Evaluation results of the new standard gas flow system at VMI: Piston prover system

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Abstract

Currently many National Metrology Institute (NMIs) as well as advanced calibration laboratories are using piston gas flow standards with mercury sealing method for gas flow calibration in the low pressure range. The flow in the small range is about cc/min. In this work, the low pressure gas flow calibration system at VMI is presented, designed and manufactured by the Taiwan National Metrology Institute, CMS/ITRI. The flow range is within (0.002 ÷ 24) L/min. The uncertainty of the reference system is assessed against the ISO/IEC Guide 98-3:2008 document, Uncertainty is evaluated from individual influence sources such as category A and B assessments. The standard uncertainty/relative standard uncertainty and degrees of freedom of the sources can be evaluated individually and then combined to produce a composite standard uncertainty/combined relative standard uncertainty and an effective degree of freedom. Finally, the relative expanded uncertainty is obtained by multiplying the relative standard uncertainty associated with a coverage factor at the 95% confidence level of the measurement result.

1. Introduction

The previous standard gas flow standard system that VMI is using is the bell prover, PVTt standards with volumes of 50L, 100L, 1000L. However, one limitation of these reference systems is that the minimum flowrate they can achieve is 0.3 L/min.
A necessity in metrology management, in order to control flow meters with a flow range of cc/min, it is necessary for VMI to equip a standard system with a flow range of cc/min.

VMI's Piston Prover - low pressure primary gas flow calibration system consists of five precision engineered cylinders of different diameters accompanied by a laser doppler scale fixed on a moving object, which can measure distances between the starting and ending points of the piston. (Currently the position of the begin and end points determined by the fiber-optic sensor on the tube has been fixed – Figure 1)

Before calibrating mass flow or volume flow, it is necessary to evaluate the volume and associated uncertainty of the piston. The volume of the piston is the product of the average of the inside diameter of the reference cylinder and the displacement of the piston as measured by a Doppler Laser scale. To evaluate the volume, the inside diameter of each reference cylinder is measured and the associated uncertainty is estimated. The inner diameters of the cylinders were determined by direct comparison with ring gauges of equivalent diameter and matched to the diameter of each cylinder through a Linear Variable Differential Transformer (LVDT) comparator. The temperature and flow pressure when inside the standard cylinder are measured by 4-wire resistance thermometers (PT100), digital pressure CPG 2500 continuously during the piston movement from the starting point to the ending point. The piston travel time is measured by the Keysight 53220A frequency counter. All devices are controlled by the computer through an intuitive software. The reference system is operated in a room with controlled ambient conditions within (23 ± 1) °C.

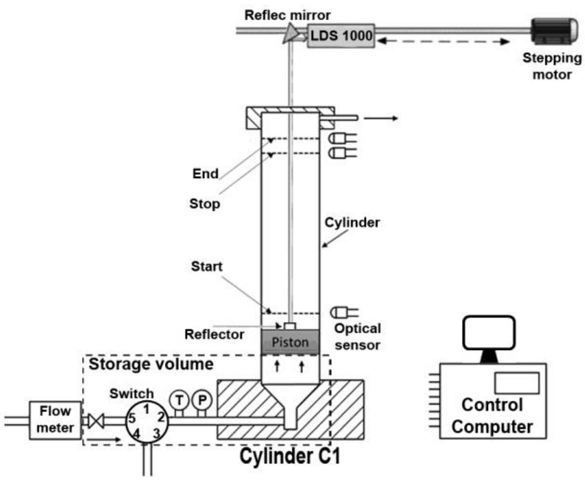

Figure 1: Schematic diagram of piston prover

2. Standard flow setting

As introduced in part 1, the piston prover at VMI consists of 5 cylinders with diameters of 16.5-27-45-80-160 mm respectively. The flow that can be controlled in by 5 cylinders is (0.002 ÷ 24) L/min. To vary the measuring range between the cylinders, the Laser interferometer fixed to the top of the cylinders can be moved parallel to the line through the center of the cylinders by means of a stepper motor.

Prior to flow assessment, component measuring devices (Pt-100, pressure sensor, laser interferometer, diameter reference ring gauge...) must be periodically calibrated in qualified laboratories. The cylinder reference tubes are internally cleaned to ensure that the inside diameter measurement is accurate and that the piston moves evenly. The collected volume (V_C) inside the cylinders is determined by formula (1).

$$V_c = \frac{\pi}{4} \times D^2 \times L \qquad (1)$$

Where, D is the average diameter of each cylinder [m], L is the displacement of the piston [m]. D is determined by averaging 10 diameter values at 10 relatively equidistant locations along the cylinder from the beginning to the end.

Prior to setting the reference flow rate, the cylinder gas shall be maintained in working condition and sufficient to maintain the required flow during the set-up without additional addition. The system of standard cylinders must ensure tightness, the leakage gas flow must not exceed the values specified in Table 1.

Table 1: Leakage flow limit

Cylinder	C1	C2	C3	C4	C5
q_L (ccm)	0,001	0,004	0.010	0,020	0,100

The flow rate of gas entering the cylinder is determined by formula (2).

$$q = \frac{V_c \times \rho_e}{t} + q_{m_l} \qquad (2)$$

with ρ_e determined by formula (3).

$$\rho_e = \frac{P_e.M}{Z_{(P_e,T_e)} \times R \times T_e} \qquad (3)$$

Where, ρ_e is air density in the cylinder at the end position [kg/m^3]; t is the time the piston moves from the beginning to the end [s]; q_{m_l} is the leak gas flow [kg/m^3].

3. Uncertainty evaluation

The uncertainty of the piston reference system is evaluated according to the guidance of ISO/IEC Guide 98-3:2008 [1] consisting of uncertainty type A and uncertainty type B. Class A is the uncertainty was determined by statistical analysis of a series of observed values. Class B is measurement uncertainty determined through past measurement data of component devices, data provided during adjustment and certification, etc.

The combined standard uncertainty u_C is the square root of the sum of the squares of u_A and u_B determined by formula (4).

$$u_C = \sqrt{\sum u_{Ai}^2 + \sum u_{Bi}^2} \qquad (4)$$

To determine the extended uncertainty U, the confidence level is expressed by the coverage coefficient k, which is the coefficient obtained from the Student's distribution through the number of degrees of freedom calculated by the Welch–Satterthwaite formula.

$$\vartheta_{eff} = \frac{u_C^4(y_i)}{\sum_{i=1}^{N} \frac{[c_i u(x_i)]^4}{\vartheta_i}} \qquad (5)$$

$$U = k.u_C \qquad (6)$$

3.1 The mathematical model

A typical system using a Piston gas flow standard to calibration the gas flow meter consists of the gas meter under test, the connecting pipes and the switching valve connected to the inlet of the volume reference pipe. Determine the control volume inside the pipe (V) from the outlet position of the test meter to the end position of the piston. Based on the law of conservation of mass produces the following formula:

$$0 = \frac{\partial}{\partial t}\int_V \rho.dV + \int_A \rho.\vec{v}.d\vec{A} \qquad (7)$$

Where: $\frac{\partial}{\partial t}$ is partial differential to time; ρ is air density in the control volume; $\vec{v}$ is the velocity of flow [m/s]; V is the control volume [m^3]; $d\vec{A}$ is the unit vector of control surface.

Based on the obtained data of the component standards are the cylinder pipe diameter, distance and time of piston displacement, temperature, gas pressure inside the cylinder... the flow rate q_m passing through the gas meter is calculated, determine according to formula (8).

$$q_m = -\int_A \rho.\vec{v}.d\vec{A} = \frac{\partial}{\partial t}\int_V \rho.dV = q_0 + q$$

$$q_m = \frac{1}{t}(V_c.\rho_e + V_0.\rho_e - V_0.\rho_b) + \rho_e.q_{V,l}$$

$$q_m = \frac{\pi.D^2.L.\rho_e}{4.t} + \frac{V_0.\Delta\rho_{eb}}{t} + \rho_e.q_{V,l} \qquad (8)$$

Where: q_0 is gas flow into volume V_0 in time t; V_0 is storage volume - the inventory volume between the exit of the gas meter and the start location on the cylinder; ρ_b is air density in the cylinder at the beginning of calibration [kg/m^3]; ρ_e is air density in the cylinder at the end of calibration [kg/m^3], $\Delta\rho_{eb}$ is the density change of the gas in the storage volume ($\Delta\rho_{eb} = \rho_e - \rho_b$) [kg/m^3]; $q_{V,l}$ is leakage gas volume flow [m^3/s]; t is collection time.

3.2 Evaluation of the measurement uncertainty of component standards

The measurement uncertainty of the mass flow rate according to (8) is determined:

$$u(q_m) = [C_D{}^2 u_D{}^2 + C_L{}^2 u_L{}^2 + C_{\rho_e}{}^2 u_{\rho_e}{}^2 + C_{V_0}{}^2 u_{V_0}{}^2 + C_{\Delta\rho_{eb}}{}^2 u_{\Delta\rho_{eb}}{}^2 + C_t{}^2 u_t{}^2 + C_{q_{V,l}}{}^2 u_{q_{V,l}}{}^2]^{1/2} \qquad (9)$$

The sensitivity coefficients can be obtained by differentiation of Eq. (8). Thus the relative sensitivity coefficients are, omitting units:

$$C_D = 2.\frac{\pi.D.L.\rho_e}{4.t}; \quad C_L = \frac{\pi.D^2.\rho_e}{4.t};$$
$$C_{\rho_e} = \frac{\pi.D^2.L}{4.t} + q_{V,l}; \quad C_{V_0} = \frac{\Delta\rho_{eb}}{t}$$
$$C_{\Delta\rho_{eb}} = \frac{V_0}{t}; \quad C_{q_{V,l}} = \rho_e \qquad (10)$$
$$C_t = -\frac{1}{t^2}\left(\frac{\pi.D^2.L.\rho_e}{4} + V_0.\Delta\rho_{eb}\right)$$

The relative uncertainty of mass flow rate is expressed as follows:

$$\left[\frac{u(q_m)}{q_m}\right]^2 = \left[\frac{u(q_m)}{\frac{\pi.D^2.L.\rho_e}{4.t} + \frac{V_0.\Delta\rho_{eb}}{t} + \rho_e.q_{V,l}}\right]^2$$

$$\left[\frac{u(q_m)}{q_m}\right]^2 \cong \left[\frac{u(q_m)}{\frac{\pi.D^2.L.\rho_e}{4.t}}\right]^2 \qquad (11)$$

Combine Eq. (9) with Eqs. (10) and (11) is reduced to:

$$\left[\frac{u(q_m)}{q_m}\right]^2 \cong \left(2.\frac{u_D}{D}\right)^2 + \left(\frac{u_L}{L}\right)^2 + \left(\frac{u_{\rho_e}}{\rho_e}\right)^2 + \left(\frac{4.\Delta\rho_{eb}.u_{V_0}}{\rho_e.\pi.D^2.L}\right)^2 + \left(\frac{4V_0.u_{\Delta\rho_{eb}}}{\pi.D^2.L.\rho_e}\right)^2 + \left(-\frac{u_t}{t}\right)^2 + \left(4.t.\frac{u_{q_{V,l}}}{\pi.D^2.L}\right)^2 \qquad (12)$$

3.2.1 Uncertainty of cylinder diameter $\frac{u_D}{D}$

Cylinder diameter was measured at 23 °C by comparison method with reference rings gauge. Each reference tube is measured in diameter at 10 points relatively equidistant from the starting position to the end point. At each measuring position, two orthogonal diameters were determined and measured in duplicate five times. The diameter of each cylinder tube is the average of the measurements.

The u_D uncertainty is evaluated by influencing factors: repeatability measurement, standard used, thermal expansion of the reference rings gauge and cylinder, temperature measurement. The results of the measurements are in Table 2.

Table 2: Cylinder diameters and uncertainties.

Cylinder	**D** (mm)	**u$_D$** (µm)	**2u$_D$/D** (%)
C1	16.5074	3.209	0.039
C2	27.0013	3.608	0.027
C3	44.9841	3.735	0.017
C4	79.9839	3.814	0.010
C5	160.0264	4.327	0.006

3.2.2 Uncertainty of piston stroke $\frac{u_L}{L}$

The travel of the piston from the starting point to the end point is measured with a laser doppler scale. The uncertainty of the standard used from the calibration results is 0.5 µm (<0.0001%). The uncertainty of displacement measurement is taken as the maximum value of the laser doppler calibration result deviation of 0.7 µm and calculated according to a rectangular distribution. The results are in Table 3.

Table 3: Piston displacement and uncertainty.

Cylinder	**L (mm)**	**u$_L$ (µm)**	**u$_L$/L (%)**
C1	25.0410	0.7	0.003
C2	599.5775	0.7	< 0.001
C3	600.7166	0.7	< 0.001
C4	599.2048	0.7	< 0.001
C5	579.4260	0.7	< 0.001

3.2.3 Uncertainty of the air density $\frac{u_{\rho e}}{\rho_e}$

The density of the gas in the cylinder is calculated according to formula (3) from the measurement results of gas temperature and pressure. From (3), the uncertainty $\frac{u_{\rho e}}{\rho_e}$ is determined by formula (13).

$$\left(\frac{u_{\rho_e}}{\rho_e}\right)^2 = \left(\frac{u_{T_e}}{T_e}\right)^2 + \left(\frac{u_{P_e}}{P_e}\right)^2 + \left(\frac{u_M}{M}\right)^2 + \left(\frac{u_R}{R}\right)^2 + \left[\frac{u_{Z_{(P_e,T_e)}}}{Z_{(P_e,T_e)}}\right]^2 \qquad (13)$$

1) *Relative uncertainty of temperature* $\frac{u_{T_e}}{T_e}$

The measurement uncertainty of the temperature measurement is determined by the influencing factors: the standard used, the stability, the uniformity, the difference when performing the linearization of the calibration results.

- Calibration results of a standard thermometer with an uncertainty of 0.015 ºC
- Temperature sensor stability is estimated ±0.01 ºC, assuming a rectangular distribution: 0.01 ºC/ √ 3 = 0.007 ºC.
- The non-uniform of gas and cylinder surface temperatures during calibration is ± 0.10 ºC, assumed to be a rectangular distribution: 0.10 ºC/ √ 3 = 0.071 ºC.
- Linearize the calibration results of the standard thermometer to the form T_e = a.T + b, the difference is calculated by the method of least squares, the result is 0.006 °C.

The combined uncertainty is 0.072 ºC. The working temperature of the gas is 295.15 K (23 ºC), the relative standard uncertainty of temperature is 0.024 %.

2) *Relative uncertainty of pressure* $\frac{u_{P_e}}{P_e}$

The uncertainty of the pressure measurement is determined by the influencing factors: the standard used, the stability of the pressure during piston movement, the difference when performing the linearization of the calibration results.

- Calibrated standard uncertainty of pressure meter is 3.80 Pa
- The fluctuation of the pressure in the cylinder is taken as the maximum allowable value of 10 Pa, assuming a rectangular distribution: 10 Pa/(2. √ 3) = 3.54 Pa.
- Linearize the digital pressure calibration result to the form P_e = a.P + b, the difference is calculated by the method of least squares, the result is 7.37 Pa.

The combined uncertainty is 9.02 Pa. The lowest operating pressure Pe = 100 kPa, that was employed to evaluate the relative standard uncertainty. Therefore, the relative standard uncertainty is 0.009 %.

3) *Relative uncertainty of the gas molecular mass* $\frac{u_M}{M}$

The molecular mass of gas is referred to publication of NIST [6]. The relative standard uncertainty of air is less than 0.019 %. The relative standard uncertainty of the other gases are less than 0.002 %

4) *Relative uncertainty of the universal gas constant* $\frac{u_R}{R}$:

The universal gas constant is a fixed number, thus the uncertainty can be ignored.

5) *Relative uncertainty of compression coefficient of the gas* $\frac{u_{Z_{(P_e,T_e)}}}{Z_{(P_e,T_e)}}$

The uncertainty of gas compressibility constant Z derived from NASA TN D-2565 [7] is 0.02%.

The relative standard uncertainty due to the measurement of gas density $\frac{u_{\rho e}}{\rho_e}$ = 0.033 %.

3.2.4 Uncertainty of the density change in the storage volume $\frac{u_{\Delta\rho_{eb}}}{\rho_e}$

The change in density of gas in the storage volume is the change in mass in that volume. $\frac{u_{\Delta\rho_{eb}}}{\rho_e}$ will be obtained by changing the mass of the storage volume with the mass of gas in the cylinder.

$$\frac{u_{\Delta\rho_{eb}}}{\rho_e} \cong \frac{\Delta m_{eb}}{\mathrm{m}_C} = \frac{V_0 \times \left(\frac{P_e}{T_e} - \frac{P_b}{T_b}\right) \times T_e}{V_C \times P_e} \qquad (14)$$

The evaluation results for each cylinder are shown in Table 4.

Table 4:

Cylinder	V_0 (m^3)	$\frac{u_{\Delta\rho_{eb}}}{\rho_e}$ (%)	$\frac{4V_0}{\pi.D^2.L} \times \frac{u_{\Delta\rho_{eb}}}{\rho_e}$ (%)
C1	2.0x10^{-5}	0.028	0.010
C2	2.7x10^{-4}	0,019	0.015
C3	4.0x10^{-4}	0.014	0.006
C4	1.2x10^{-3}	0.008	0.003
C5	2.5x10^{-3}	0.002	0.000

3.2.5 Uncertainty of the storage volume $\frac{u_{V_0}}{V_C}$

Each standard pipe has a different storage volume, this volume is determined by the algebraic method of adding the volumes of the pipes, valve chambers, connectors… The error of this measurement is estimated to be 10 %. The results are shown in Table 5.

3.2.6 Uncertainty of time measurement $\frac{u_t}{t}$

The uncertainty of the time measurement is considered by the individual influencing factors: the reference device, the synchronization, the resolution, the bias of the timer.

Table 5:

Cylinder	V_0 (m^3)	V_C (m^3)	$\frac{u_{V_0}}{V_C}$ (%)	$\frac{4.\Delta\rho_{eb}.u_{V_0}}{\rho_e.\pi.D^2.L}$ (%)
C1	$2.0x10^{-5}$	$5.34x\ 10^{-5}$	0.037	0.001
C2	$2.7x10^{-4}$	$3.43x10^{-4}$	0.079	0.001
C3	$4.0x10^{-4}$	$9.54x10^{-4}$	0.042	0.001
C4	$1.2x10^{-3}$	$3.01x10^{-3}$	0.040	0.000
C5	$2.5x10^{-3}$	$1.16x10^{-2}$	0.021	0.000

- The timer KEYSIGHT/53220A is used to measure the diversion time. The relative uncertainty of the instrument is $6x10^{-8}$. Thus, the uncertainty of this item can be ignored.
- System timer and counter synchronization test showing that the maximum error is less than is 0.001 s. It is assumed as a rectangular uncertainty distribution. The air collection time of the piston prover is at least 30 s, the relative standard uncertainty is 0.001 s/(30x √3) = 0.002 %.
- The bias of the timer is 0.03 μHz/Hz, so the uncertainty is $3x10^{-7}$ %, can be neglected.
- The resolution of timer is 0.0001 seconds, and it is assumed as a rectangular uncertainty distribution, the relative standard uncertainty is 0.0001 s/(30x2 √3) = 10^{-4} %.

Combined relative standard uncertainty of time measurement $\frac{u_t}{t}$ =0.002%

3.2.7 Uncertainty of the leakage of volume

The maximum estimated leak gas flow rate is as shown in Table 1. Relative uncertainty is calculated for the worst case at the minimum flow rate of each cylinder. The results are in Table 6.

Table 6:

Cylinder	q_L (ccm)	Q_{min} (ccm)	$u(q_L)$ (ccm)	$u(q_L)/Q_{min}$ (%)
C1	0.001	2	0.001	0.029
C2	0.004	25	0.002	0.009
C3	0.010	100	0.006	0.006
C4	0.020	400	0.012	0.003
C5	0.100	2000	0.058	0.003

3.2.8 Expanded uncertainty

The expanded uncertainty at the 95% confidence level is obtained by multiplying the combined standard uncertainty with the coverage factor. In this study, k = 2 is taken as the coverage factor without the need to calculate the number of degrees of freedom [2]. Table 7, the uncertainty budget of cylinder 1 is the largest value of 0.12 % with 95 % confidence level.

With the same evaluation procedure, the uncertainties for the other cylinders are given in Table 8. The largest value of uncertainty is in cylinder 1 at a minimum flow rate of 2 ccm.

Table 7: Uncertainty budget of the cylinder 1

Components of uncertainty	$\frac{u(x_i)}{x_i}$ (%)	c_i	$c_i \times \frac{u(x_i)}{x_i}$ (%)
u_D	0.019	2	0.039
u_L	0.003	1	0.003
u_{ρ_e}	0.033	1	0.033
u_{T_e}	0.024	1	0.024
u_{P_e}	0.009	1	0.009
u_M	0.019	1	0.019
u_R	0	1	0
$u_{Z(P_e,T_e)}$	0.002	1	0.002
$u_{\Delta\rho_{eb}}$	0.028	0.270	0.010
u_{V_0}	0.037	0.027	0.001
u_t	0.002	1	0.002
$u_{q_{V,l}}$	0.029	1	0.029
u_C			0.060
k			2
U			0.120

Table 8: Expanded uncertainty of cylinders.

Cylinder	C1	C2	C3	C4	C5
u_C	0.060	0.052	0.046	0.044	0.044
k	2.00	2.00	2.00	2.00	2.00
U	0.12	0.10	0.09	0.09	0.09

7. Conclusion

The prymary piston prover system that VMI has equipped to extend flow capacity down to 2 ccm. This system will help Vietnam strengthen its state management of metrology and integrate with the world on the ability to measure small gas flows. The system is evaluated according to the guidelines of ISO 25. In this study, the evaluation method and the calculation results show that the piston prover system with the largest uncertainty is cylinder 1 with open uncertainty. The relative breadth is 0.12 % with a 95 % confidence level.

References

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