A PRACTICAL METHOD TO DETECT THE REPEATABILITY OF A GRAVIMETRIC FLOW TEST RIG BY USING THE TIMING ERROR

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Abstract

The present paper presents an improved method for the diverter error determination method according to the international recommendation ISO4185. By extending this test it is possible to make a qualitative test of the overall status of a gravimetric flow test rig. The proposed method has been successfully applied to results of diverter tests of the large water flow calibration facilities of the PTB in Braunschweig and Berlin. It is shown that by applying this method also the repeatability of the flow meter used, and of the test bench alone can be estimated.

1 Introduction- Motivation

The validation of the uncertainty estimations of any flow test rig used for research or traceability purposes is of central importance. The estimation methods are well standardized and the main components as the weighing system, the water density determination, the timing error, the flow conditions and the repeatability of the Meter Under Test (MUT) have been subject of several studies and publications.

The most recognized way of validating the declared uncertainties is trough interlaboratory comparisons. For the relevant case, where the reference value is defined during the test, its analysis is not trivial. In order for the reported data of every laboratory to be comparable and consistent, it is necessary for all participants to measure exactly the same measurand; i.e. all relevant measurement conditions have to be the same. Considering that most of the participants of a comparison are dealing with this exercise at this condition or range for the first time, it is very likely that one or more participants will have undetected biases even larger than their declared uncertainty. This is not reproachable, since interlaboratory comparisons are many times the only way to detect biases.

There are two ways to solve this problematic. The first option is to take advantage of an external reference. This can be done by performing more than one round in a comparison; the first round should give an initial view onto the individual performance of the participants, and give them the possibility to roughly recognize their flaws and to correct them if possible. After the problems are detected and solved, the second comparison round can be performed. The efforts needed to perform a double comparison are huge, therefore a different option would be preferred.

The optimal solution should allow participants to make a qualitative check of their systems in order to detect major problems before the comparison is performed.

This paper presents a method to detect qualitatively if important systems of the flow test rig are working properly. This method is an extended version of the timing error detection test proposed by the ISO 4185. This new approach acts as a quality control to proof the proper function of the flow test rig from a metrological point of view. It presents additionally a way to quantify the repeatability of the test flow measurement device, and consequently also the repeatability of the test bench alone.

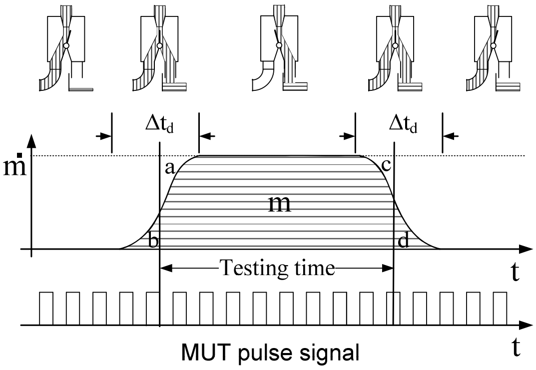

Figure 1: The ideal testing time is defined as the time in which the dashed area is equal to the area defined by the testing time limits and the dotted line. The pulse trail shown below represents schematically the uninterrupted flow rate perceived by the meter under test.

2 Model of the diverter error

The uncertainty contribution of the diverter error is only relevant if long measurement times are not possible. For most gravimetric test rigs, all measurements taking less than one minute and where an expanded uncertainty of under 0.1% is aimed should be generally analyzed in detail. The time the diverter takes to change from one position to the other Δt_d can be from a few milliseconds up to one second. During this time, the flow rate directed into the weighing tank increases from zero to the actual flow rate; the shape of the incrementing flow rate within Δt_d is not known.

There are two ways of estimating the diverter error: by means of modeling and measuring the diverting system, i.e. at least the blade dynamics and the water jet, or by modeling the effects that are caused indirectly by the diverter. The first method delivers also the uncertainty of the diverting phenomena itself, but requires large modeling and measuring efforts including flow profile measurements at the outlet of the nozzle using LDV for example, and characterization of the motion dynamics of the blades. This analysis has been firstly presented by Engel et al [3]. The second method is empirical and has bin introduced by the international standard ISO 4185. The advantage of this method is its universal applicability, since no additional measurement systems are required.

In order for a measurement to be error free, it is necessary that the area defined by the testing time limits and the flow rate is equal to the dashed area (see Figure 1). If this is the case, the diverter would be error free. For this to be true, it is necessary that:

$$a + c = b + d. \tag{1}$$

If the flow rate can not be sustained adequately, and the diverting time Δt_d is large, it might is advisable to apply following conditions:

$$a = b\,;\;\; c = d \tag{2}$$

The reason for this is that if only the first condition is used to adjust the trigger point, the measurement would be error free only if the flow rate is exactly the same for both diverting actions; even symmetric diverters like the unidiverter from the NIST [2] or the double wing diverter from the NMIJ [5] have to ensure very stable flow rates at the beginning and at the end of the measurement. But if also condition 2 is fulfilled even if the flow rate changes considerably, the diverter would be able to cancel its error automatically..

2.1 The corrected ISO 4185 diverter equation

Every time a diverting action is performed, if the trigger point is not error free, a constant time error $(-\Delta t)$ is added to the time measurement. If measurements with different amount of diverting actions are performed, it is possible to calculate the constant time error that is being added per diverting action.

For this, two measurements are necessary. One with only one filling interval, i.e. a regular calibration, and one with n filling intervals. The time is only measured while the diverter is directing the water into the weighing tank. For further details on the procedure please refer to the ISO 4185. The equation proposed to estimate the diverter error based on these two measurements is:

$$\Delta t = \frac{t'}{(n-1)}\left(\frac{\frac{m_i}{t'_i}}{q_i/q\frac{m}{t'}} - 1\right) \tag{3}$$

Where the symbols t, Δt, q and m, represent the time, the diverter error, the volumetric flow rate and the mass respectively; subindex i refers to interval measurements and superindex $'$ refers to apparent magnitudes.

Equation 3 has some limitations: the remaining diverter error has to be neglectably small $\Delta t << t$, the filling times have to be exactly the same, and density or temperature variations are not considered. In order to compensate for those phenomena, an additional term k_t for different filling times, and a term k_f for different mass flow rates that include the density variation effect have been introduced. The corrected formulation is:

$$\boxed{\Delta t = \frac{\Delta t + t'}{\left(\frac{n}{k_t} - 1\right)}\left(\frac{\frac{m_i}{t'_i}}{k_f\frac{m}{t'}} - 1\right)} \tag{4}$$

$$k_t = t_i/t \tag{5}$$

$$k_f = \dot{m}_i/\dot{m} \tag{6}$$

NOTE *For a detailed explanation on the derivation of equation 4 please reffer to the Appendix A. Appendix B includes the derivation of the second method proposed by the ISO4185 for determining the diverter error*

In [2] and [4] there are different timing error equations presented, if compared with the formulation proposed by the ISO4185, there are some differences. These differences vanish if equation 4 is used.

For equation 4 to be valid, it is necessary that following requirements are fulfilled:

- Regarding the flow rate:
  - the mean flow rate is kept constant
  - the diverter does not influence the flow rate
  - the flow rate during one diverting action is constant
  - the flow measurement device is highly repeatable
  - the flow profile at the diverter inlet does not change
- Regarding the measurement systems:
  - The weighing drift is neglectable for the duration of the measurements
  - Nearly the same absolute weighing load is used for both measurements
  - All density, pressure and temperature measurements are functioning properly
- Regarding the diverting system:
  - The duration and the velocity of the diverting action has a normal distribution
  - The position of the trigger point is fixed

Most of these assumptions are applicable also for calibration measurements. This is the basic principle of the flow test rig testing method: If the flow test rig and the deduced model behave consistently, this means that all assumptions have been met, and no major problems are expected during an interlaboratorial comparison. If no consistency is given, at least one of the assumptions is not being fulfilled.

2.2 Variance determination

If the diverter error test is repeated several times, slightly different timing errors will be found. In a similar way, if the number of filling intervals is increased, the spread of the different calculated timing errors varies. The typical dependence of the standard deviation (20 samples) of the single tests on the amount of intervals n used is schematized in figure 2.

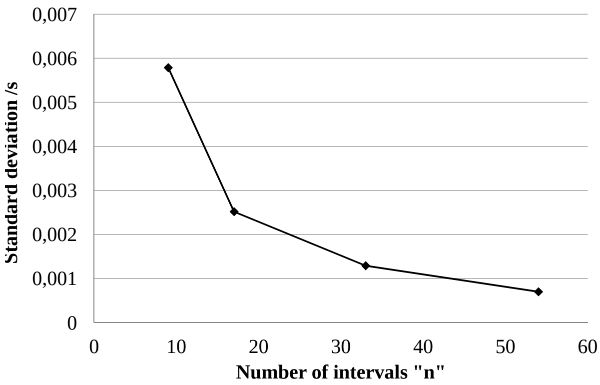

Figure 2: Variation of the standard deviation of the timing error, as a function of the amount of intervals used for the test. 20 samples have been measured.

The shape of the curve in figure 2 can be explained by making an error analysis of equation 4.

Note. *Equation 4 represents the method used for determining the value of Δt. Its variance $\sigma_{\Delta t}$ is describing the behaviour of the method, in contrast to the variance σ_δ that is describing the diverter variability and representing an additional contribution to $\sigma_{\Delta t}$.*

By introducing the dimensionless variables 7

$$\epsilon = \frac{\Delta t}{t}; \quad k_m = \frac{m_i}{m}; \quad k_f = \frac{\dot{m}_i}{\dot{m}}; \quad k_t = \frac{t_i}{t} \tag{7}$$

in equation 4 we obtain:

$$\epsilon = \frac{1}{n - k_t}\left(\frac{k_m}{k_f} - k_t\right) \tag{8}$$

Its associated variance is given by:

$$\sigma_\epsilon^2 = \left(\frac{k_f}{n - k_t}\right)^2 \sigma_{km}^2 + \left(\frac{k_m}{n - k_t}\right)^2 \sigma_{kf}^2 \tag{9}$$

Since time measurements are performed normally at an accuracy better than $10^{-6}s$ we cann assume that

$$\sigma_{kt} \approx 0 \tag{10}$$

The value of σ_{kf} can not be neglected. Depending on the level of correlation r the value of $\sigma_{\dot{m}}$ would be

$$\sigma_{\dot{m}} = \frac{\sigma_{kf}}{\sqrt{2 - r}} \tag{11}$$

Equation 11 will be used in the next section to estimate the value of the repeatability of the flow test rig alone. For k_m a special treatment is necessary. If the accumulated masses have been the same during the test with one filling interval and the corresponding test with n filling intervals, and assuming also that they have been performed under repeatability conditions[1], the variations of k_m can be attributed only to the randomness of the diverter motion σ_δ. By rewriting k_m as a function of the flow rate and the time we can include the effects of the randomness of the diverter motion:

$$k_m = k_f \frac{\left(t_i' + n\Delta t\right)}{(t' + \Delta t)} \tag{12}$$

Considering the effect of averaging on $n\Delta t$ the variance of σ_{km} would be given by:

$$\sigma_{km}^2 = \frac{\left(t_i' + n\Delta t\right)^2}{(t' + \Delta t)^4}\sigma_\delta^2 + \frac{n^2}{(t' + \Delta t)^2}\sigma_\delta^2 \frac{1}{n}$$

Assuming also that

$$\frac{\left(t_i' + n\Delta t\right)}{(t' + \Delta t)} \approx k_t \approx 1 \quad and \quad k_f \approx 1 \tag{13}$$

Finally, the variance of consecutive mass measurements caused by the diverter variations would be given by

$$\sigma_{km}^2 = \frac{1 + n}{t^2}\sigma_\delta^2 \tag{14}$$

Even if equation 14 would allow to explicitly calculate σ_δ, given that the determination of σ_{km} is based on the repeatability of the flow meter the obtained results would be inaccurate and not usable.

By introducing equation 14 and applying the conditions (13) into equation 9 we obtain

$$\sigma_\epsilon^2 = \left(\frac{1}{n - 1}\right)^2 \frac{1 + n}{t^2}\sigma_\delta^2 + \left(\frac{1}{n - 1}\right)^2 \sigma_{kf}^2 \tag{15}$$

Reintroducing $\sigma_{\Delta t}$ and rearranging the terms:

$$\boxed{\sigma_{\Delta t}^2 = \frac{n + 1}{(n - 1)^2}\sigma_\delta^2 + \frac{t^2}{(n - 1)^2}\sigma_{kf}^2} \tag{16}$$

This formulation describes the typical shape shown for example in figure 2. Consequently, it should be possible to extract the values of σ_{kf} and $\sigma_{\Delta t}$ based on repeated tests according to the ISO4185.

As we can see in equation 16, the resulting value of $\sigma_{\Delta t}$ depends largely on the duration of the tests and on σ_{kf}, as a consequence, it is not possible to attribute the value of $\sigma_{\Delta t}$ to σ_δ.

If the value of σ_δ can be neglected we can estimate the value of the repeatability of the flow meter using only one set of measurements by applying:

$$\boxed{\sigma_{\Delta t} = \frac{t}{n - 1}\sigma_{kf}} \tag{17}$$

3 Measurements at the PTB

Exhaustive diverter tests have been performed at the heat laboratory of the PTB-Berlin and the flow test facility of the PTB-Braunschweig. Some of these results have been previously published by Mathies and Lederer in [4] or by Engel in [3]. The configuration of the diverter installed at the PTB-Berlin has some different characteristics as typical diverters for cold water. The reason is that water temperatures up to 90^oC are used to test heat flow meters. Due to the high evaporation rates expected at temperatures beyond 40 ^oC open water surfaces have to be avoided. The geometry can be observed

[1]Repeatability conditions include mainly the temperature, and a short period of time between measurements

in figure 3. The working principle is the same as for single blade diverters; the difference is that instead of only one blade, several radial positioned edges are installed. In spite of being almost complete encapsulated, the diverter still works contactless. The actuators are pneumatic ensuring a fast dynamic; the position is logged using a radial incremental encoder.

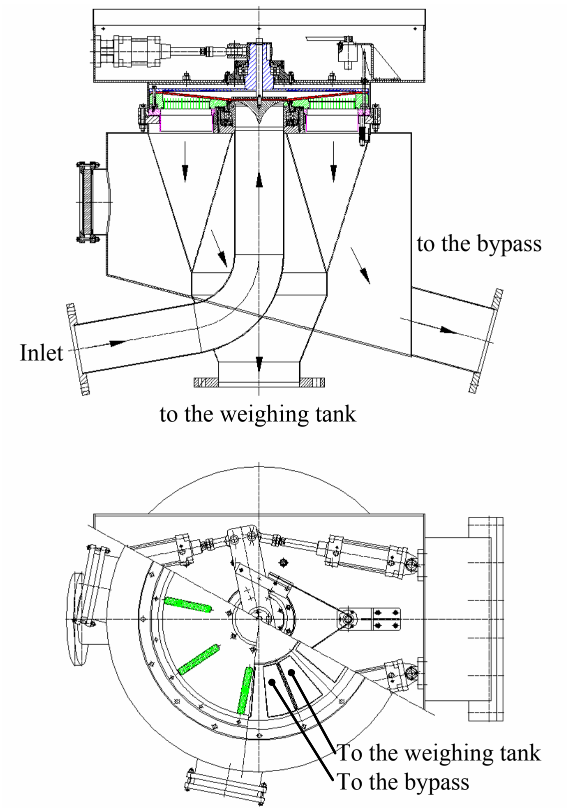

Figure 3: Scheme of the Diverter installed in the PTB-Berlin. The working principle is the same as for single blade diverters; the difference is that instead of only one blade, several radial positioned edges are installed.

The tests have been performed according to table 1.

Table 1: Diverter test conditions

	PTB-Braunschweig	PTB-Berlin
Temperature	20^oC	50^oC
Flowrate	$90\ m^3/h$	$199\ m^3/h$
Intervals n	7, 14, 26, 51	8, 17, 33, 54
Duration	$100\ s$	$360\ s$
Mass	$2500\ kg$	$19900\ kg$
Repetitions	15	20
Diverter error	4.6 ms (n=51)	8.6 ms(n=54)
Repeat. K-factor[a]	0.016%	0,009%

[a] The k-factor of the refference flow meter

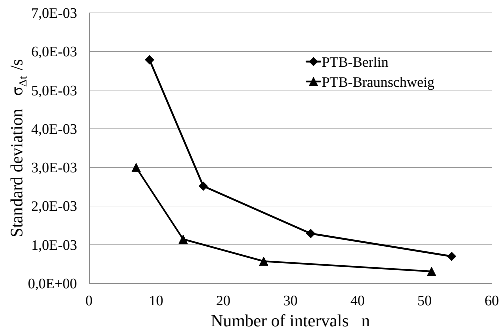

Figure 4: Standard deviation of the timing error, for different amounts of filling intervals during the tests in the PTB-Braunschweig and PTB-Berlin

As we can see in figure 4, the determination of the diverter error can be better than $1ms$ for $n > 50$. For $n < 10$ given to the large uncertainties associated with poor sampling, only values for $n > 10$ will be processed. The standard deviation for $n < 10$ is very large and at the same order of magnitude of the timing error itself.

3.1 Estimation of the parameters

By introducing the parameters φ and λ into equation 16

$$\varphi = \sigma_{\Delta t}^2 \frac{(n-1)^2}{(n+1)}$$

$$\lambda = \frac{1}{(n+1)}$$

we obtain

$$\varphi = t^2 \sigma_{kf}^2 \lambda + \sigma_\delta^2 \qquad (18)$$

It is possible to fit the values of σ_δ and σ_{kf} in equation 18 based on the experimental data. Affter fitting, considering the different filling times of PTB-Braunschweig and PTB-Berlin, following figures can be drawn:

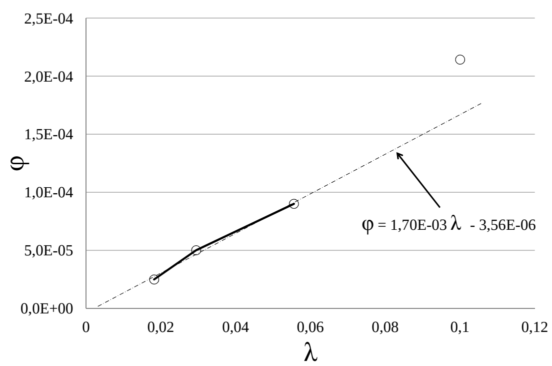

Figure 5: Linear fit function of the parameters φ and λ for the tests made in the PTB-Berlin. The experimental data is imposed to the fitted data (dashed lines)The value for $n < 10$ is included

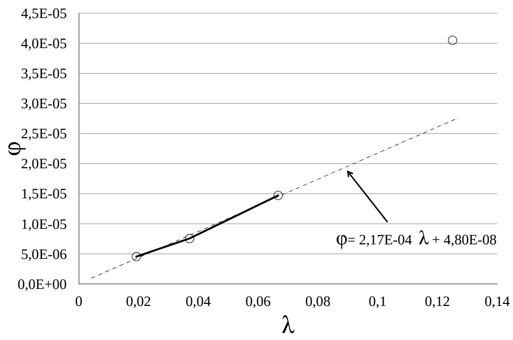

Figure 6: Linear fit function of the parameters φ and λ for the tests made in the PTB-Braunschweig. The experimental data is imposed to the fitted data (dashed lines). The value for $n < 10$ is also included

Considering the values extracted from figures 5 and 6 following table can be obtained:

Table 2: Fitting results for σ_δ and σ_{kf}

	PTB-Braunschweig	PTB-Berlin
σ_δ^2	4,80E-8	-3,56E-6
$t^2\sigma_{kf}^2$	2,17E-4	1,70E-3
σ_δ	0,22 ms^a	—[a]
σ_{kf}	0,015%	0,012%

[a]These values are not accurate and should not be used

It can be seen that the resolution offered by this method is not enough to resolve the magnitude of the diverter variability σ_δ. The value of σ_δ can be considered to be neglectable if compared with σ_{kf} If this condition is true, we can apply equation 17 and obtain excelent results, as seen in figure 7

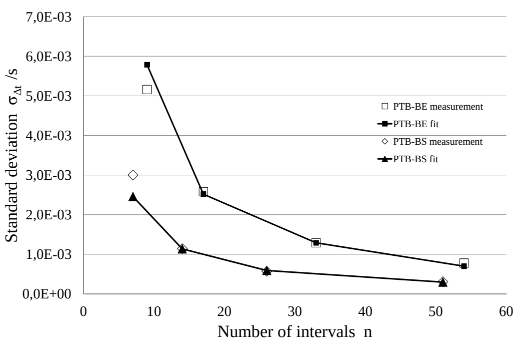

Figure 7: Measured and fitted standard deviation of the timing error, for different amounts of filling intervals during the tests in the PTB-Braunschweig and PTB-Berlin.

3.2 Repeatability of the test bench

In the former section we have shown that for $n > 10$ using equation 18 or 17 it is possible to estimate the value of σ_{kf}. The relation 11 defines the relation between σ_{kf} and $\sigma_{\dot{m}}$. If the result of the calibration is given by the k-factor, then the repeatability of the test bench alone would be approximately given by:

$$\sigma_{ref} = \sqrt{\sigma_k^2 - \sigma_{\dot{m}}^2} \tag{19}$$

Assuming correlation can be ignored in equation 11 following results can be obtained:

Table 3: Fitting results for σ_{ref}

	σ_k	$\sigma_{\dot{m}}$	σ_{ref}
PTB-Berlin	0,016%	0,008%	0,014%
PTB-Braunschweig	0,008%	0,011%	—

4 Discussion

As it can be seen in table 2 the method is not able to resolve the value of the diverter repeatability σ_δ for the presented example; its value seems to be neglectable compared to the uncertainty introduced by σ_{kf}. Therefore, in this case it seems to be enough to apply equation 17; this equation is defined only by one parameter; consequently only one measurement with preferable $n > 14$ would suffice for future testing. Given the good consistency observed in figure 2 this procedure seems to be acceptable on these measurement sites; nevertheless, this might be different at other test facilities. It is recommended to start the calculation by using the parameters φ and λ of equation 18 and if required to use equation 17.

In the case of the results presented in table 3 it can be seen that for the tests performed at the PTB-Braunschweig, the repeatability of the calibration of the k-factor of the master meter has an amazingly low value of 0,009%. Since this value contains the repeatability of the test rig, and also the repeatability of the mater meter itself, the estimated value for the flow meter alone must be lower than 0,009%. In this case the value was about 0,011% indicating that while performing the extended diverter test, probably repeatability conditions have not been met at the required level. But since the model is in good agreement with the measurement results as shown in figure 7 and the encountered difference is very low (under 0,003%) it can be concluded, that the flow test rig is functioning properly.

The presented results confirm that the proposed method is applicable to test the overall quality of a flow test rig. It provides a tool to detect if the main components are working properly.

Considering that the diverting systems, as every mechanical system, are prone to aging it is recommended to keep the amount of tests performed with the diverter at a minimum.

5 Conclusions

The proposed method is very sensitive to variations on the ideal conditions; if repeatability is not given, the proposed model will not fit the measurement data. This indication should suffice to detect the sources of errors and avoid early and unnecessary efforts to be invested in interlaboratory comparisons. If very good and stable conditions are given, it is foreseen that more detailed information regarding the diverter, the master meter, and the test rig itself can be estimated.

Appendix

Appendix A: Derivation of ISO4185 Method 2

Consider Figures 8 and 9. The timing error shown in the horizontal axis can be also interpreted as an aparent flow rate error. The dashed area is the mass error that would be originated by a timing error Δt.

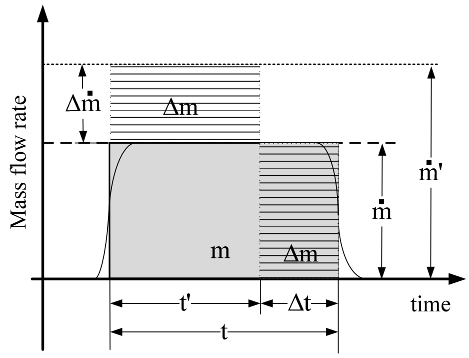

Figure 8: The figure shows that the diverter error in the time measurement shown in the horizontal axis, can be also seen as an error on the vertical axis as a flow rate error.

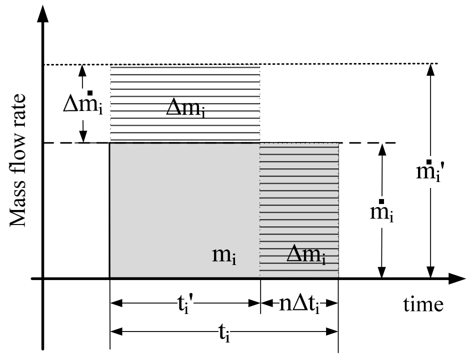

Figure 9: The figure shows that also for an interval filling measurement the diverter error, can be also seen as an error on the vertical axis as a flow rate error. The scheme is simplified, off-times, where water is flowing to the bypass are not shown.

Considering figure 8 and 9 we have the following relations. For a continuous test run:

$$\Delta m = \Delta \dot{m} \cdot t' \tag{20}$$

$$\Delta m = \Delta t \cdot \dot{m} \tag{21}$$

For a discontinuous test run:

$$\Delta m_i = n \cdot \Delta t \cdot \dot{m}_i \tag{22}$$

$$\Delta m_i = t'_i \cdot \Delta \dot{m}_i \tag{23}$$

eliminating Δm and Δm_i from equations 20, 21, 22 and 23 we obtain the apparent flow rate variation:

$$\Delta \dot{m} = \frac{\Delta t}{t'} \dot{m}$$

$$\Delta \dot{m}_i = \frac{n \cdot \Delta t}{t'_i} \dot{m}_i$$

Considering a continuous test run we obtain the following relations:

$$\dot{m}' = \frac{m}{t'}$$

$$\dot{m} = \dot{m}' - \Delta \dot{m}$$

$$\dot{m} = \frac{m}{t'} - \frac{\Delta t}{t'} \dot{m} \tag{24}$$

Analogously, considering a discontinuous test run we obtain:

$$\dot{m}_i = \frac{m_i}{t'_i} - \frac{n \cdot \Delta t}{t'_i} \dot{m}_i \tag{25}$$

Ideally, the continuous test run, and the one with interval filling periods should be performed at the same conditions, i.e. Exactly the same flow rates should be installed, and the same filling times should be aimed; but when this is not possible or not wanted, a correcting factor k_q can to take account for small flow rate variations; similarly we apply k_t to compensate for different filling times:

$$k_t = t'_i / t' \tag{5}$$

$$k_q = \dot{m}_i / \dot{m} \tag{26}$$

By inserting equations 24 and 25 into equation 26 and applying 5 we finally obtain:

$$\boxed{\Delta t = \frac{\Delta t + t'}{\left(\frac{n}{k_t} - 1\right)} \left(\frac{\frac{m_i}{t'_i}}{k \frac{m}{t'}} - 1 \right)} \tag{4}$$

The equation recommended by the ISO 4185 does not consider a factor k_t for different filling times, assumes $\Delta t << t$, and does not consider density variations. The simplifications made in the equation proposed by the ISO 4185 might be adequate for most of the cases, but under certain circumstances they can produce a considerable bias on the diverter error. **In order to avoid this, it is strongly recommended to replace the equation proposed in the ISO 4185 by equation 4.**

Appendix B: Derivation of ISO4185 Method 1

The second approach introduced by the ISO 4185 is also based on the comparison of two calibrations, but instead of varying the number of diverting actions, only the duration of the test is changed. Since the diverter error does not change, the effect on the resulting apparent flow rate allows the determination of the diverting error.

By rewriting equation 24 in terms of the apparent flow rate for two measurements in different conditions $\dot{m}'_1$ and $\dot{m}'_2$:

$$\dot{m}'_1 = \left(\frac{\Delta t}{t_1} \dot{m}_1 + \dot{m}_1 \right)$$

$$\dot{m}'_2 = \left(\frac{\Delta t}{t_2} \dot{m}_2 + \dot{m}_2 \right)$$

Substracting term by term and reordering we obtain:

$$\boxed{\Delta t \left(\frac{\dot{m}_1}{t'_1} - \frac{\dot{m}_2}{t'_2} \right) = (\dot{m}'_1 - \dot{m}'_2) - (\dot{m}_1 - \dot{m}_2)} \tag{27}$$

dividing by $\dot{m}'_2$ we obtain a similar equation presented by the ISO4185.

$$\frac{\Delta t}{\dot{m}'_2}\left(\frac{\dot{m}_1}{t'_1} - \frac{\dot{m}_2}{t'_2}\right) = \frac{\left(\dot{m}'_1 - \dot{m}'_2\right) - \left(\dot{m}_1 - \dot{m}_2\right)}{\dot{m}'_2}$$

As seen in equation 27, this method is strongly dependent on the accuracy of the used flow meters. In order to reduce this random error, several measurements have to be made [2]; Nevertheless, this method has the advantage that no additional special measurements have to be performed, it is basically possible to include every measurement made by the test rig to confirm or to monitor the diverter error, provided that the used reference flow meters are stable and repeatable.

References

[1] International Standard ISO 4185, *Measurement of liquid flow in closed conduits - Weighing method*, first edition, ISO (1980).

[2] Iryna V. Marfenko, Tsyh T. Yeh, John D. Wright *Diverter Uncertainty Less Than 0.01 percent for Water Flow Calibrations* International Symposium for Fluid Flow Measurement Conference Proceedings, Queretaro (2006).

[3] Rainer Engel and Hans Baade *Model-based flow diverter analysis for an improved uncertainty determination in liquid flow calibration facilities*, Measurement Science and Technology, Volume 21, Issue 2, 2010.

[4] N. Mathies, T. Lederer *Neue Methode zur Bestimmung des Zeitfehlers für Umschalteinrichtungen von gravimetrischen Messanlagen (New Way for Estimating the Diverter Timing Error for Gravimetric Flow Calibration Systems)* tm - Technisches Messen, Volume 74, Issue 6, June 2007

[5] T. Shimada, S. Oda, Y. Terao, M. Takamoto *Development of a new diverter system for liquid flow calibration facilities*, Flow Measurement and Instrumentation, Volume 14, Issue 3, June 2003

[2]Considering that Δt is the slope of the equation of a line drawn by equation 27, the International recommendations suggest to estimate its value by means of a linear regression.