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Power spectrum measurement of high frequency angle modulated digital signal from output power spectrum of a digital divider

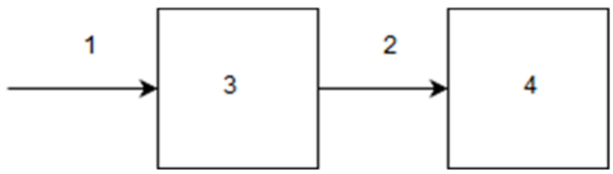
Leonidas Niyonkuru[1], Prof. Dr. ir. Gerd Vandersteen[2] and Prof. Dr. ir Leo Van Biesen[3]

[123]ELEC Department, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussels, Belgium
[1] niyonkuru.leonidas@vub.be
[2]gerd.vandersteen@vub.be
[3] lvbiesen@vub.be

Abstract **This paper shows how to measure high frequency angle modulated digital signals using a relatively low frequency spectrum analyzer and a digital frequency divider. The mathematical proof is based on comparison of the signal spectrum before and after the frequency divider. The spectral scaling of angle modulated digital signal by the divider is predictable mathematically implying that the spectrum at the input of the divider can be deduced from the spectrum at the digital output of the frequency divider. The proposed method was confirmed by measurement.**
Keywords **– frequency divider, power spectrum, angle modulated signal**

I. INTRODUCTION

High frequency spectrum analyzers can be expensive and are not always available to the experimenter [1]. Down converting the high frequency signal to be measured enables the usage of lower frequency spectrum analyzer[2]. A first, classical possibility is to perform a frequency down conversion using analogue mixers and a stable high-frequency source [3]. This paper develops an alternative method to measure the power spectrum of a high frequency angle modulated digital signal using a frequency divider.
Figure 1 shows the functional scheme of that method in which the digital divider do not contain any analogue component (e.g. analog mixer or oscillator). The programmability of such digital frequency dividers leads to a high flexibility [4].
For angle modulated digital signals, we show that a digital frequency divider can be used to down convert the spectrum of angle modulated signal to a lower frequency band which can be more convenient from measurement point of view. The mathematical proof of the proposed method is based on the analysis of the angle modulated signal at the input and the output of the digital divider. This mathematical framework is afterwards confirmed using measurements of an angle modulated digital signal spectrum at input and output of a digital divider with a variable division factor (N=3,4,5,6,12,25,39). Finally, we compare the input and output spectra for various division factors N.



1. High frequency angle modulated digital signal
2. Lower frequency angle modulated digital signal
3. Digital frequency divider
4. Lower frequency spectrum analyzer

Fig. 1 Down conversion of high frequency angle modulated digital signal using digital divider before the low frequency spectrum analyzer.

II. DESCRIPTION OF THE METHOD

This paragraph gives the mathematical description of the angle modulated digital signal that is down converted using a digital frequency divider.
The mathematical description starts by first decomposing the digital (square wave) signal in its fundamental and its harmonic components. The power spectrum at the output of the frequency divider will be measured around a single carrier.
Consider an angle modulated signal, $A(t)$, at the input of the frequency divider by integer N and let $A_d(t)$ be the signal at the output of the mentioned divider (Fig. 2).

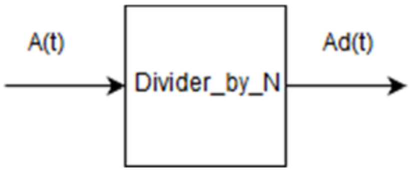


Fig. 2 Divider with angle modulated signal at his input

The general expression of an angle modulated signal can be written as:

$$A(t) = Acos(\omega_o t + mS(t)) \quad (1)$$

with A the carrier amplitude, ω_o the carrier frequency, m the modulation index, and $S(t)$ the phase modulating of the signal.

For the ease of analyze, suppose that

$$S(t) = \sin(\omega_m t) \quad (2)$$

Expression (1) then becomes

$$A(t) = Acos(\omega_o t + m\sin(\omega_m t)) \quad (3)$$

The instantaneous (radial) frequency, which equals the derivative of the phase, is given by

$$\omega_o + m\omega_m \cos(\omega_m t). \quad (4)$$

The output frequency at frequency divider by N then yield a frequency

$$\frac{\omega_o}{N} + \frac{m\omega_m}{N}\cos(\omega_m t) \quad (5)$$

The modulated signal at the divider output therefore equals

$$A_d(t) = A_d\cos(\int_0^t (\frac{\omega_o}{N} + \frac{m\omega_m}{N}\cos(\omega_m t))dt)$$

and hence

$$A_d(t) = A_d\cos(\frac{\omega_o}{N} t + \frac{m}{N}\sin(\omega_m t)) \quad (6)$$

Expression (3) and (6) are quite similar, except that both the carrier amplitude and the modulation index have changed. Note that the modulation frequency, ω_m, remains constant in both expressions.

Expressions (3) and (6) can be written as [6]

$$A(t) = A\sum_{n=-\infty}^{\infty} J_n(m)\cos(\omega_o - n\omega_m)\,t$$

and

$$A_d(t) = A_d\sum_{n=-\infty}^{\infty} J_n(\frac{m}{N})\cos\left(\frac{\omega_o}{N} - n\omega_m\right)t$$

where $J_n(m)$ and $J_n(\frac{m}{N})$ are the first kind Bessel function, m and $\frac{m}{N}$ the modulation index of the signal at the input and output of the frequency divider respectively. Obviously $A(t)$ and $A_d(t)$ have similar spectral components. The relative amplitude of components at $\frac{\omega_o}{N} \pm n\omega_m$ frequency will be scaled by

$$J_n(\frac{m}{N})/J_n(m)$$

which can be approximated by $1/N$ for a small modulation index m. Hence, the relative power will decrease by $20\ log\ N$.

Hence measuring the angle modulated signal spectrum with small modulation index at the output of the divider enables us to calculate the signal's input power spectrum. However, the purpose is spectrum measurement of angle modulated digital signal and therefore digital divider should be used. The above analyze can be extended to digital signals as square wave can be considered as a sum of sinusoidal signals. The input of the CMOS divider in next section is therefore a logic signal.The next section provides the experimental validation of the proposed method.

III. MEASUREMENTS

A divider based on CD74HC4059 frequency divider chip with an integer division factor N between 3 and 15999 was used. Measurements were done for N=3,4,5,6,12,25 and 39.

While the frequency divider is fed with an angle modulated digital signal, the power spectrum is measured at the output of the digital divider. The input signal spectrum is shown on Fig. 3, while the spectrum at the divider's output is shown in Fig. 4 and Fig. 5 for a division ratio of 3 and 5 respectively. Fig. 6 shows the output signal spectrum for different division ratio N=3,4,5,6,12,25 and 39. For each division ratio, the graph is plotted at a relative frequency from the carrier frequency taken as reference. The spectrum was measured for a frequency band of 1kHz around the carrier for every division ration N. The top curve in Fig. 6 is the spectrum of the input signal while the others represent the output signal spectrum for N=3,4,5,6,12,25 and 39 (from top to bottom).

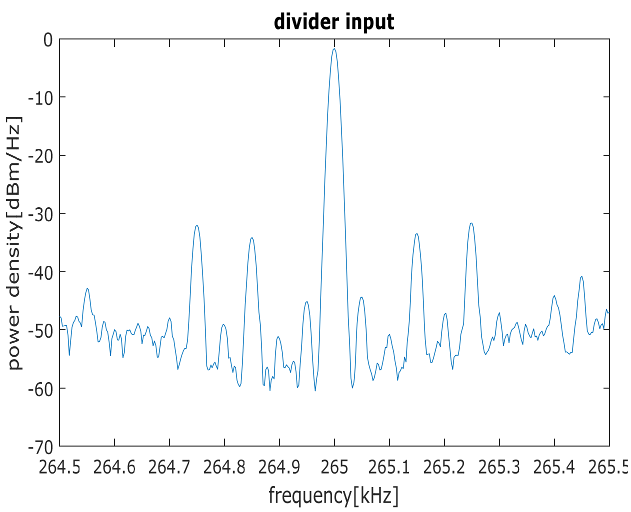


Fig. 3. divider input spectrum

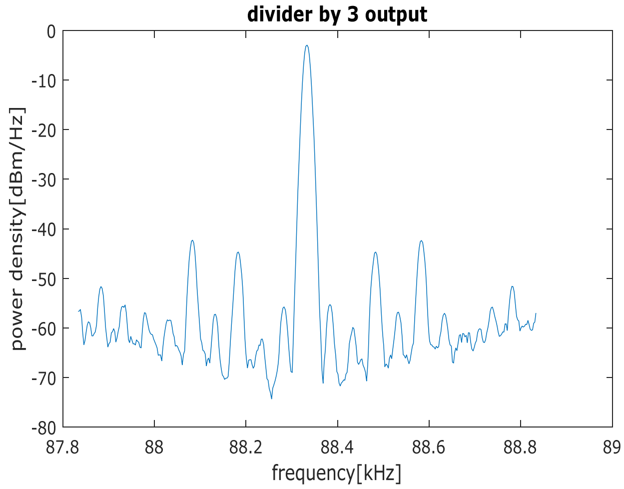


Fig. 4. Output power spectrum for division ration of 3

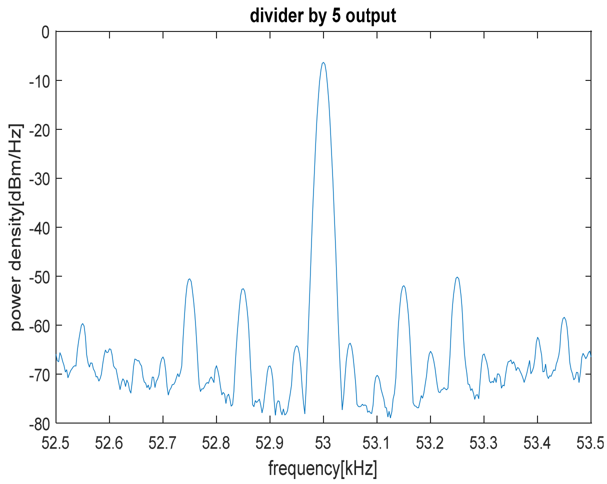


Fig. 5. Output power spectrum for division ration of 5

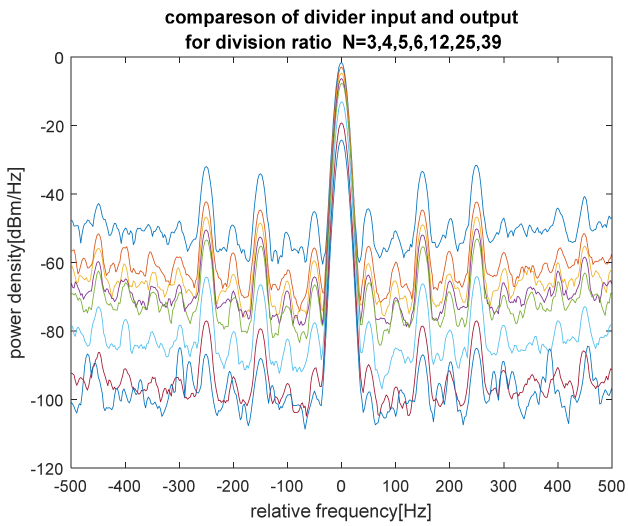


Fig. 6. Input (top curve) and output power spectra for division ratio of 3,4,5,6,12,25,39 (from top to bottom)

IV. RESULTS DISCUSSION

Fig. 6 shows that the measured spectrum with different division factor N are quite similar and that they follow a relative attenuation of $20logN$ as predicted in the previous section for small modulation indexes.

Table 1 shows the measurements at the frequencies where peaks are observed. The first row represents the input spectra, while the other rows represent the output spectra of the divider for different division ratio N.

To compare different outputs, the following calculations were preformed: for each frequency in Table 1 and for each division factor N, we take the difference of power (in dB) at the input and the output of the divider and subtract the difference of carrier power at input and output. The carrier power is at frequency zero (see Fig. 6). The calculation result is shown in Table 2.

Example: for the division factor N=3 and relative frequency -250 Hz, the output power of the divider is -42.28dBm and the input power is -32.00dBm. The difference of input and out power is -32.00dBm-(-42.28dBm) = 10.28dBm. The difference of carrier power at input and output for N=3 is -1.64dBm-(-2.96dBm) = 1.32dBm. Hence, the relative difference is 10.28dBm-1.32dBm = 8.96dB (shown in Table 2). This result is to be compared with 20logN=20log3=9.54dB shown in last column.

Note that the attenuation of 20logN is valid only for small modulation index m and $\frac{m}{N}$. For larger modulation index, the value of Bessel function for each components n determines the output amplitude.

f	-250	-150	0	150	250
Input	-32.00	-34.12	-1.64	-33.42	-31.62
N=3	-42.28	-44.68	-2.96	-44.68	-42.35
N=4	-46.80	-48.48	-4.73	-48.99	-46.74
N=5	-50.53	-52.56	-6.33	-51.94	-50.17
N=6	-53.39	-55.22	-7.72	-55.36	-53.11
N=12	-64.25	-66.49	-13.10	-66.22	-64.10
N=25	-77.03	-79.93	-19.27	-78.54	-77.05
N=39	-86.82	-88.00	-24.27	-86.90	-85.05

Table 1: Samples of measurement at some peaks from Fig. 6.

N \ f	-250	-150	150	250	20log N
3	8,96	9,24	9,94	9,41	9,54
4	11,71	11,27	12,48	12,03	12,04
5	13,84	13,75	13,83	13,86	13,98
6	15,31	15,02	15,86	15,41	15,56
12	20,79	20,91	21,34	21,17	21,58
25	27,4	28,18	27,49	27,8	27,95
39	32,19	31,25	30,85	30,8	31,82

Table 2 Relative attenuation of angle modulated signal due to digital divider at different frequencies compared with 20 logN

The table 2 shows that the attenuation by the digital divider for each frequency and for each division ratio N was comparable to $20logN$. The digital frequency divider can be used to down convert the frequency of an angle modulated digital signal and can therefore be used for its spectrum measurement using a lower frequency spectrum analyzer.

V. CONCLUSIONS

This work shows the possibility to measure the power spectrum of an angle modulated digital signal using a digital frequency divider and a spectral measurement using a lower frequency spectrum analyzer. The method is based on the fact that the frequency and amplitude at the divider's output are predictable and hence by measuring the divider output spectrum, we can deduce the input spectrum. The add-value of this method is the possibility of measurement of a high frequency signal using relatively low frequency spectrum analyzer and frequency divider, and this would be less expensive than a high frequency spectrum analyzer. The results were supported and demonstrated through measurements.

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