Simulation Study on Measurement Method of Flow Fluctuation Signal Based on Chaotic Oscillator

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Abstract

Based on the initial sensitivity of chaotic oscillator which is sensitive to regular signal and immune to noise simultaneously，the chaotic signal detection method can detect the signal submerged in the noise stronger than the measured signal, and relevant research has been applied to measure weak signals submerged by large noise. In this paper, chaotic oscillator is applied in flow fluctuation signal detection of a flow standard device, and a new detection model is proposed to detect the fluctuation signal using the maximum Lyapunov exponent to obtain the threshold value and monitor motions transition of detection system. The Holmes Duffing oscillator is chosen to work as the detection system. Fourth-order Runge-Kutta algorithm is used to solve the function of Holmes Duffing oscillator and to calculate maximum Lyapunov exponent. Simulation experiment on different frequencies fluctuation signal and under different levels of noises is conducted. Simulation results show that detection model can effectively detect the flow fluctuation signal output by flowmeter, and it is immune to the noises.

1. Introduction

Flow standard devices are key equipment to provide accurate stable flow values for calibrating and evaluating flowmeter performance [1]. As one of the important indicators of flow standard devices, flow stability refers to the fluctuation degree of flow over a period of time and flow stability has received more and more attention. Flow stability not only affects the flow accuracy of water flow standard device, but also brings large uncertainty to calibration results [2]. In order to improve the flow stability of the flow standard device, it is necessary to accurately measure the flow fluctuation.

In general flow standard device, flow fluctuation signals can be regarded as sinusoidal signals [2]. Pulsation sources known in water flow standard devices, such as pump rotation, valve vibration, etc., fluctuate at a frequency of about 0.1-100 Hz. For existing flow stability measurement method, a single flow meter is generally using, and the standard deviation of measurement results output by the flow meter is used to characterize flow fluctuation. However, it is difficult to directly and accurately measure the flow fluctuation because of the complex mechanism of the fluctuation, the variety of the fluctuation types and the large background noise in most cases [2].

In addition to the unsteady performance of the flowmeter, which may lead to the distortion of the measurement results, flow fluctuation is also difficult to distinguish from noise, pipeline vibration and electrical interference [3]. Especially, the noise signal of flow meter or measuring system is superimposed in the flow fluctuation signal, or even stronger than the flow fluctuation signal, which makes the measurement result of flow fluctuation inaccurate.

The flow fluctuation signal and the noise of the flow meter or measurement system are uncertain, and the noise is usually considered as white noise. Although noise reduction methods such as narrowband filtering and time-domain averaging can eliminate noise to a certain extent, the flow fluctuation signal will also be attenuated[4]. When the fluctuation signal is completely drowned out by noise, the signal becomes more difficult to process.

Therefore, a method to detect flow fluctuation signal without noise interference is needed. Based on the initial sensitivity of chaotic oscillator, the chaotic weak signal detection system can accurately detect target sinusoidal signals submerged in noise . Relevant studies have been applied to measure weak signals submerged by large noise [5]. This method provides the possibility for accurate detection of flow fluctuation signals.

In this paper, chaotic oscillator is applied to the fluctuation signal detection of a flow standard device. Based on The Holmes-Duffing oscillator, a chaotic weak signal detection model is designed, and the detection threshold is calculated by using the maximum Lyapunov exponent to monitor the state transition of chaotic system. The fourth order Runge-Kutta algorithm is used to solve the Holmes-Duffing oscillator function and QR decomposition is used to calculate the maximum Lyapunov exponent of the system. Simulation experiments are carried out for signals with different frequency fluctuation and different noise levels. The simulation results show that the chaotic detection model can detect the fluctuation signal of flow standard device effectively and is not disturbed by noise.

2. Chaotic weak signal detection principle

2.1 The development of chaotic weak signal detection

In 1992, Brown et al. proposed for the first time to apply the initial sensitivity of Duffing oscillator to the construction of sensors, initiating the application of chaos theory in weak signal detection [6]. In the same year, Birx et al. proposed a method to detect weak signals through chaotic oscillators. These studies have laid a foundation for measuring the amplitude of weak signals with known frequencies[7]. On the basis of previous studies, some researchers have made great efforts[7]. They applied the initial sensitivity of Duffing oscillator to the detection of amplitude, frequency and phase of weak sinusoidal signals respectively, and focused on the use of Duffing oscillator or its improved form to improve the detection performance.

2.2 Methods to distinguish the state of chaotic systems

Weak signal detection based on chaotic system requires a method of chaos discrimination. Because the core idea of weak signal detection through chaotic system is to judge whether weak signal is introduced according to whether the state of the system has changed, so the means to judge the system state has become an important content of this detection method. At present, the methods to judge the state of chaotic system can be roughly divided into two categories, the first is image analysis method, the second is quantitative analysis method.Image analysis method includes phase trajectory graph, time history method, Poincare section method, flicker sampling method and power spectrum method. In this paper, phase trajectory diagram is used to observe the state of the system. Quantitative analysis methods include Kolmogorov entropy, fractal dimension method, Melnikov method, Lyapunov characteristic exponent method and so on. Since quantitative analysis is to judge system state by numerical values rather than images, it has good accuracy.Therefore, in recent years, scholars have gradually turned to quantitative analysis. In quantitative analysis, Lyapunov characteristic exponent is widely used as the basis of chaos discrimination. In this paper, the maximum Lyapunov exponential method is used to judge the system state.

2.3 Holmes-Duffing equation and its solution

In nonlinear theory, Duffing equation is a commonly used equation with characteristics of bifurcation and chaos. In practical engineering, many mathematical model structures of nonlinear vibration problems can eventually be transformed into the expression of Duffing equation, and the Duffing system is immune to noise, but very sensitive to periodic signals. The phase trajectory of the system may also change due to the slight change of periodic force. Therefore, the study and discussion of Duffing equation is of great significance in weak signal detection. The expression of the Holmes-Duffing equation can be written as follows:

$$\ddot{x}(t) + k\dot{x}(t) - x(t) + x^3(t) = \gamma\cos(\omega t) \quad (1)$$

Where k is the damping ratio, γ is the amplitude of the periodic policy force, ω is the angular frequency of the policy force, and the relation between it and the frequency is $\omega=2\pi f$,f is the signal frequency.

For ordinary differential equations:

$$\begin{cases} y' = f(x,y) \\ y(x_0) = y_0 \end{cases} \quad (2)$$

Its solution is y=y(x), Taylor expansion of Y, Runge-Kutta method is an approximation method for the application of Taylor expansion, using the linear combination of function values f(x,y) on a series of points to construct the approximation function.After solving the Holmes-Duffing equation with the fourth order Runge-Kutta algorithm, the Lyapunov exponent of Duffing was calculated by QR decomposition algorithm based on Householder transform.

Consider the Holmes-Duffing system:

$$\ddot{x} + k\dot{x} + ax + bx^3 = \gamma\cos(\omega t) \quad (3)$$

Where k is the damping ratio, $ax+bx^3$ is the nonlinear restoring force, γcos(ωt) is the periodic policy force, γ is the periodic policy force amplitude ,ω is the periodic policy force angular frequency. Since the Duffing system belongs to a two-dimensional non-autonomous system, it can be transformed into a three-dimensional autonomous system. Suppose:

$$y_1(t) = x(t), y_2(t) = \dot{x}(t), y_3(t) = t \quad (4)$$

There are:

$$\begin{cases} \dot{y}_1(t) = y_2(t) \\ \dot{y}_2(t) = -ky_2(t) - ay_1(t) - by_1{}^3(t) + Y\cos(\omega t) \\ \dot{y}_3(t) = 1 \end{cases} \quad (5)$$

That is:

$$\dot{Y}(t) = J(t)Y(t), Y(0) = I_3 \quad (6)$$

I_3 is the third-order identity matrix, J(t) is the Jacobia matrix of the system, Y(t) is the basis solution matrix. Where J(t) is:

$$J(t)=\begin{bmatrix} 0 & 1 & 0 \\ -a-3bx^2 & -k & -f\omega\sin(\omega t) \\ 0 & 0 & 0 \end{bmatrix} \tag{7}$$

QR decomposition of Y(t) is as follows:

$$Y(t)=Q(t)R(t) \tag{8}$$

Among them:

$$Q(t)=\begin{bmatrix} Q_{11}(t) & Q_{12}(t) & Q_{13}(t) \\ Q_{21}(t) & Q_{22}(T) & Q_{23}(t) \\ 0 & 0 & 0 \end{bmatrix}$$
$$R(\mathrm{t})=\begin{bmatrix} R_{11}(t) & R_{12}(t) & R_{13}(t) \\ 0 & R_{22}(t) & R_{23}(t) \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

According to the calculation formula of Lyapunov exponent:

$$\lambda_{\mathrm{i}}=\lim_{t\to+\infty}\frac{1}{t}\lambda_i(t)=\lim_{t\to+\infty}\frac{1}{t}\ln(R_{ii}(t)),\quad i=1,2,3 \tag{10}$$

The above Runge-Kutta algorithm was used to calculate the three-dimensional autonomous system of Duffing equation and the fundamental solution matrix Y(T) of the subsystem under the three-dimensional autonomous system at T/ K points in time T. QR decomposition was performed for each Y(T) to calculate the change trend of Lyapunov exponent with respect to time T. When k =0.5, a =-1,b=1,γ 0.3, 0.6 and 0.9 respectively, as shown in the following figures:

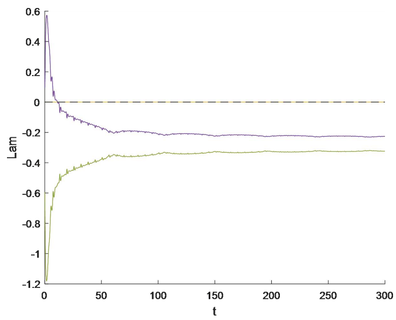

Figure 1: Lyapunov exponential diagram at γ=0.3.

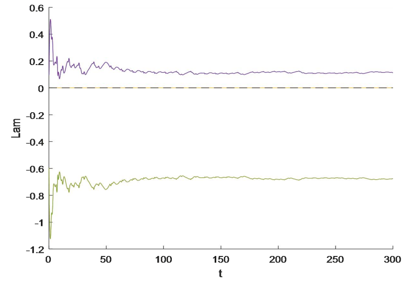

Figure 2: Lyapunov exponential diagram at γ=0.6.

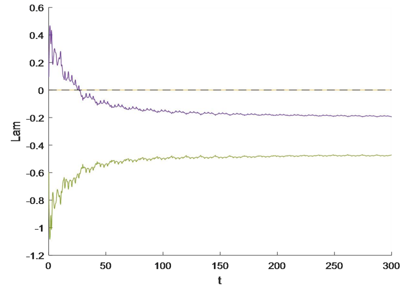

Figure 3: Lyapunov exponential diagram at γ=0.9.

As can be seen from the figures, when γ=0.3, the maximum Lyapunov exponent of Duffing system is negative, and the system is in a periodic state. When γ=0.6, the maximum Lyapunov exponent of the system is positive, and the Duffing system is in chaos state. When γ=0.9, the maximum Lyapunov exponent of the Duffing system is negative, and the Duffing system is in a periodic state. To sum up, the system experienced changes from periodic state to chaotic state and then to periodic state with the change of the amplitude of the policy force. Phase trajectory diagrams of these three states were drawn for verification:

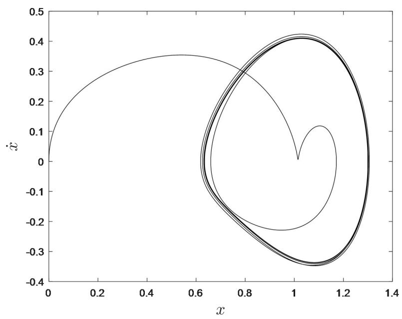

Figure 4: Phase trajectory diagram at γ=0.3.

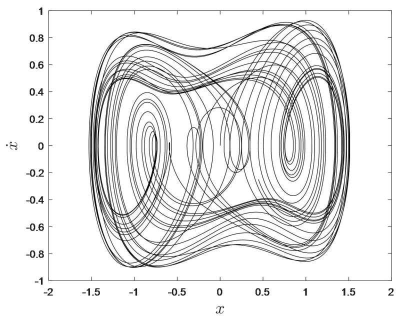

Figure 5: Phase trajectory diagram at γ=0.6.

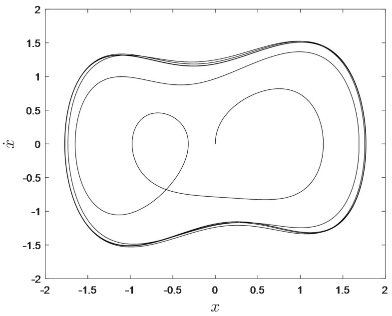

Figure 6: Phase trajectory diagram at γ=0.9.

According to the image, it can be seen that the judgment result of the phase trajectory diagram of the system is consistent with the judgment result of the system state obtained by calculating the value of Lyapunov characteristic exponent.

3. Simulation Experiment

3.1 Calculating the System threshold of Duffing

According to the state of the control system with the coefficient γ, the corresponding γ value of Duffing system from chaotic state to large periodic state is called the threshold of state transition YD. In order to understand the change of Lyapunov exponent of the system after the coefficient γ changes, when ω=1, k=0.5, starting from γ=0, step size is 0.1, to γ=2. According to Lyapunov exponent of each point, the relationship between coefficient γ and Lyapunov exponent is drawn as follows:

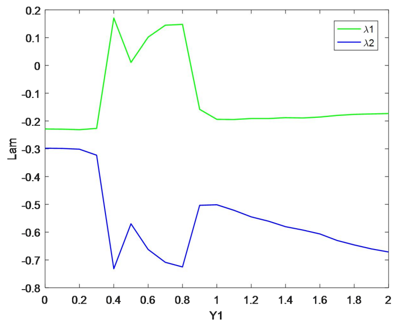

Figure 7: Relationship between coefficient γ and Lyapunov exponent at ω=1.

As can be seen from the figure above, when γ=0.3, 0.6 and 0.9, the maximum L exponent of the system is negative, positive and negative respectively, which is consistent with the verification results above. The system changes from chaotic state to stable periodic state after γ=0.8. In order to obtain the threshold of chaotic system, the maximum Lyapunov exponent (hereinafter referred to as the maximum L exponent) of the system from γ=0.8 to γ=0.9 should be calculated. The calculation step is 0.01. Since Lyapunov exponent is unstable at the early stage of iteration, the mean value of Lyapunov exponent corresponding to the last 30 times of each iteration is taken as the maximum Lyapunov exponent of an iteration,then the corresponding value of γ and the maximum Lyapunov exponent is shown in Table 1:

Table 1: The coefficient γ corresponds to the maximum Lyapunov exponent.

γ	0.8	0.81	0.82	0.83	0.84	0.85
L	1.11E-01	1.32E-01	1.63E-01	-3.56E-02	-1.01E-01	-1.76E-01

In the table, we take a representative portion of the data , L is the maximum L exponent.As the amplitude of flow fluctuation in the water flow standard device is on the order of 0.1, the threshold is YD=0.8.

3.2 Detection of fixed frequency weak signal amplitude

According to the above calculation, the angular frequency of periodic policy force is ω=1rad/s, and the frequency is about 0.16 Hz, within the fluctuation range of 0.1-100 Hz of water flow standard device. When k =0.5, threshold YD=0.8, and simulation signal is added, and the simulation signal is cosine signal superimposed with Gaussian white noise. The amplitude of cosine signal is from 0.01 to 0.04, the step size is 0.01, and the noise variance is from 0.00001 to 1. Meanwhile, the SNR of the added signal and noise is calculated as follows:

$$SNR = 10\lg\frac{A_S^{\ 2}}{2\sigma^2} \tag{11}$$

Where, A_S is cosine signal amplitude and σ^2 is noise variance.

When the amplitude of simulation signal is 0.01 to 0.04, the maximum L exponent corresponding to different variance noises is shown in the following tables:

Table 2: When the signal amplitude is 0.01, different variance noises correspond to the maximum L exponent table.

The noise power	L	SNR
0.00001	1.15E-01	6.99
0.0001	1.75E-01	-3.01
0.001	1.72E-01	-13.01
0.01	1.23E-01	-23.01
0.1	1.44E-01	-33.01
1	1.43E-01	-43.01

Table 3: When the signal amplitude is 0.02, different variance noises correspond to the maximum L exponent table.

The noise power	L	SNR
0.00001	1.36E-01	13.01
0.0001	1.53E-01	3.01
0.001	1.41E-01	-6.99
0.01	1.30E-01	-16.99
0.1	1.24E-01	-26.99
1	1.45E-01	-36.99

Table 4: When the signal amplitude is 0.03, different variance noises correspond to the maximum L exponent table.

The noise power	L	SNR
0.00001	-3.56E-02	16.53
0.0001	-3.56E-02	6.53
0.001	-4.34E-02	-3.47
0.01	-3.19E-02	-13.47
0.1	-3.50E-02	-23.47
1	-2.22E-02	-33.47

Table 5: When the signal amplitude is 0.04, different variance noises correspond to the maximum L exponent table.

The noise power	L	SNR
0.00001	-1.01E-01	19.03
0.0001	-1.01E-01	9.03
0.001	-1.01E-01	-0.97
0.01	-1.02E-01	-10.97
0.1	-1.12E-01	-20.97
1	-9.07E-02	-30.97

According to the data analysis in the above table, it can be seen that: When threshold YD = 0.8, add angular frequency ω = 1 rad/s of fixed frequency signals, as long as the signal amplitude exceeds 0.02, the biggest L exponent system of symbols will be from positive to negative, and the water flow standard device of flow fluctuation amplitude in 0.1 magnitude, that system has the ability of detection of flow fluctuation signal, and the system has good noise immunity. The signal-to-noise ratio of signal to noise can reach -33.47dB.

In order to capture more flow fluctuation frequency of the water flow standard device, the angular frequency of periodic policy force is adjusted to ω=1.1rad/s, the frequency is about 0.18Hz, and k=0.5. The threshold value of the system at this time is calculated. The relationship between coefficient γ and Lyapunov exponent is shown as follows:

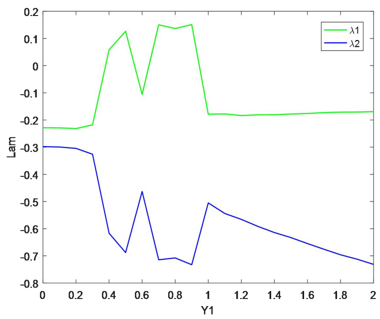

Figure 8: Relationship between coefficient γ and Lyapunov exponent at ω=1.1.

The system changes from chaotic state to stable periodic state after γ=0.9. In order to obtain the threshold value of chaotic system, the maximum L exponent of the system from γ=0.9 to γ=1.0 should be calculated, and the calculation step is 0.01.

Table 6: The coefficient γ corresponds to the maximum Lyapunov exponent.

γ	0.92	0.94	0.95	0.96	0.97	0.98
L	1.25E-01	1.75E-01	1.48E-01	-6.22E-02	-1.29E-01	-1.57E-01

When the threshold YD=0.9, the simulation signal is added, and the amplitude of cosine signal is from 0.04 to 0.07. Other parameters are the same as those of ω=1. When the amplitude of simulation signal is 0.04 to 0.07, the maximum L exponent corresponding to different variance noises is shown in the following tables:

Table 7: When the signal amplitude is 0.04, different variance noises correspond to the maximum L exponent table.

The noise power	L	SNR
0.00001	1.66E-01	19.03
0.0001	1.53E-01	9.03
0.001	1.46E-01	-0.97
0.01	1.31E-01	-10.97
0.1	1.71E-01	-20.97
1	1.24E-01	-30.97

Table 8: When the signal amplitude is 0.05, different variance noises correspond to the maximum L exponent table.

The noise power	L	SNR
0.00001	1.15E-01	20.97
0.0001	1.61E-01	10.97
0.001	1.33E-01	0.97
0.01	1.24E-01	-9.03
0.1	1.60E-01	-19.03
1	1.28E-01	-29.03

Table 9: When the signal amplitude is 0.06, different variance noises correspond to the maximum L exponent table.

The noise power	L	SNR
0.00001	-6.22E-02	22.55
0.0001	-6.22E-02	12.55
0.001	-6.25E-02	2.557
0.01	-5.94E-02	-7.45
0.1	-5.61E-02	-17.45
1	-1.05E-01	-27.45

Table 10: When the signal amplitude is 0.07, different variance noises correspond to the maximum L exponent table.

The noise power	L	SNR
0.00001	-1.29E-01	23.89
0.0001	-1.28E-01	13.89
0.001	-9.80E-02	3.89
0.01	-1.22E-01	-6.11
0.1	-1.11E-01	-16.11
1	-1.05E-01	-26.11

According to the data analysis in the above table, it can be seen that: When the threshold YD=0.9, the fixed frequency signal with angular frequency ω=1.1rad/s is added, as long as the signal amplitude exceeds 0.05, the sign of the maximum L exponent of the system will change from positive to negative, and the flow fluctuation amplitude of the water flow standard device is on the order of 0.1, indicating that the system has the ability to detect the flow fluctuation signal. Moreover, the system has good noise immunity, and the signal-to-noise ratio can reach -27.45dB.

Similarly, take the angular frequency of periodic policy force as ω=1.2rad/s, frequency about 0.19Hz, k =0.5, calculate the system threshold at this time, and the relationship diagram between coefficient γ and Lyapunov exponent is as follows:

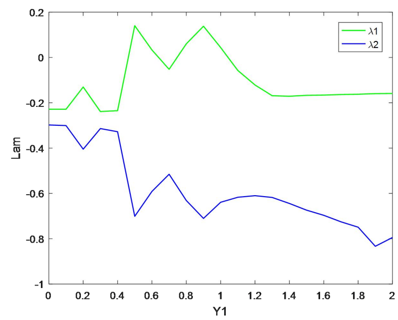

Figure 9: Relationship between coefficient γ and Lyapunov exponent at ω=1.2.

The system changes from chaotic state to stable periodic state after γ=1.1. In order to obtain the threshold of chaotic system, the maximum L exponent of the system from γ=1.1 to γ=1.2 should be calculated, and the calculation step is 0.01.

Table 11: The coefficient γ corresponds to the maximum Lyapunov exponent.

γ	1.1	1.11	1.12	1.13	1.14	1.15
L	1.74E-01	-1.29E-01	-1.19E-01	-1.58E-01	-1.38E-01	-1.38E-01

When the threshold YD=1.1, the simulation signal is added, and the amplitude of cosine signal is from 0.01 to 0.04. Other parameters are the same as those of ω=1. When the amplitude of simulation signal is 0.01 to 0.04, the maximum L exponent corresponding to different variance noises is shown in the following tables:

Table 12: When the signal amplitude is 0.01, different variance noises correspond to the maximum L exponent table.

The noise power	L	SNR
0.00001	-1.29E-01	6.99
0.0001	-1.29E-01	-3.01
0.001	-1.29E-01	-13.01
0.01	-6.87E-02	-23.01
0.1	-1.74E-03	-33.01
1	-6.43E-02	-43.01

Table 13: When the signal amplitude is 0.02, different variance noises correspond to the maximum L exponent table.

The noise power	L	SNR
0.00001	-1.19E-01	13.01
0.0001	-1.19E-01	3.01
0.001	-1.20E-01	-6.99
0.01	-1.29E-01	-16.99
0.1	-7.16E-02	-26.99
1	-9.14E-02	-36.99

Table 14: When the signal amplitude is 0.03, different variance noises correspond to the maximum L exponent table.

The noise power	L	SNR
0.00001	-1.58E-01	16.53
0.0001	-1.58E-01	6.53
0.001	-1.58E-01	-3.47
0.01	-1.59E-01	-13.47
0.1	-1.61E-01	-23.47
1	-1.62E-01	-33.47

Table 15: When the signal amplitude is 0.04, different variance noises correspond to the maximum L exponent table.

The noise power	L	SNR
0.00001	-1.38E-01	19.03
0.0001	-1.38E-01	9.03
0.001	-1.37E-01	-0.97
0.01	-1.36E-01	-10.97
0.1	-1.55E-01	-20.97
1	-8.64E-02	-30.97

According to the data analysis in the above table, it can be seen that: When the threshold YD=1.1, the fixed frequency signal with angular frequency ω=1.2rad/s is added, as long as the signal amplitude is not less than 0.01, the symbol of the maximum L exponent of the system will change from positive to negative, and the flow fluctuation amplitude of the water flow standard device is on the order of 0.1, indicating that the system has the ability to detect the flow fluctuation signal. Moreover, the system has good noise immunity, and the signal-to-noise ratio can reach -43.01dB.

4. Conclusion

In this paper, chaotic oscillator is applied to the detection of flow fluctuation signal of a flow standard device, and a detection model of flow fluctuation signal based on Holmes-Duffing oscillator is proposed. By using the maximum Lyapunov exponent the detection threshold is calculated and the state transition of chaotic detection system is monitored. Fourth order Runge-Kutta algorithm was used to solve the Holmes-Duffing oscillator function, and the maximum Lyapunov exponent was calculated by QR decomposition. Simulation experiments were carried out for different frequencies flow fluctuation signals and different noise levels, and simulation results show that the chaotic detection model can detect the fluctuation signal of flow standard device effectively and is not disturbed by noise.

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