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Bias-induced Impedance of Current-carrying Conductors: Measurements Simulation Using FRA

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***Abstract* – The paper presents mainly the simulation of impedance measurements utilizing a frequency response analyzer (FRA) for the study of the current-carrying conductors under the influence of bias. It is vital to have confidence in the results of impedance spectroscopy because one of the informative quantities has very little impact on the resulting impedance. During the passage of an electric current, the studied objects have an additional low-frequency impedance called the bias-induced impedance effect (*Z_{BI}*-effect*) [1]. An emphasis was placed on explaining how the *Z_{BI}*-effect* acts as an amplifier for temperature modulation of a resistance caused by the test signal. A description of the investigated object is based on electrical models whose parameters provide useful information for further research. Several studies have been done to assess the consistency of electrical parameters. A silver wire was used in research as an object under study and as a representative of pure metals.**

***Keywords* – current-carrying conductors, bias-induced impedance, impedance spectroscopy, FRA**

I. INTRODUCTION

In previous work [1], the bias induced impedance effect (*Z_{BI}-effect*) was described. Experimental results related to the study of current-carrying conductors under bias using impedance spectroscopy were mainly presented.

In this study, we provide a more detailed overview and modeling of the measurement process. First, it is worth making a few remarks. The interpretation of the measurement results in [1] was restricted to electrical models - equivalent circuits. A possibility was expressed, but a thermophysical interpretation was not made since the author lacks the necessary knowledge, and such a task was not outlined. The following introduction will highlight the main works associated with this interpretation.

The first mention of the analysis of the response of a conductor with DC and AC currents was found in [2]. A platinum resistance heater/thermometer (PRT) is fabricated on two membranes. One membrane can be Joule heated to cause heat conduction through the sample to the other membrane. An SR830 lock-in amplifier is used to measure the first harmonic component of the voltage drop across the heating PRT, yielding the differential resistance.

It is worth paying attention to the work [3]. This paper illustrates the reaction of a conductor with current to various combinations of electrical influences known in the literature as the 1ω, 2ω, and 3ω methods. It has established a theoretical basis that links the parameters of the transfer function to the parameters of the object under study and the electrical impact on it. Note that what is referred to as the electrical transfer function (see, for example, Table 1 in the paper cited above) is actually a thermal transfer function denoted by K/W units of measurement - the same units as the thermal resistance R_{th} that is a parameter in the following equations in this paper. An undeniable advantage of the article is the possibility to interpret electrical measurements directly into thermophysical parameters of the objects under study - a specific thermal conductivity and a heat capacity.

An important article [4] deals with the practical aspects of developing resistive thermal probes for scanning thermal microscopy. It is based mainly on the theoretical contributions of [3]. Yet, some analytical expressions are more transparent and, in some respects, differ from the one stated in [3]. The expressions in this work also link the thermal characteristics and electrical parameters of the research object, which is an undoubted advantage.

We emphasize here that all of our work in [1] and here focuses on electrical parameters. The measuring method will be followed without considering the thermophysical properties of the studied objects. Our research has produced "output products" in the form of electrical parameters that can be effectively applied by specialists within the framework of thermophysical theories.

In the papers [2] - [4] and the works following them, the Lock-in Amplifier class devices are used as meters for measuring electrical parameters. These devices do not directly measure the impedance of the objects under investigation (especially using bias), but with some additions, they can implement this function.

Our research employs commercially available instruments that directly measure the impedance of objects under bias conditions. Potentiostat and Galvanostat are commonly used terms to describe these devices. Initially, they were designed to operate in the time domain, but in recent decades, they have begun to be equipped with impedance modules with decent metrological properties. The Potentiostat/Galvanostat Biologic SP-240 (BioLogic Sciences Instruments) was used as the measuring instrument [5]. In our earlier paper [1], we considered the nature of the Z_{BI}-*effect* in sufficient detail, using various materials to evaluate it in a broad context. A promising result associated with the dynamic nature of the temperature coefficient of resistance (TCR) has been revealed. This result may explain the behavior of the alloys. In this work, we will simulate the measurement process using one of the representatives of pure metals as an object under test.

Fig. 1 shows the impedance characteristics of a silver wire (Aldrich Chem. Co. 99.99+%) 500 mm long and 0.25 mm in diameter in two modes of current exposure: without bias I_{dc} = 0 A and with bias I_{dc} = 3 A. The amplitude of the test signal I_{ac} was 100 mA, which met the small-signal requirements.

The measurements were conducted by operating the Biologic SP-240 in galvanostatic mode. Data were verified in some cases using the Gamry Reference 3000 potentiostat/galvanostat (Gamry Instruments). We used a 4-wire sample holder [1] inside the Faraday cage. The experiment was conducted at a temperature of about 20 °C in a room environment.

Without bias, the impedance of the investigated conductor is well described by the three-element model, Fig. 2 (a). The R_p parameter in this model is responsible for the skin effect in the high-frequency region. The remaining two parameters, L_p and R_s, relate to the size and material of the object. In the low-frequency region, imaginary part noise is natural (in this region, ReZ/ImZ > 10^3) and depends, in particular, on the sensitivity of the measuring device. The use of a Faraday cage can reduce the level of this noise significantly.

In the low-frequency part of the spectrum, it is immediately observed that the nature of the reactance changes dramatically with presenting bias: from inductive to capacitive. To deal with experimental data in the case of bias, it is necessary to incorporate in the previous model other elements which are responsible for the Z_{BI}-*effect*: parallel R_{BI} and C_{BI}, Fig. 2 (b).
As you can see from the fitting results, bias has a very small effect on the high-frequency part of the impedance spectrum (R_p and L_p). Throughout the entire frequency range, there is a noticeable change in the R_s – one of the real components of the impedance. Bias causes a shift in the temperature of the conductor, resulting in this change. The effect is common. Generally, model (b) well approximates the impedance of the object under study over the entire experimental frequency range. The effect of bias on the high-frequency part of the spectrum is not significant, and we are not concerned with it. We will further study the low-frequency part of the spectrum - approximately frequencies below 10 Hz.

In the following section, the simulation of the measurement process will be examined to confirm the validity of the results obtained using our instruments.

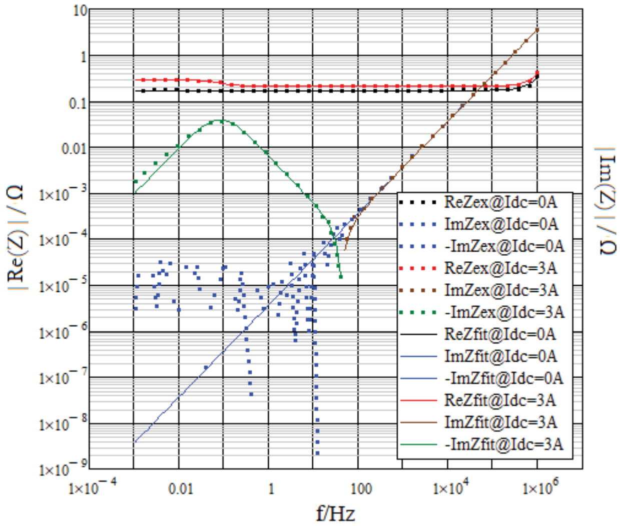

Fig. 1. Experimental Re*(Z),* Im*(Z) and fitted results; silver wire; bias: I_{dc} = 0* A *and I_{dc} = 3* A*; length 500* mm *and diameter 0.25* mm*; frequency range 1* MHz *... 0.001* Hz*.*

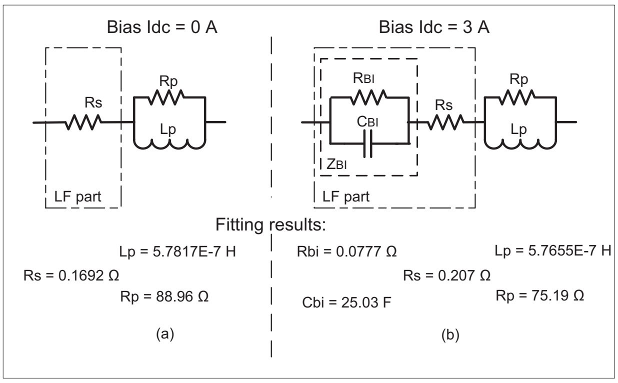

Fig 2. Electrical models and their parameters used to fit experimental data presented in Fig.1; (a) - results at zero bias (I_{dc} = 0 A*); (b) - results at I_{dc} = 3* A*.*

II. IMPEDANCE MEASUREMENTS SIMULATION UTILIZING FRA

The modern impedance measuring instruments have several basic structures [6]. We will simulate a structure called a Frequency Response Analyzer (FRA). An impedance part of the instruments used in this work is centered around the FRA.

To understand the measurement process, we must first build a model of the object under study – the current-carrying conductor under the influence of the superposition of the test signal and bias. Here, we will examine how the blocks act upon the object in the process of considering the operation of the blocks. These blocks are included in the FRA structure.

The work [6] presents a simplified block diagram of FRA and explains how such a structure operates. Here are some details of this block to simulate and verify the measurement results. Fig. 3 illustrates the FRA that underlies the measurement procedure. Let us choose the galvanostatic mode as the option of operating for definiteness. It means that the sample under test will receive a sum of the bias current I_{dc} and a sinusoidal test signal with an amplitude I_{ac} having a circular frequency $\omega = 2\pi f$:

$$I_{act}(t) = I_{dc} + I_{ac}\sin(\omega t) \tag{1}$$

The total current causes Joule heating $Q = I^2R$ and therefore leads to a change in the resistance of the conductor due to its weak but existing nonlinearity. The square of the current will have three components:

$$I_{act}^2(t) = I_{dc}^2 + 2I_{dc}I_{ac}\sin(\omega t) + I_{ac}{}^2\sin(\omega t)^2 \tag{2}$$

The last term of expression (2) will generate the 2nd harmonic in the heating of the conductor and, accordingly, this component will modulate the second harmonic of the conductor's resistance. In this work, we are considering the small-signal condition ($I_{ac} << I_{dc}$) and a weak nonlinearity of the object under study. Thus, ignoring the second and higher harmonics, the expression for the resistance of the conductor under the influence of the bias and test signal can be approximately described as

$$R_{rsp}(t) \approx R_{wp} + R_{ac}(\omega)\sin(\omega t + \psi(\omega)) \tag{3}$$

where: $R_{wp} \propto I_{dc}^2$ - static resistance at working point; $R_{ac} \propto I_{dc}I_{ac}$ - amplitude of modulation part of the resistance. The value of R_{ac} at the operating point depends on the circular frequency ω due to inertia. The phase shift ψ is also related to the test signal frequency due to inertia. Ignoring the second and higher harmonics in expression (3) is also justified by the FRA functional, which will be discussed below.

With an increase in the frequency of the test signal, the modulation value of resistance due to the test signal tends to zero, and the phase angle tends to 90 degrees. With decreasing frequency, the modulation value will tend to be maximum, and the phase angle will tend to zero. All this is due to the inertia of the object under study.

Thus, we have formed a model of the object under study (3) – the current-carrying conductor under the influence of the superposition of the test signal and bias.

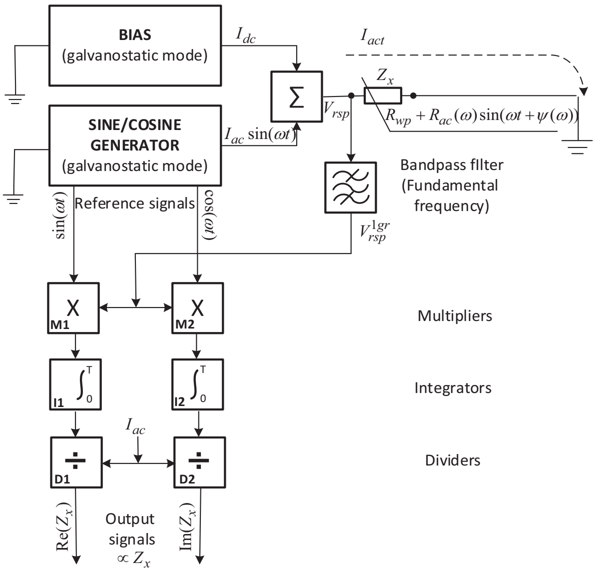

Fig. 3. Functional diagram of a frequency response analyzer.

According to Ohm's law, the voltage drops $V_{rsp}(t)$ across the object under study will be equal to the product of expression (1) and (3):

$$\begin{aligned} V_{rsp}(t) &= [I_{dc} + I_{ac}\sin(\omega t)] \times \\ &\times [R_{wp} + R_{ac}(\omega)\sin(\omega t + \psi(\omega))] \end{aligned} \tag{4}$$

Corresponding to the functionality of the FRA [6], [7], harmonics that are multiples of the fundamental will be suppressed. This also applies to the appearance of the DC component in the signal. We designate this function by introducing a bandpass filter into the circuit (Fig. 3), which only passes the fundamental frequency. Therefore, following the bandpass filter, higher harmonics and DC components are removed. In this case, the output signal will only contain the fundamental frequency (1gr):

$$V_{rsp}^{1gr}(t) = I_{ac}\sin(\omega t)R_{wp} + I_{dc}R_{ac}(\omega)\sin(\omega t + \psi(\omega)) \tag{5}$$

The expression (5) contains two components. The first term of this expression determines the static component of the investigated object. The second term defines the dynamic component generated by resistance modulation and bias. The signal (5) is fed into one of the inputs of two multipliers M1 and M2 (see Fig. 3).

Other inputs received the sinusoidal and cosine signals with unit amplitude transmitted from the generator (reference signals). As a result, the quadrature components of the output signal of the multipliers are respectively equal:

$$V_{\sin}(t) = \sin(\omega t)[I_{ac}\sin(\omega t)R_{wp} + \\ + I_{dc}R_{ac}(\omega)\sin(\omega t + \psi)] \quad (6)$$

$$V_{\cos}(t) = \cos(\omega t)[I_{ac}\sin(\omega t)R_{wp} + \\ + I_{dc}R_{ac}(\omega)\sin(\omega t + \psi)] \quad (7)$$

The outputs of the multipliers are fed into the inputs of integrators I1 (which extracts a real component of the signal) and I2 (which extracts an imaginary component of the signal). After the integrators, the signals will have the following form:

$$\operatorname{Re}V = I_{ac}R_{wp}\frac{2}{T}\int_0^T[\sin(\omega t)\sin(\omega t)]dt + \\ + I_{dc}R_{ac}(\omega)\frac{2}{T}\int_0^T[\sin(\omega t)\sin(\omega t + \psi(\omega))]dt \quad (8)$$

$$\operatorname{Im}V = I_{ac}R_{wp}\frac{2}{T}\int_0^T[\cos(\omega t)\sin(\omega t)]dt + \\ + I_{dc}R_{ac}(\omega)\frac{2}{T}\int_0^T[\cos(\omega t)\sin(\omega t + \psi(\omega))]dt \quad (9)$$

Further, we will use the equalities (10) and (11):

$$\frac{2}{T}\int_0^T[\sin(\omega t + \psi(\omega))\sin(\omega t)]dt \equiv \cos(\psi(\omega)) \quad (10)$$

$$\frac{2}{T}\int_0^T[\sin(\omega t + \psi(\omega))\cos(\omega t)]dt \equiv \sin(\psi(\omega)) \quad (11)$$

taken from [6] to form the final expression. The first parts of expressions (8) and (9) do not depend on the phase angle and only give the real component of the signal. Dividing (8) and (9) by the known amplitude of the test signal I_{ac} (D1 and D2, see Fig. 3) and considering expressions (10) and (11) noted above, we finally get:

$$Z_x = R_{wp} + \frac{I_{dc}}{I_{ac}}R_{ac}(\omega)\left\{\cos[\psi(\omega)] + j\sin[\psi(\omega)]\right\} \quad (12)$$

According to the electrical model for the low-frequency part of the impedance (see LF part in Fig.2b), we can write

$$Z_x = R_s + Z_{BI} \quad (13)$$

where $R_s \equiv R_{wp}$ and

$$Z_{BI} \equiv \frac{I_{dc}}{I_{ac}}R_{ac}(\omega)\left\{\cos[\psi(\omega)] + j\sin[\psi(\omega)]\right\} \quad (14)$$

The analysis (14) shows that the effect of R_{ac} modulation in the resulting impedance will increase with an increase in the ratio of the bias I_{dc} to the test signal I_{ac}. It is worth reminding that the R_{ac} modulation will also depend on the amplitude of the test signal (Joule heating), but their ratio (R_{ac}/I_{ac}) remains almost the same at the low signal approach. For example, it has been experimentally confirmed that at a bias I_{dc} = 3 A, a 10-fold increase in the test signal from 10 mA to 100 mA leads to an increase in this ratio by only 0.1%.

Because the inertia will be absent at extremely low frequencies, we can assume that the angle ψ will tend to zero. Therefore, the imaginary part in (14) will be disappeared. In addition, the modulation amplitude caused by the test signal will be maximized: $R_{ac}(\omega) \to R_{ac}^{\max}$. In this case, the expression (14) takes the form:

$$R_{BI} = \frac{I_{dc}}{I_{ac}}R_{ac}^{\max} \quad (15)$$

Let us estimate the value of the $R_{ac}^{\max}$ modulation. After fitting with a bias I_{dc} = 3 A and a test signal I_{ac} = 100 mA, R_{BI} = 0.0777 Ω is found (Fig. 2b). As a result of the expression (15), we arrive at the value of the amplitude of the maximum modulation $R_{ac}^{\max}$ = 0.0026 Ω. This value provides valuable information about the thermophysical properties of the object under study [3], [4]. A further useful parameter for assessing the thermophysical properties is the time constant $\tau_{BI} = R_{BI} C_{BI}$ obtained from the model in Fig. 2b.

From the value of $R_{ac}^{\max}$ above, let us estimate the temperature modulation. The well-known formula for finding resistance at a given temperature is:

$$\rho_2 = \rho_1 \cdot [1 + \alpha \cdot (T_2 - T_1)] \quad (16)$$

where $\alpha = 3.8 \cdot 10^{-3}$ Ω/Δ°C is a temperature coefficient of resistance at room temperature; ρ_1 is a silver resistivity at T_1; ρ_2 is a required value of resistivity at temperature T_2. Based on (16), considering the geometric dimensions of the conductor, to solve the inverse problem - finding the temperature from the resistance at the operating point R_s, the corresponding expression will be written as follows:

$$T_{wp} = \frac{R_s^{3A} + R_s^{0A} \cdot (T_{Room} \cdot \alpha - 1)}{\alpha \cdot R_s^{0A}} \quad (17)$$

where values are: R_s^{3A} = 0.207 Ω and R_s^{0A} = 0.1692 Ω (see Fig 2b and Fig2a accordingly). The ambient temperature T_{Room} used in the calculation according to formula (17) was approximately 20 °C. Calculation by formula (17) gave the result T_{wp} = 78.79 °C.

To find the temperature variation from the variation in resistance R_{ac}^{max} caused by the test signal, formula (17) is modified as follows:

$$T_{wp}^{ac} = \frac{\left(R_s^{3A} + R_{ac}^{\max}\right) + R_s^{0A} \cdot \left(T_{Room} \cdot \alpha - 1\right)}{\alpha \cdot R_s^{0A}} \qquad (18)$$

A calculation using formula (18) gave the result $T_{wp}^{ac} =$ 82.83. Thus, the difference between the temperature values obtained by formulas (18) and (17) is 4.04 °C. This temperature increase should be observed at extremely low frequencies when the thermal inertia of the system is close to zero.

It can be seen that the temperature modulation is small in comparison with the high temperature additive component at the operating point. Nonetheless, this value is available for direct and not very complicated measurements (without pretensions to accuracy, but only to assess the order). In the following section, we demonstrate the effectiveness of direct temperature measurement in evaluating this temperature modulation. Thus, it will be possible to confirm the value of temperature modulation of the resistance R_{ac}^{max}, found above, caused by the test signal, by an independent method.

III. CHECK OF DATA CONSISTANCY USING DIRECT TEMPERATURE MEASUREMENTS

Direct measurements of the temperature of the sample under test (SUT) are conducted as follows. A small piece of investigated conductor - about 5 mm (1% of the full length) was wrapped around the thermocouple. Using repeat impedance measurements in the same frequency range, we examined the thermocouple effect on the SUT's impedance and found no noticeable difference with or without the thermocouple. We estimated the thermocouple's time constant experimentally, and it was less than 3 s. Considering this value, a measurement range of up to 1 mHz was chosen to ignore the effect of thermocouple inertia on temperature measurements. Data were collected using a thermocouple type K and a Eurotherm 2416 controller.

For each of the measurement frequencies, three periods of the test signal were carried out. Fig. 4 shows the results of direct temperature measurements.

The initial stage of the experiment included a delay of 15 minutes at a given bias value without applying a test signal.

A temperature response was noticeable from a frequency of 0.25 Hz down to lower frequencies. The initial section of the apparent temperature response of the system is affected by both the inertial properties of the object under study and the inertia of the thermocouple. In

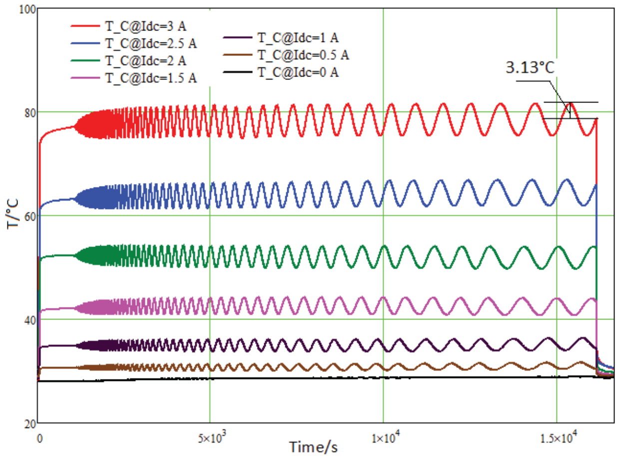

Fig. 4. Temperature of silver wire as a function of time during impedance measurement. The test signals are 100 mA *at different biases: 0 ... 3* A*; frequency range is* *1* MHz ... *1* mHz; *length 500* mm *and diameter* 0.25 mm*. The initial wait before starting impedance measurements was 15 minutes at fixed bias.*

this regard, we do not study this frequency fragment. The measurement time from the start frequency (1 MHz) to the visible response of the system to the test signal (0.25 Hz) takes approximately 3 minutes.

The subsequent measurement from 0.25 Hz to 1 mHz takes about four hours (ten points per decade with the uniform step at a logarithmic scale and three periods for each frequency).

The experiment reveals that at an extremely low frequency of 1 mHz and a bias value of 3 A, the temperature variation has an amplitude of 3.13 °C at an average temperature of 78.47 °C (see Fig. 4). The test signal was 100mA as before. The measured temperature variation by the thermocouple is relatively close to the temperature estimated by previous indirect measurements and calculations. A difference of about 20% is found between the values obtained by direct and indirect methods of measuring temperature variations. This difference can be attributed to several factors, the analysis of which is not the subject of this study. We have only estimated the order of magnitude here. The measurement of temperature variation with a thermocouple has only an assessment role.

IV. DISCUSSIONS

The current-carrying conductors are nonlinear inertial objects. Non-linearity and inertia are related to the temperature properties of the research object and its environment. In essence, these properties determine the Z_{BI}*-effect.*

Previous studies [1] mainly dealt with experiments demonstrating the Z_{BI}*-effect* of various material types. This article focuses on the measurement procedure - what is actually measured and what the measurement results can tell us about further research on examined objects. The answer to this question is as follows. The mathematical modeling of the measurement process using FRA showed that the impedance modulation caused by the test signal on fundamental frequency is amplified by the ratio of the bias to the signal amplitude when a bias is applied. This phenomenon is worthy of its name: bias induced impedance effect (Z_{BI}-*effect*).

We can say allegorically that this is an "honest cheating" of the measuring devices. The measuring device captures two components of the reaction at the fundamental frequency. One of them is caused by a test signal at the working point. The second reaction is an additional response caused indirectly. In this response, an ac-signal is generated from the applied bias because of the temperature modulation of the wire initiated by the same test signal. No matter where the fundamental frequency signal comes from, it will be perceived by the measuring device as "native" and evaluated in comparison with the excitation signal.

The estimate of the modulation component of the impedance caused by the test signal, obtained using the FRA, is credible. The measurement error of this modulation will be determined by the metrological characteristics of the instruments used for measuring the impedance.

It is important to emphasize that the higher the bias level (all other things being equal), the stronger the Z_{BI}-*effect* manifests itself. In the absence of bias, the Z_{BI}-*effect* disappears (Fig. 1). Using the results of impedance spectroscopy in the form of electrical models and calculating the modulation parameter R_{ac} can provide useful information for determining thermophysical parameters [2,3] and [4].

V. CONCLUSIONS

This work describes in detail the operation of the Frequency Response Analyzer underlying the impedance meters employed for the study of the current-carrying conductors under the influence of bias.

The electrical model of the object under study has been constructed. One of the informative parameters of this model is the temperature modulation of the resistance caused by the test signal. At low frequencies, FRA output signals provide information about the bias-induced impedance Z_{BI}. With bias applied, the Z_{BI}-*effect* acts as an amplifier for the temperature modulation of the resistance caused by the test signal. An amplifying gain is determined by the bias-to-test signal ratio.

Using impedance measurement devices, you can determine the modulation component of the resistance of the current-carrying conductor R_{ac} by knowing the levels of the test signal and bias. Together with the parameters of the electrical model (especially the Z_{BI} time constant), the R_{ac} can serve as an informative parameter for finding the thermophysical parameters of the objects being studied.

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