>0.624</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>16</td>
<td>1354.648</td>
<td>1.60%</td>
<td>0.416</td>
<td>0.366</td>
<td>0.377</td>
<td>0.391</td>
<td>0.394</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>17</td>
<td>1362.486</td>
<td>1.58%</td>
<td>0.501</td>
<td>0.449</td>
<td>0.463</td>
<td>0.475</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

Note: vₙ represents the line velocity of φₙ path from the bottom.
large and so all 8 paths were in operation. And in cases 13 to 17, the flow depths were smaller and therefore less than 8 paths could be used. The line velocities of each path were measured and calculated based on cross-paths systems, and the detailed results are shown in Table 3. To plot the line velocities as a function of the path elevation in a dimensionless form results in Fig.8. V represents the average value of all the measured line velocities in the corresponding case.

Fig.8 Line velocity upon mounting elevation in a dimensionless form

From Fig.8, it can be seen that when $h/W_i<0.45$ dimensionless velocity $v/V$ increases with the dimensionless height $h/h_i$ for all investigated cases. However, the rate of increase of $v/V$ performs differently depending on $h/h_i$. When $h/h_i$ is larger than 0.7, the rate of increase of $v/V$ gets diffused with increasing $h/h_i$. The corresponding line velocities are generally smaller than the predicted values which would be obtained by extrapolation of the rate of increase of the line velocities of the underlying paths. Therefore if both top paths are lying in the range $h/h_i<0.7$, the surface velocity by extrapolation based on the top two paths may be too large, which would partly contributed to the positive error of discharge measurements. In case 16 and case 17, only one path is lying in the range of $h/h_i>0.7$, and the extrapolation of the surface velocity using the top two paths may also generate relatively larger prediction errors. Thus at least two paths should be mounted in the range $h/h_i>0.7$ to minimize the error produced by the surface velocity estimation. Through further analysis it is believed that the above phenomenon is caused by the influence of the free surface. Yang et al. (2004) proposed a method to determine the influence scope of the free surface using Eq.11

$$h_m = \frac{1}{1 + \alpha}$$

in which, $h_m$ is the distance of the maximum velocity point from the bed, and $h_r-h_m$ is the influence depth of the free surface; $\alpha$ is the only parameter and can be estimated by Eq.12 in this paper.

$$\alpha = 1.3 \exp\left(-\frac{W_i}{2h_i}\right)$$

Substituting hydraulic parameters of Case 8~17 can obtain that $\alpha$ is lying in the range of 0.117~0.453 and the corresponding $h_m/h_i$ is within 0.688~0.895. In addition, the value of $h_m/h_i$ decreases with the increasing flow depth when the channel wide is fixed. Clearly the minimum value of $h_m/h_i$ of 0.688 (≈0.7) in these cases verifies that the influence scope of free surface is in the range of $h/h_i>0.7$ and only at least two paths mounted in the range $h/h_i>0.7$ can well reflect the free surface’s influence. For paths which are lying in the range $h/h_i<0.7$ and are not close to the bottom, $v/V$ depends approximately linearly on $h/h_i$. And theoretically the line velocities at this range could be fully derived according to the line velocities obtained by neighboring paths. In another word, only limited paths need to be arranged in this range. By contrast, for the range where $h/h_i$ approaches 0, $v/V$ is influenced not only by neighboring path velocities, but also by the boundary layer at the bottom. In order to estimate the velocity profile in this range, maybe more paths can be mounted or the integration algorithm can be improved. For the cases of $h/W_i>0.45$, the critical value of 0.7 in this physical model tests might be improper and should be reevaluated. However, the division methods of the flow is still valid, as the flow mechanism keeps unchanged. If the method proposed by Yang et al. (2004) is used, $\alpha$ can be increased to the maximum of 1.3 and therefore the critical value can be decreased to the minimum of 0.435. However, the absolute value can be obtained only when $h/W_i$ is determined.

In addition, transverse velocities were also unavoidably included in the path velocities due to diagonal planes, such as Case 8 with its transverse velocity distribution shown in Fig.9. In order to indicate the influence of the transverse velocity, Fig.10 is presented with $v_x$ representing the transverse velocity and $v_y$ representing the longitudinal velocity. Clearly the velocity along the path line should be measured as $v_x \cos \phi$ assuming $v_y=0$. However, the transverse velocity will exert an influence on the line velocity measurement. For the transducers of Group A, the measured line velocity will be added by $v_y \sin \phi$. When calculating
longitudinal velocity \( v_A \), the influence of transverse velocity will be divided by \( \cos \phi \), which is shown as Eq.13. For the transducers of Group B, \( v_B \) can be obtained similarly as Eq.14.

\[
\begin{align*}
v_A &= \left( v_x \cos \phi + v_y \sin \phi \right) / \cos \phi \\
&= v_x + v_y \tan \phi
\end{align*}
\]  

Clearly the flow velocities derived from the measured line velocities are not accurate enough, and unavoidably the influence of transverse velocity is included. However, if transducers of Group B are used as in Fig.10, the calculated longitudinal velocity \( v_B \) can be expressed as Eq.14. Combining Eq.13 and Eq.14 can derive Eq.15, and \( v_x \) can be obtained with influence of \( v_y \) being counteracted.

\[
\begin{align*}
v_B &= \left( v_x \cos \phi - v_y \sin \phi \right) / \cos \phi \\
&= v_x - v_y \tan \phi
\end{align*}
\]

In the actual fact, the transverse velocities are always large, especially the inflow conditions are not good enough. For Case 8, the largest transverse line velocity has a maximum value of 3.7% of the corresponding average line velocity in flow direction. Thus a crossed-path system is suggested if possible.

\[
\begin{align*}
v_x &= \left( v_A + v_B \right) / 2
\end{align*}
\]  

In this figure, case 9 and case 15 are taken as examples for further analysis. The black and red solid polylines represent the line velocity profiles obtained by the mean-section method and parameter \( K_t \) was set to 0.1 while \( K_b \) to 0.4. The surface and bottom velocities were not obtained by measuring, but extrapolated using the neighbor line velocities. The blue and grey dashed curves represent approximated velocity profiles based on Giordano Law using the least square method. It can be seen clearly that in the intermediate layers, the velocity profiles obtained by the mean-section method and the Giordano Law approximation can fit well with each other. The main differences mainly occur in the top and the bottom layers. The approximations of the top layer are lying above the corresponding top paths while the bottom layer are lying below the corresponding lowest paths. Both integrated methods produce different velocity profiles in these two layers, thus producing different integration errors.
Line-velocity profile characteristics vary depending on the elevations. Therefore when conducting an error analysis for the discharge evaluation, the total flow depth can be divided into three parts. The flow below the lowest path is named Part I. The flow above the top path is named Part III. Part II is lying between Part I and Part III. In order to make a comprehensive analysis, all the 39 cases are used.

For Part II, both the mean-section method and the Giordano Law method produce accurate sub-discharge determinations. However, as the integration methods are different, the sub-discharges calculated by these two methods are not exactly the same. In this section, the sub-discharges of Part II $Q_{II}$ were calculated for all the 39 cases using both integration algorithms. The results, as shown in Fig. 12, indicate that the sub-discharge differences $\Delta_{II}$ are very small, less than 0.1% of the total discharges in the corresponding cases. And more paths produce smaller sub-discharge differences $\Delta_{II}$ between two integration methods. The reason for the small errors in three-path cases might result from compensation effects of the approximation.

For Part I, the Giordano Law method can give well-defined sub-discharges while the mean-section method will provide a series of values for each case depending on the choice of $K_b$. Thus, the results of both methods might differ more. In this section, both methods are used. Especially for the mean-section method, $K_b$ commonly takes values in the range of 0–1 according to the ANSI 6416 norm (ISO 6416 2005). Thus 9 values between 0.1 and 0.9 with an interval of 0.1 between them are chosen. The results are all shown in Fig. 13, in which $\Delta_I$ represents the differences of the calculated results by both methods and $\Delta_I/Q$ indicates their relative measurement errors over corresponding total discharge. Clearly, the more paths are mounted, the smaller the differences of both algorithms are. And $\Delta_I/Q$ is strongly related with $K_b$. For the investigated cases, the differences between the two algorithms are getting smaller with the increasing $K_b$, with $K_b$ evaluated as 0.1. When $K_b$ takes an inappropriate value, such as 0.1, mean-section method and Giordano Law could generate differences larger than 10% of the corresponding total discharges. For the tested cases, values of $K_b$ around 0.9 show relatively small differences between the mean-section method and Giordano Law in evaluating sub-discharges of Part I. In a word, the accuracy of $Q_I$’s evaluation can greatly influence the determination of total discharges, thus $K_b$ should be optimized when using the mean section method.

Fig. 11 shows the line-velocities and the paths’ elevations. Clearly the lowest path is still in the main flow region. And its line-velocity may not reflect the characteristics of line velocities in Part I. So choosing $K_b$ was done by experience. Thus in order to evaluate $K_b$ as well as the sub-discharge of Part I as well as possible, the lowest path position should be lowered and the line-velocities in the boundary at the bottom should be known. The lowering must however take into account that reflection on the bottom might lower the reliability of determining the lowest path.

For Part I, the Giordano Law method can give well-defined sub-discharges while the mean-section method will provide a series of values for each case depending on the choice of $K_b$. Thus, the results of both methods might differ more. In this section, both methods are used. Especially for the mean-section method, $K_b$ commonly takes values in the range of 0–1 according to the ANSI 6416 norm (ISO 6416 2005). Thus 9 values between 0.1 and 0.9 with an interval of 0.1 between them are chosen. The results are all shown in Fig. 13, in which $\Delta_I$ represents the differences of the calculated results by both methods and $\Delta_I/Q$ indicates their relative measurement errors over corresponding total discharge. Clearly, the more paths are mounted, the smaller the differences of both algorithms are. And $\Delta_I/Q$ is strongly related with $K_b$. For the investigated cases, the differences between the two algorithms are getting smaller with the increasing $K_b$, with $K_b$ evaluated as 0.1. When $K_b$ takes an inappropriate value, such as 0.1, mean-section method and Giordano Law could generate differences larger than 10% of the corresponding total discharges. For the tested cases, values of $K_b$ around 0.9 show relatively small differences between the mean-section method and Giordano Law in evaluating sub-discharges of Part I. In a word, the accuracy of $Q_I$’s evaluation can greatly influence the determination of total discharges, thus $K_b$ should be optimized when using the mean section method.
For Part III, the Giordano Law uses the line-velocity profiles above the top path according to the fitted curve by all installed paths, and the sub-discharges in Part III are also obtained by numerical integration of the analytical profile (equation 8). In contrast, the mean-section method uses surface velocity by linear extrapolation based on the top two paths. Therefore, the line-velocity profiles of the two integration methods are different, leading to different sub-discharges of Part III. Additionally, as the flow surface varies depending on the height difference $H-h_t$, the choice of $K_t$ is difficult and might be made dependent on $H-h_t$. In this measurement campaign, $K_t$ is evaluated in the range of 0.1 to 1.0, and all the 39 cases are used to further analysis. The results are shown in Fig.14, and $\Delta III$ represent the sub-discharge differences of both methods for each case.

From Fig.14, it can be seen that if $(H-h_t)/H$ is small, the relative error $\Delta III/Q$ was also small and the choice of $K_t$ has little influence on $\Delta III/Q$ (see $(H-h_t)/H<0.15$). That’s because for these cases, the water level is close to the installation height of the top path: the top layer is always rather thin and the sub-discharge of Part III is only a small portion of the total flow. As $(H-h_t)/H$ gets larger, the top layer gets thicker, and the corresponding sub-discharge of Part III gets larger. Any small changes of $K_t$ produce in these cases, relatively large errors $\Delta III/Q$.

Taking the conclusions obtained by Fig.8 into considerations, one can conclude that in order to obtain relative accurate measurements of sub-discharge of Part III, enough paths should be arranged in the depth range $h/H>0.7$ and $(H-h_t)/H$ should be minimized if possible. Here too, the top path cannot be too close to the water level due to the ultrasonic reflexion on the water surface. If the water level is not constant during the operation of the channel, switch on and off strategies have to be implemented.

Comparing Part I, Part II and Part III, it can be concluded that the sub-discharges differences in Part II are small between Giordano Law and mean section method. However, for Part III, the sub-discharges differences in Part III are a little larger. In comparison, the sub-discharges in Part I show substantial differences between two methods. Improper evaluation for $K_b$ in mean section method can produce as large discharge differences as 12% from Giordano Law, especially in cases of small discharges. Thus, it was very important to properly determine $K_b$.

5 Conclusions

The Ultrasonic transit-time method is widely used for the discharge determination in open channels. In order to verify the performance of ultrasonic transit-
time flow meters and to analyze their errors, a testing system was designed and 39 cases were tested. The main conclusions are as follows:

1) The Ultrasonic transit-time method can be used to determine the discharge in open channels.

2) The transducers of ultrasonic open channel flow meters always protrude into the channel and therefore disturb the local velocity field, which introduces some errors in the determination of the discharge. Thus the transducers should be made as small as possible, and be mounted into the corresponding side walls if possible. This effect is more dominant in smaller channels as the one used in this test study. Compensation methods are complicated and need detailed flow field measurements or simulation around the transducers.

3) More paths produce smaller errors if properly mounted. It is better to mount at least two paths in the range of \( h/H > 0.7 \) in order to accurately extrapolate the line velocity profile at the surface, and the height difference \( H-h_i \) should not be too large.

4) Integration methods play an important part in the determination of the discharge in open channels. In this paper, the Giordano Law and mean-section methods (ISO6416) were used for further analysis. The results of the tests show that the sub-discharges between the top path and the lowest path are largely identical but with minor differences between two methods. The sub-discharges differences above the top path can be narrowed if the distance between surface and top path height is reduced. However, the Giordano Law and mean-section methods show substantial differences in the determination of sub-discharges below the lowest path. Especially the sub-discharge determination by the mean-section method varies greatly with \( K_b \). Commonly large \( K_b \), for example 0.9, could produce small difference from the Giordano Law.

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Reference


