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# OPTIMAL NON-LINEAR SEARCH METHOD FOR CAMERA CALIBRATION

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**Abstract** − Extracting three-dimensional information from two-dimensional image coordinates acquired by video cameras is a essential problem in computer vision. Prerequisite is a camera calibration procedure. For the purpose of camera calibration both linear and iterative techniques have been developed. The iterative methods utilize non-linear camera models, i.e. non-linear search technique. Three different popular non-linear search techniques (Newton method, Gauss-Newton method, Levenberg-Marquardt method) were chosen and compared in order to find out which one gave the best results for the purpose of camera calibration and 3D reconstruction. Levenberg-Marquardt method has been proven to be superior which was in accordance to theoretical expectation of its features. It exercised robustness and convergent sets of camera model parameters in all experimental trials. Moreover, it justified intuitive expectation of improving reconstruction accuracy by augmenting linear camera model with additional non-linear lens distortion parameters.

Keywords: Levenberg-Marquardt, camera calibration

## 1. INTRODUCTION

Biomechanical analysis of human movement utilizes very often data of subject's position in space. A convenient way for 3D reconstruction of points in space is based on processing images acquired by video cameras. Camera calibration is a necessary step in order to extract metric information from 2D images. In the context of three-dimensional machine vision camera calibration is the process of determining the internal camera geometric and optical characteristics (intrinsic parameters) and/or 3D position and orientation of the camera frame relative to a certain world coordinate system (external parameters) [1]. Part of the calibration procedure is a observation of calibration object whose geometry in 3D space is known with a very good precision. Such an object provides us with points of known positions in space, which are then used for calibration and possibly later for accuracy reconstruction analysis. The entire process of projecting points from 3D space to camera image plane can be modelled by the so-called camera model function [2]. Consequently, camera model is a function of the above mentioned camera internal and external parameters.

As a rule, the more complex camera model function is (i.e. more parameters are involved), the more realistic is description of projecting points from 3D space to image plane. Theoretically, the final reconstruction accuracy should be also improved by using comprehensive camera model [3]. In practice situation is somewhat different. One of the major drawbacks of using too many parameters is computational burden. Namely, nature of camera model can vary from simple linear functions to highly non-linear ones [4]. The former offers a closed form solution and is easily solved during the calibration procedure, while the later ones requires some non-linear search technique [5]. The aim of this paper is to take commonly used non-linear camera model and try to find optimal numerical procedure for calibrating it, i.e. solving its parameters.

## 2. METHODS

The camera model used here is effectively the same as those used in photogrammetry for a long time [6]. Throughout the time many variations of it have been proposed, however all of them are based on the same pinhole model augmented with a means of compensating for non-linear lens distortion. Specifically, we have used widely spread linear model known as Direct Linear Transformation (DLT) [7]. That model ignores non-linear lens distortion and assures relatively compact and fast solution for model parameters by solving set of linear equations. Then the original linear DLT model was build up with four additional parameters, two for tangential distortion and two for radial distortion. Furthermore inclusion of lens distortion parameters asks for some use of non-linear estimation technique. Three different non-linear search techniques were investigated. In particular those were Newton method, Gauss-Newton method and Levenberg-Marquardt method. It is a simple matter of finding out [8]. Theoretically the most promising and advanced was Levenberg-Marquardt method.

Nevertheless, the basic idea common to all three different minimization techniques will be explained in brief here. We start from the assumption that N dimensional function *f* can be approximated in vicinity of N dimensional solution $\mathbf{x_j} = [x_1, x_2, \ldots x_N]$ by Taylor series (1):

$$f(\mathbf{x}) = f(\mathbf{x_j}) + (\mathbf{x} - \mathbf{x_j}) \cdot \nabla f(\mathbf{x_j}) + \frac{1}{2} \cdot (\mathbf{x} - \mathbf{x_j}) \cdot \mathbf{H} \cdot (\mathbf{x} - \mathbf{x_j}) \qquad (1)$$

where **H** is so-called Hessian matrix consisted of second partial derivations of function *f* in point $\mathbf{x_j}$. All three different methods finds the minimum of function $f$ using Newton method for finding roots of function gradient $\nabla f$ (2):

$$\nabla f(\mathbf{x}) = \nabla f(\mathbf{x_j}) + \mathbf{H}\cdot(\mathbf{x}-\mathbf{x_j}) = 0 \tag{2}$$

Therefore function $f$ has its extrem for some value $\Delta\mathbf{x}$,. known as Newton step, calculated from (3):

$$\mathbf{x}-\mathbf{x_j} = \Delta\mathbf{x} = -\mathbf{H}^{-1}\cdot\nabla f(\mathbf{x_j}) \tag{3}$$

During the camera calibration function $f$ that we are actually minimizing is sum of squares differences between theoretical camera model function $y_m(\mathbf{x})$ and values obtained in practice $y_m$ (4):

$$f(\mathbf{x}) = \sum_{m=1}^{M}\left[y_m - y_m(\mathbf{x})\right]^2 = \sum_{m=1}^{M} g_m(\mathbf{x})^2 \tag{4}$$

where $M$ is a number of measurements – calibration points. Introduction of matrix **A** simplifies expressions for function gradient $\nabla f$ and Hessian matrix **H** (5):

$$\mathbf{A} = \begin{bmatrix} \frac{\partial y_1(\mathbf{x})}{\partial x_1} & \frac{\partial y_1(\mathbf{x})}{\partial x_2} & : & \frac{\partial y_1(\mathbf{x})}{\partial x_N} \\ : & : & : & : \\ \frac{\partial y_M(\mathbf{x})}{\partial x_1} & \frac{\partial y_M(\mathbf{x})}{\partial x_2} & : & \frac{\partial y_M(\mathbf{x})}{\partial x_N} \end{bmatrix}$$
$$\nabla f(\mathbf{x_j}) = -2\cdot\mathbf{A}^{\mathbf{T}}\cdot\mathbf{g} \quad \mathbf{g} = \left[g_1 \;\; : \;\; g_M\right]$$
$$\mathbf{H} = 2\cdot\mathbf{A}^{\mathbf{T}}\cdot\mathbf{A} - 2\cdot\sum_{m=1}^{M}\left[y_m - y_m(\mathbf{x})\right]\cdot\frac{\partial^2 y_m(\mathbf{x})}{\partial x_i\cdot\partial x_j} \tag{5}$$

Finally, by combining (3) and (5) we are able to derive expression (6) that will reveal us key differences between three different non-linear search methods, i.e. Newton method, Gauss-Newton method and Levenberg-Marquardt method.

$$\left[2\cdot\mathbf{A}^{\mathbf{T}}\cdot\mathbf{A} - 2\cdot\sum_{m=1}^{M}\left[y_m - y_m(\mathbf{x})\right]\cdot\frac{\partial^2 y_m(\mathbf{x})}{\partial x_i\cdot\partial x_j}\right]\cdot\Delta\mathbf{x} = 2\cdot\mathbf{A}^{\mathbf{T}}\cdot\mathbf{g}$$
$$\mathbf{x}-\mathbf{x_j} = \Delta\mathbf{x} = \left[\mathbf{A}^{\mathbf{T}}\cdot\mathbf{A} - \sum_{m=1}^{M}\left[y_m - y_m(\mathbf{x})\right]\cdot\frac{\partial^2 y_m(\mathbf{x})}{\partial x_i\cdot\partial x_j}\right]^{-1}\cdot\mathbf{A}^{\mathbf{T}}\cdot\mathbf{g} \tag{6}$$

Equations (3) and (6) represent in effect systems of linear equations which vector of solutions $\Delta\mathbf{x}$ can be calculated rather simply. However, methods differ in calculation of Hessian matrix. Newton method calculates Hessian matrix as it is according to (5) by taking both summands. It has been shown [8] that calculation of second partial derivates can have introduce instability during the iterative procedure. Furthermore, second summand can be usually neglected with respect to the first one, and is actually zero for the linear models. That's why Gauss-Newton method discards second summand and uses approximate expression (7) for calculation of Hessian matrix **H**:

$$\mathbf{H} \cong 2\cdot\mathbf{A}^{\mathbf{T}}\cdot\mathbf{A}$$
$$\mathbf{x}-\mathbf{x_j} = \Delta\mathbf{x} = -\mathbf{H}^{-1}\cdot\mathbf{A}^{\mathbf{T}}\cdot\mathbf{g} \tag{7}$$

This is a moment to recall our initial assumption of function approximation in the vicinity of solution (convergence area) by Taylor serious. If that assumption is fulfilled during the iterative procedure Gauss-Newton method should work just fine. However there is a possibility that Newton step, which we take between iterations to refine out present set of solution, to take us out of convergence area where Taylor series approximation is no longer valid. Eventually we could either end up with no convergent set of solution or we could converge to incorrect set of solution [8] (for instance problem of local minimum). Thus, it is better to take only one part of Newton step between some iterations. This will give us guarantee that we will stay on the track of function minimization (principle of steepest descent method), and at the same time stay in convergence area. Levenberg-Marquardt method has neatly embedded mechanism, which checks after each iteration should we refine our set of solution with full Newton step (i.e. Gauss-Newton) or only with part of it. Levnberg-Marquardt as such has been proven in practice very successful and has become *de facto* a standard for many researches (applications).

Calibration object used can take various forms and shapes. Traditional approach is 3-dimensional rigid structure (also known as 3D calibration cage) on which are clearly marked 3-D positions of certain number of points/calibration points. In this work calibration object used was a black plane with 6 × 11 white circled markers painted on) 18 cm apart (Fig. 1), giving 0.9 m × 1.92 m useable plane size. In addition, plane was moveable to precise locations, parallel to each other, throughout the depth of calibration volume (volume of interest), providing calibration points data as it was true 3D calibration cage used.

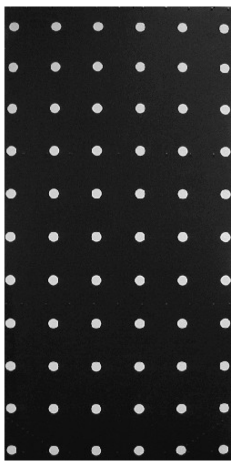

Fig. 1. Black calibration plane with 6 × 11 white circled markers painted on

Table 1. Values of camera parameters during the iterative steps procedure using Levenberg-Marquardt method

| **Internal camera parameters** | | | | | | |
|---|---|---|---|---|---|---|
| Focal length | Coordinates of principal points | | Tangential distortion parameters | | Radial distortion parameters | |
| 6,8757 | 773,4695 | 596,7679 | 0 | 0 | 0 | 0 |
| 6,8897 | 773,4510 | 598,4645 | -0,0054 | 0,0001 | -0,0002 | -0,0003 |
| 6,8174 | 772,4198 | 606,3472 | -0,0056 | 0,0001 | -0,0001 | -0,0005 |
| 6,8072 | 771,4905 | 632,3975 | -0,0055 | 0,0001 | 0,0001 | -0,0005 |
| 6,8080 | 773,7084 | 645,1850 | -0,0054 | 0,0001 | 0,0002 | -0,0005 |
| 6,8082 | 774,9434 | 647,2386 | -0,0054 | 0,0001 | 0,0002 | -0,0004 |
| 6,8083 | 775,1475 | 647,9322 | -0,0054 | 0,0001 | 0,0003 | -0,0004 |
| 6,8081 | 775,2688 | 648,0200 | -0,0054 | 0,0001 | 0,0003 | -0,0004 |
| 6,8082 | 775,2896 | 647,8898 | -0,0054 | 0,0001 | 0,0003 | -0,0004 |
| 6,8082 | 775,2896 | 647,8898 | -0,0054 | 0,0001 | 0,0003 | -0,0004 |
| **External camera parameters** | | | | | | |
| Camera position in space | | | Euler angles | | | |
| -155,3785 | 81,2688 | -236,9163 | -179,0290 | 34,1015 | -179,4358 | |
| -153,4085 | 81,0602 | -234,1792 | -179,0451 | 34,0787 | -179,4241 | |
| -151,2113 | 81,1115 | -230,8358 | -179,3623 | 34,0534 | -179,5983 | |
| -150,9431 | 81,0833 | -230,4192 | -180,4230 | 34,0293 | -180,1838 | |
| -150,9502 | 81,0630 | -230,4496 | -180,9420 | 34,0978 | -180,4723 | |
| -150,9571 | 81,0613 | -230,4706 | -181,0260 | 34,1374 | -180,5204 | |
| -150,9598 | 81,0603 | -230,4766 | -181,0542 | 34,1439 | -180,5364 | |
| -150,9558 | 81,0604 | -230,4715 | -181,0579 | 34,1480 | -180,5385 | |
| -150,9571 | 81,0605 | -230,4737 | -181,0526 | 34,1487 | -180,5356 | |
| -150,9571 | 81,0605 | -230,4737 | -181,0526 | 34,1487 | -180,5356 | |

Table 2. Values of camera parameters during the iterative steps procedure using Gauss Newton method

| **Internal camera parameters** | | | | | | |
|---|---|---|---|---|---|---|
| Focal length | Coordinates of principal points | | Tangential distortion parameters | | Radial distortion parameters | |
| 0,95008 | 594,22 | 5707,6 | -0,51902 | 0,012208 | 0,13207 | -0,054962 |
| 0,8835 | 592,2 | 5765,1 | -0,52485 | 0,012345 | 0,13356 | -0,05558 |
| 0,81692 | 590,19 | 5822,5 | -0,53068 | 0,012482 | 0,13504 | -0,056197 |
| 0,75034 | 588,17 | 5879,9 | -0,53651 | 0,01262 | 0,13653 | -0,056815 |
| 0,68376 | 586,16 | 5937,3 | -0,54234 | 0,012757 | 0,13801 | -0,057432 |
| 0,61718 | 584,15 | 5994,8 | -0,54818 | 0,012894 | 0,13949 | -0,05805 |
| 0,5506 | 582,13 | 6052,2 | -0,55401 | 0,013031 | 0,14098 | -0,058668 |
| 0,48402 | 580,12 | 6109,6 | -0,55984 | 0,013168 | 0,14246 | -0,059285 |
| **External camera parameters** | | | | | | |
| Camera position in space | | | Euler angles | | | |
| 243,03 | 63,958 | -347,03 | 380,47 | -28,471 | -109,46 | |
| 247,5 | 63,764 | -353,59 | 382,73 | -28,408 | -110,7 | |
| 251,98 | 63,569 | -360,15 | 384,99 | -28,344 | -111,94 | |
| 256,46 | 63,375 | -366,71 | 387,26 | -28,281 | -113,17 | |
| 260,93 | 63,18 | -373,27 | 389,52 | -28,218 | -114,41 | |
| 265,41 | 62,986 | -379,83 | 391,78 | -28,155 | -115,64 | |
| 269,88 | 62,791 | -386,39 | 394,05 | -28,091 | -116,88 | |
| 274,36 | 62,597 | -392,96 | 396,31 | -28,028 | -118,12 | |

Table 3. RMS values of 'unknown' points reconstruction using linear camera model (DLT);

| Par kamera | xrms | yrms | zrms | xsre | ysre | zsre | Nk | Nr |
|---|---|---|---|---|---|---|---|---|
| L45, D45 | **0,285** | **0,366** | **0,308** | **0,226** | **0,287** | **0,233** | **29** | **1027** |
| L30, D30 | **0,247** | **0,344** | **0,340** | **0,194** | **0,272** | **0,271** | **29** | **1027** |

Table 4. RMS values of 'unknown' points reconstruction using non-linear camera model (Levenberg-Marquardt method)

| Par kamera | xrms | yrms | zrms | xsre | ysre | zsre | Nk | Nr |
|---|---|---|---|---|---|---|---|---|
| L45, D45 | 0,165 | 0,132 | 0,197 | 0,134 | 0,104 | 0,151 | 29 | 1027 |
| L30, D30 | 0,154 | 0,142 | 0,296 | 0,121 | 0,112 | 0,220 | 29 | 1027 |

## 3. RESULTS

Some of the key features of any non-linear search technique is accuracy of set of solution and speed of convergence, if any. Thus, special attention was paid to it when comparing those three different non-linear search techniques. In particular Table 1 and Table 2 are showing typical outcome of minimization algorithms for Levenberg-Marquardt and Gauss-Newton method respectively.

As a final indicator certain number of ‘unknown points’ in space was reconstructed after camera pair was calibrated using each of the non-linear search technique. Since Newton method and Gauss-Newton method failed to give convergent set of camera parameters solution, accuracy analysis reconstruction for non-linear camera model, incorporating Levenberg-Marquardt method, and DLT camera model (linear model) was preformed. Specifically, accuracy analysis was preformed by calculating root mean square values (RMS) between predicted points position by calibrated camera model and its true positions [9]. RMS values were calculated separately for all three axes of spatial coordinate system. The points used for calibration were not used later for accuracy analysis in order to acquire more realistic results about system performance. In order to check repeatability of obtained results several experimental trials were preformed, each time with differently chosen calibration and reconstruction points. Table 3 and Table 4 represent typical accuracy reconstruction results when using linear camera model and non-linear camera model (Levenbreg-Marquardt method used for solving camera parameters) respectively. *Nk* column shows number of calibration points and *Nr* column represents number of ‘unknown’ reconstructed points. Cameras were in such positions that angles between its optical axes were approximately 60° (L30, D30) and 90° (L45, D45).

## 4. DISCUSSION AND CONCLUSION

It came out that among three non-linear methods tested, Levenberg-Marquardt method is the only effective method for non-linear camera parameter search. Based on the linear model used (DLT), Levenberg-Marquardt method gave in all experimental trials convergent sets of parameters solutions, which is not the case for the other two methods (Table 1 and Table 2). Hence, stability and robustness of non-linear search method highly depends how close is initial set of solution to the final one, rectified after certain number of iterations. Therefore, if we were using different linear camera model, as mean of providing initial sets of solutions during calibration, it is a possibility that Newton and/or Gauss-Newton method give the same if not better results than Levenberg-Marquardt.

On the top of all that accuracy reconstruction results (RMS values) when using Levenberg Marquardt method justified the inclusion of additional lens distortion parameters. In other words reconstruction accuracy when using only linear camera model (DLT) is evidently inferior to non-linear camera models used, as it was theoretically implicated (Table 3 and Table 4)

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