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PRBS non-idealities affecting Random Demodulation Analog-to-Information Converters

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Abstract **–This paper aims to the characterization of non-idealities of the Pseudo Random Binary Sequence (PRBS) mixing the signal input in a Random Demodulation Analog-to-Information Converter (RD AIC). Once the PRBS has been characterized, a simulation analysis has been carried out in order to evaluate the error that each component of the jitter affecting the PRBS causes on the AIC output. Results show that significant error is also caused by deterministic jitter components that could be included in the AIC model.**

Keywords **– Compressive Sampling, Analog-to-Information Converter, Pseudo-Random-Binary-Sequence, Jitter.**

I. INTRODUCTION

Nowadays, available technologies do not meet easily the demand of acquisition in the case of wideband spectrum signals. In fact, sampling theory is constructed on the Nyquist-Shannon theorem, which imposes that a band-limited analog signal can be acquired with a sampling frequency at least equal to twice its bandwidth. Therefore, a huge number of samples is required to represent even short time frames of wideband signals.

In the last decade, the desire to circumvent this limitation has prompted a new signal acquisition technique, the Compressive Sampling (CS) [1], as an alternative paradigm to the traditional sampling in case of compressible signals, i.e. signals with a few large coefficients and many small coefficients in a certain domain. CS has attracted considerable attention in signal acquisition field because it allows compressible signals to be acquired at sub-Nyquist rate and consequently to be represented by fewer samples.

The new sub-Nyquist sampling structures that try to implement the CS theory are called Analog-to-Information Converters (AICs) to distinguish them from the conventional Analog-to-Digital Converters (ADCs). Compared to the traditional signal acquisition architectures they operates at lower speed reducing the load in the front-end system and transferring it to the back-end system.

The operating principle of the most common AIC architectures (i.e. random [2] and polyphase random [3] demodulator, random [4] and nonuniform [5] sampler, modulated wideband converter [6], compressive multiplexer, [7] and time encoding machine [8]) is based on the use of a Pseudo Random Binary Sequence (PRBS). A PRBS is a sequence of random symbols in the set {-1, +1}, which should ideally alternate at a fixed symbol rate and it is consequently modelled as Dirac pulses or as zero-order hold. In practice, due to a variety of factors, it has not an ideal trend but it is affected by the jitter phenomenon.

Some Authors have discussed the effects produced by PRBS jitter used in the mixing step of AICs, in terms of Signal-to-Noise Ratio [9] and Effective Number of Bits [10]. These analyses have been conducted considering only the jitter stochastic component.

Anyway, approximating the PRBS trend to an ideal one and including any jitter systematic components determine inaccuracy on the AIC output. In [11], a numerical analysis has been conducted in order to evaluate the error caused by modelling the PRBS with an ideal law, which does not take the PRBS actual trend and any kind of jitter into account.

This work has the double aim of: (i) characterizing the PRBS non-idealities that can be present in a circuit implementing a Random Demodulation (RD) AIC, and (ii) evaluating the effects of such non-idealities on the AIC output. This latter goal has been pursued by means of a simulation analysis where the difference between the AIC output achieved using a jittered PRBS and that achieved using an unjittered PRBS, has been evaluated.

The paper is organized as it follows. In Section II, a depiction of the RD AIC is presented. Section III provides an overview on the various jitter components that influence the PRBS. Section IV describes the procedure used to characterize experimentally the PRBS non-idealities. Section V is dedicated to the analysis of the jitter effects in simulation explaining the adopted test strategy and discussing the obtained results.

II. RD ARCHITECTURE

The architecture taken into account in this work is the RD AIC. It is an evolution of the traditional ADC architecture. Specifically, it consists of a mixer connected to an anti-aliasing filter followed by an ADC and a processing unit [12, 13].

The RD AIC operating principle is discussed in the following. First, the input signal $x(t)$ is mixed with a broadband PRBS $p(t)$. Then, the modulated output is low-pass filtered and sampled uniformly by a low sampling frequency ADC, obtaining a compressed vector $\mathbf{y}$. Finally, an estimation of the vector of the samples of the input signal $\hat{\mathbf{x}}$ is obtained through a reconstruction algorithm based on an l_1-norm minimization. The signal can be recovered provided that: (i) the hypotheses of CS are fulfilled, (ii) the system implementing the RD AIC has been correctly modelled, and (iii) the right reconstruction algorithm has been chosen.

The advantages of the AIC operation can be better understood in the frequency domain. Let a compressible signal in the frequency domain have its spectral content exceeding the ADC Nyquist band. If the signal were acquired by a traditional ADC, part of its spectral content would be cut off by the anti-aliasing filter positioned before the ADC, maintaining only the part of information within the Nyquist band. Instead, in the case of an AIC, since the PRBS has a spectrum with a large bandwidth, the mixing causes the spreading of each frequency component of the input signal, such that part of the information about each frequency component of the input signal is moved to low frequencies. Therefore, part of the original information has not been lost after the conversion (see the block diagram in Fig. 1). It is very important to underline that the bit-rate of the PRBS determines the maximum frequency component the AIC is able to acquire.

III. JITTERED PRBS

As previously described, the PRBS is a fundamental signal for the mixing step in a RD AIC architecture.

The first, simplest and most used PRBS model is an ideal sequence of square symbols. In fact, each symbol of the PRBS ideally should last an exactly predetermined time interval and it should be constant during such interval. In practice, both systematic and random components influence the duration of each symbol, causing the jitter phenomenon. The difference Δt between the ideal time interval of the symbol and the acquired (actual) time interval is called Time Interval Error (TIE) and it is directly related to jitter [14].

Jitter consists of a stochastic component, i.e. Random Jitter (RJ), and a systematic component, i.e. Deterministic Jitter (DJ). The preponderance of a component on the other one depends on the specific application.

The TIE probability density function (pdf) for the random component is unbounded, which means that it increases as the number of samples increases. The TIE pdf for the deterministic component is, instead, bounded, which means that it does not increase beyond certain limits, regardless of the number of samples. It necessarily follows that two jitter components come from phenomena to consider in totally different ways. In fact, RJ is measured and analysed time after time without, however, being corrected. On the contrary, there is some cause that

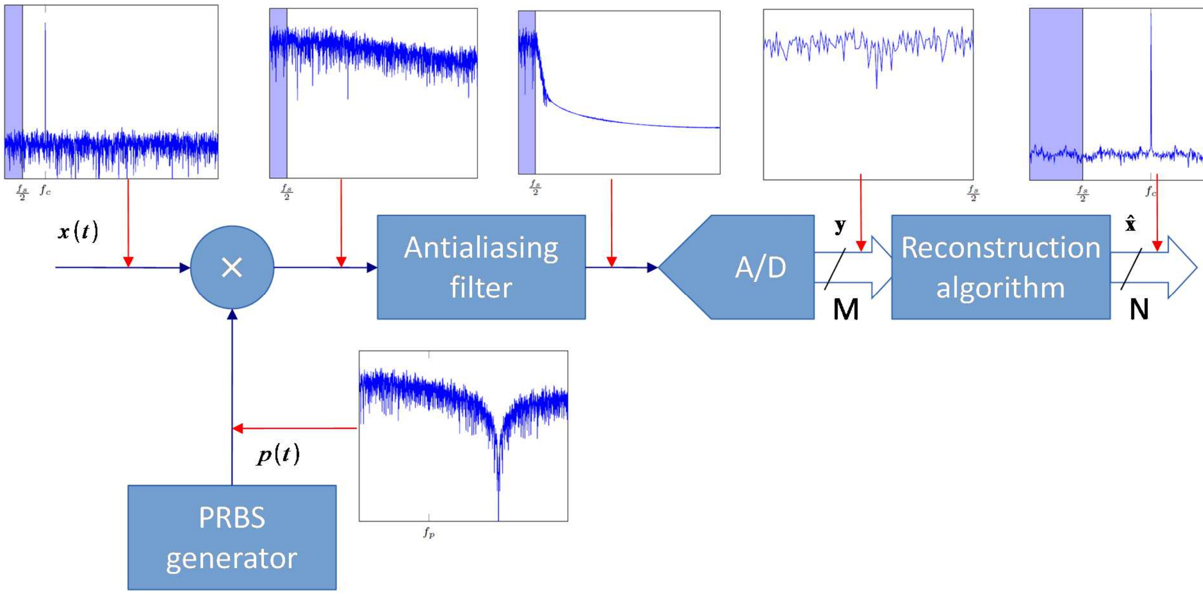

Fig. 1. Block diagram of the RD AIC.

triggers DJ in a systematic way and, as a consequence of this, it is possible to realize a model able to eliminate this jitter component in order to reproduce the input signal more accurately.

In case of RJ, the TIE is modelled by a Gaussian pdf:

$$f_{RJ}(\Delta t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\Delta t^2}{2\sigma^2}} \quad (1)$$

where, σ represents the value of RJ. The main reason for the Gaussian assumption is that TIE is caused by the superimposition of a large number of random noise sources and, according to the central limit theorem, the sum of many random variables converges to a Gaussian distribution, whatever the distributions of the individual sources are. Another reason is that the most significant source of random noise is thermal noise, well known to have a Gaussian distribution.

DJ consists of one or more of the following classes: Periodic Jitter (PJ), Data Dependent Jitter (DDJ) and Bounded Uncorrelated Jitter (BUJ).

PJ is jitter related to TIE repeating periodically with time. In case of PJ, the TIE pdf can be approximated to a pair of Dirac delta functions:

$$f_{PJ}(\Delta t) = \frac{1}{2}\left(\delta(\Delta t - P) + \delta(\Delta t + P)\right) \quad (2)$$

with P the value of PJ.

DDJ is jitter correlated to its data pattern: specifically, it indicates that the current bit transition time is influenced by the previous bit transition time and influences the future bit transition time. In turn, DDJ contains two subcomponents: Duty Cycle Distortion (DCD) and Inter Symbol Interference (ISI).

DCD is the deviation of the value of the duty cycle that is the ratio of the pulse width (either positive -which requires a sequence 010 - or negative -which requires a sequence 101) to the period defined by a clock signal: therefore, shorter or longer pulse widths determine DCD. Also in case of DCD, the TIE pdf is a dual-Dirac delta function:

$$f_{DCD}(\Delta t) = \frac{1}{2}\left(\delta\left(\Delta t - \frac{W}{2}\right) + \delta\left(\Delta t + \frac{W}{2}\right)\right) \quad (3)$$

with W the value of DCD.

ISI represents the component due to switching between 1 and 0: in fact, each transition has a finite time duration and if the current transition happens before the previous transition reaches the designated level a deviation of both time and level occurs on the bit. In case of ISI, the TIE pdf has the generic form:

$$f_{ISI}(\Delta t) = \sum_{k=1}^{K} p_k \delta(\Delta t - D_k) \quad (4)$$

where p_k is the probability for the ISI fraction value D_k. Clearly, the sum of all the probabilities need to satisfy the normalization property: $\sum_{k=1}^{K} p_k = 1$. In this work only the bit prior to the transition has been considered and the cases of current bit equal to previous bit (001 for transition 01, 110 for transition 10) and the cases of current bit different from previous bit (001 for transition 01, 110 for transition 10) have been assumed equiprobable and equidistant. Ultimately, the values in (4) are $p_k = \frac{1}{4}$ and $D_k \in \left\{\mp\frac{I}{2}, \mp\frac{I}{6}\right\}$, with I the value of ISI.

Finally, BUJ is jitter that is not function of the data pattern changes. It is caused by cross-talk between two transmission channels or, more generally, by electromagnetic interference. The TIE pdf is a truncated Gaussian. Anyway, in the present work BUJ has not been considered because the PRBS generator output flows through a single channel.

In conclusion, the TIE related to the total jitter is characterized by a pdf equal to the convolution of all the individual distributions:

$$f_{TJ}(\Delta t) = f_{RJ}(\Delta t) * f_{PJ}(\Delta t) * f_{DCD}(\Delta t) * f_{ISI}(\Delta t). \quad (5)$$

In [9, 10] the PRBS has been described as the sum of an ideal PRBS and a jitter error. In particular, in the latter term, jitter widths come into play as elements of a Gaussian distribution with zero mean and standard deviation equal to the jitter root-mean-square. On the contrary, less attention has been paid to the deterministic component. In [11] DJ has been considered assuming it comprised only of pure DCD. However, if properly characterized, DJ in all its components can be included in the AIC model, thus improving the reconstruction of the original signal.

IV. EXPERIMENTAL CHARACTERIZATION OF PRBS NON IDEALITIES

The PRBS non-idealities of the Tektronix AWG420 Waveform Generator, used in the AIC prototype as a PRBS generator [11,12,13], have been characterized experimentally by means of a LeCroy SDA 6000A Serial Data Analyzer (i) acquiring the PRBS actual trend, and (ii) measuring the values of PRBS jitter components.

The PRBS trend has been acquired according to the following procedure. PRBS codes have been generated with a length of N=512 in MATLAB environment and downloaded to the Tektronix AWG420by means of a GPIB interface. Then, the actual PRBS signal has been generated by the Tektronix AWG420 at a rate of 200 Mbit/s and the related ISI–plot has been evaluated by means of the LeCroy SDA 6000A. The LeCroy SDA 6000A measures all the input sequences of fixed length for a predetermined acquisition window and provides, for each possible sequence, the waveform averaged on all the acquired occurrences of that sequence. In the case of the

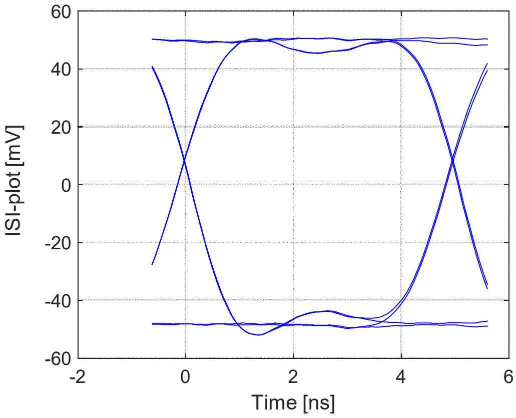

Fig. 2. ISI-plot for the PRBS generated by means of the Tektronix AWG420 signal generator.

current work, the ISI-plot has been measured related to 3-bit long sequences and for an acquisition window equal to 100 µs. This graph is reported in Fig. 2.

Additionally, the characterization of the PRBS jitter generated by the Tektronix AWG420 has been achieved by measuring its deterministic and random components. Fig. 3 shows the histogram of the TIE, where the contribution of jitter components can be appreciated. Specifically, measured jitter values are the following: RJ = 5,4 ps, PJ = 151,2 ps, ISI = 25 ps, DCD = 216 ps.

Measured jitter values suggest that, in this particular case, the DJ influence is greater than the RJ and that the preponderant deterministic components are DCD and PJ. Hence, a simplified model to describe the jittered PRBS in an AIC, as used in [9, 10], is too restrictive and also the influence of the deterministic component needs to be investigated.

V. EVALUATION OF JITTER EFFECTS

A. Test strategy

Once the PRBS signal has been characterized, by evaluating the amount of the different jitter components, a simulation analysis has been realized in order to quantify the effect of each jitter component on the AIC output. In particular, the analysis has been carried out in terms of the error in decibels committed comparing the AIC output obtained with a jittered PRBS $\mathbf{y}_{\mathrm{J}}$ to the AIC output obtained with a PRBS where no jitter has been included $\mathbf{y}_{\mathrm{NJ}}$:

$$Err = 10\log\frac{(\mathrm{y_J}-\mathrm{y_{NJ}})_{rms}}{\mathrm{y_J}_{\boldsymbol{rms}}}. \tag{6}$$

Simulations have been conducted considering in turn all types of jitter influencing the PRBS in the prototype, i.e. RJ, DCD, ISI, PJ.

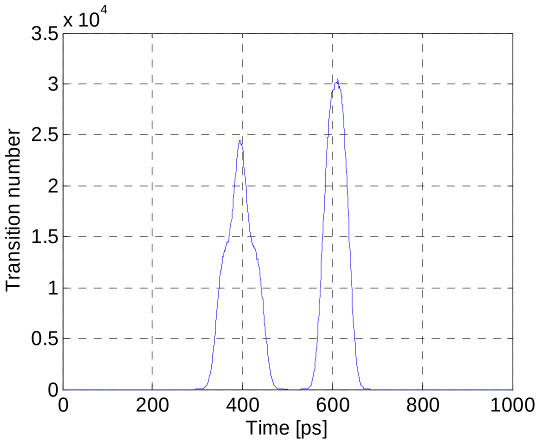

Fig. 3. TIE Histogram.

B. Simulation analysis

In order to evaluate the jitter effects on the AIC output, the RD AIC has been subdivided in blocks modelling the circuital elements of the architecture. The input analog signal *x(t)* and all analog blocks (see Fig. 1) have been digitally represented with a simulation step of 1 ps.

The mixing step has been modelled with a multiplication between the input signal and the signal *p(t)* coming from the PRBS generator. Such signal is a Pulse Amplitude Modulated one, where the modulating sequence is a PRBS with a rate of 200 Mbit/s, generated in MATLAB environment and the waveform has been reproduced according to that acquired as described in Section IV.

The filter has been simulated making use of the actual parameters of the ADRF6510 low-pass filter of the RD architecture. The ADRF6510 is a dual-channel programmable filter and its cut-off frequency (tunable in the range [1-30] MHz) was set at 19 MHz. For this purpose, the waveform generator Tektronix AWG420 was connected to the filter and sine waves were generated at a frequency $f = Jf_p/N$, with $J = 1, 2, \dots N/2$, $f_p =$ 200 MHz and $N = 512$. The output of the filter has been acquired at a sampling frequency of 200 MSamples/s by the AD9230 Evaluation Board, connected to the computer through a USB cable, and the amplitude and phase of the frequency response have been evaluated in MATLAB environment.

The simulation phase has been carried out as it follows. All the sequences of the ISI-plot previously acquired and characterized by a transition have been modified such to give a null jitter. Then, all jitter components influencing the PRBS signal, defined according to (1), (2), (3), (4) respectively, have been considered individually and their effects have been evaluated in turn in terms of the error defined in (6).

The analysis of the DJ components has been conducted moving sequence transitions towards the

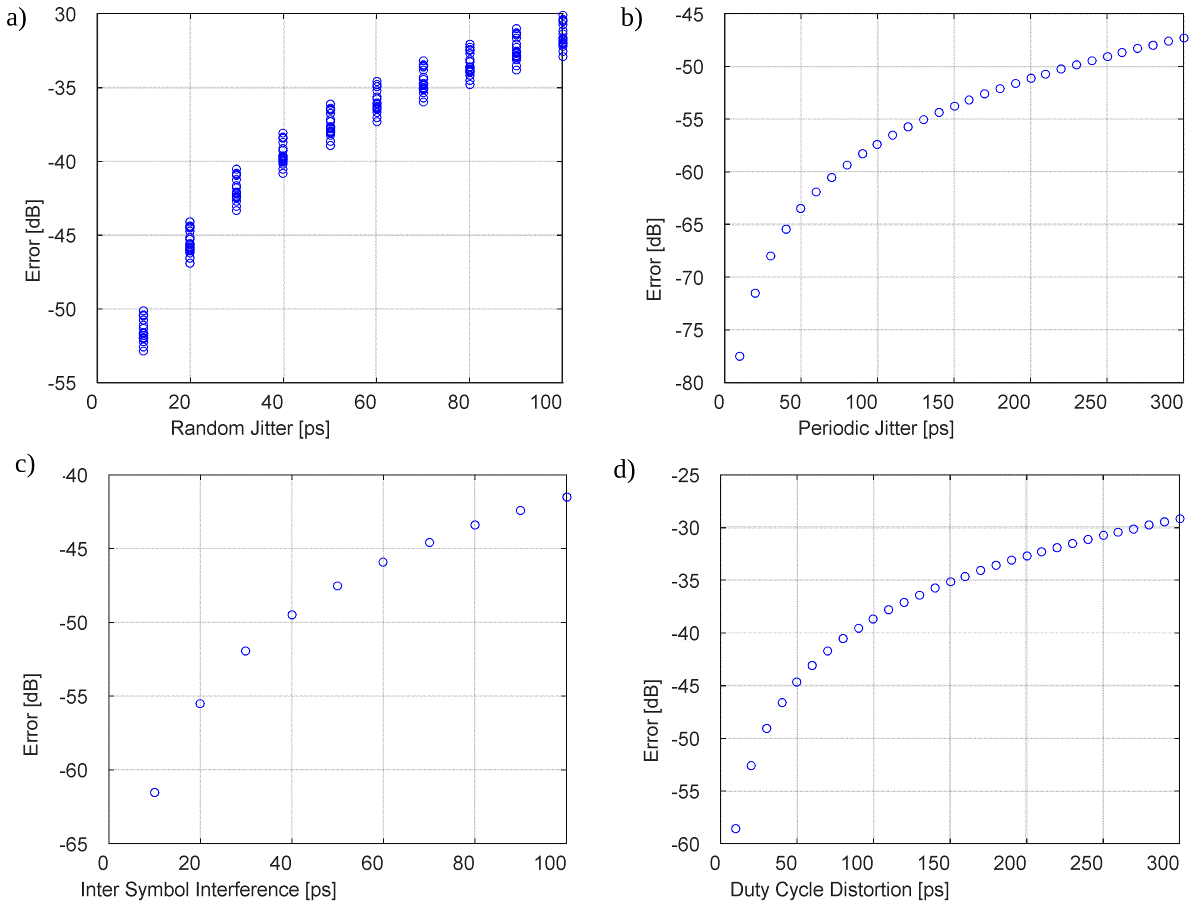

Fig. 4. Simulated error vs. Random Jitter (a), Periodic Jitter (b), Inter Symbol Interference (c), Duty Cycle Distortion (d).

desired value. A similar approach has been followed to generate the RJ: selected values of the standard deviation of the Gaussian distribution have determined TIEs of the PRBS signal and consequently the sequence transitions.

In adherence to the measured experimental data the range used for PJ and DCD is [0, 300] ps, while the range used for ISI and RJ is [0, 100] ps. Anyway, in all cases the values in the range have been taken uniformly with a step of 10 ps.

C. Simulation results

The obtained results concerning the error values have been reported for RJ and different DJ components.

Fig. 4.a shows the trend of the error reporting the results of twenty different random iterations as a function of RJ in the range [0, 100] ps. The error increases progressively both in average and standard deviation.

Other figures represent the trend for DJ. Specifically, Fig. 4.b reports the error as a function of PJ in the range [0, 300] ps, while Fig. 3.c and 3.d report the error as a function of DDJ components in the range [0, 100] ps for ISI and [0, 300] ps for DCD.

Obviously, for all four cases the value observed in absence of jitter is minus infinity and then increases progressively.

It is worth noting that, considering equal values for the jitter components, the most significant incidence is due to RJ and DCD, because of their different error trend. As an example, the error corresponding to the jitter value 100 ps is about -32 dB for RJ, -57 dB for PJ, -42 dB for ISI, -38 dB for DCD.

Anyway, it is very important to take into account that the jitter components have not the same order of magnitude. Specifically, in the considered case, DCD and PJ weight more than ISI and RJ. Therefore, in this case, jitter components affect the AIC output in different way. In fact, considering the experimentally measured jitter the effective committed error is about -57 dB for RJ, -54 dB for PJ, -53 for ISI, -32 for DCD. Consequently, it can be deduced that only error value for DCD is as high as expected. Instead, the RJ weight is quite trivial, while PJ has the same incidence of ISI.

VI. CONCLUSIONS AND FURTHER WORK

In this paper, the PRBS non-idealities in a RD AIC have been characterized by means of a Serial Data Analyzer. Furthermore, the measured PRBS waveform have been used to simulate the effects of its jitter components. Results showed an increase of the error not only due to the random component but also to the deterministic component. This latter, however, could be included in the AIC model and corrected, if adequately characterized.

Future work is devoted to the extension of the AIC model in order to include the effects of PRBS jitter such as all the non-idealities/non-linearities of other blocks of the RD AIC architecture. Such extension is interesting not only for testing purposes, allowing a deeper characterization of the RD AIC, but it could also contribute to an improvement of the performance of the reconstruction stage.

Finally, an experimental investigation on an AIC prototype is planned, in order to verify the coherence of the presented simulation analysis on an actual implementation.

REFERENCES

[1] E.J.Candès, “Compressive Sampling”, Proceedings of the International Congress of Mathematicians, pp. 1433-1452, Madrid, 2006. DOI: 10.4171/022-3/69

[2] S.Kirolos, J.Laska, M.Wakin, M.Duarte, D.Baron, T.Ragheb, Y.Massoud, R.Baraniuk, “Analog-to-Information Conversion via Random Demodulation”, Proceedings of the 2006 IEEE Dallas/CAS Workshop on Design, Applications, Integration and Software, pp. 71-74, Richardson, 2006. DOI: 10.1109/DCAS.2006.321036

[3] J.N.Laska, J.P.Slavinsky, R.G.Baraniuk, "The polyphase random demodulator for wideband compressive sensing", Proceedings of the Forty Fifth Asilomar Conference on Signals, Systems and Computers, pp. 515-519, Pacific Grove, 2011. DOI: 10.1109/ACSSC.2011.6190054

[4] J.Laska, S.Kirolos, Y.Massoud, R.Baraniuk, A.Gilbert, M.Iwen, M.Strauss, “Random Sampling for Analog-to-Information Conversion of Wideband Signals”, Proceedings of the 2006 IEEE Dallas/CAS Workshop on Design, Applications, Integration and Software, pp. 119-122, Richardson, 2006. DOI: 10.1109/DCAS.2006.321048

[5] M.Wakin, S.Becker, E.Nakamura, M.Grant, E.Sovero, D.Ching, J.Yoo, J.Romberg, A.Emami-Neyestanak, E.Candes, “A Nonuniform Sampler for Wideband Spectrally-Sparse Environments”, IEEE Journal on Emerging and Selected Topics in Circuits and Systems, vol.2, No.3, pp. 516-529, 2012. DOI: 10.1109/JETCAS.2012.2214635

[6] M.Mishali, Y.C.Eldar, “From Theory to Practice: Sub-Nyquist Sampling of Sparse Wideband Analog Signals”, IEEE J. Sel. Top. Signal Process., vol.4, No.2, pp. 375–391, Apr. 2010. DOI: 10.1109/JSTSP.2010.2042414

[7] J.P.Slavinsky, J.N.Laska, M.A.Davenport, R. G.Baraniuk, "The compressive multiplexer for multi-channel compressive sensing", Proceedings of the International Conference on Acoustics, Speech and Signal Processing, pp. 3980-3983, Prague, 2011. DOI: 10.1109/ICASSP.2011.5947224

[8] X.Kong, P.Petre, R.Matic, A.C.Gilbert, M.J.Strauss, "An analog-to-information converter for wideband signals using a time encoding machine", Proceedings of the Digital Signal Processing Workshop and IEEE Signal Processing Education Workshop, pp. 414-419, Sedona, 2011. DOI: 10.1109/DSP-SPE.2011.5739250

[9] X.Chen, Z.Yu, S.Hoyos, B.M.Sadler, J.Silva-Martinez, "A Sub-Nyquist Rate Sampling Receiver Exploiting Compressive Sensing", IEEE Transactions on Circuits and Systems I: Regular Papers, vol.58, No.3, pp. 507-520, March 2011. DOI: 10.1109/TCSI.2010.2072430

[10] O.Abari, F.Lim, F.Chen, V.Stojanović, “Why Analog-to-Information Converters Suffer in High-Bandwidth Sparse Signal Applications”, IEEE Transactions on Circuits and Systems I: Regular Papers, vol.60, No.9, pp. 2273-2284, Sept. 2013. DOI: 10.1109/TCSI.2013.2246212

[11] P.Daponte, L.De Vito, G.Iadarola, S.Rapuano, “Effects of PRBS Jitter on Random Demodulation Analog-to-Information Converters”, Proceedings of the 3rd IEEE International Workshop on Metrology for AeroSpace, pp. 643-648, Florence, 2016.

[12] D.Bao, P.Daponte, L.De Vito, S.Rapuano, “Frequency-domain characterization of random demodulation analog-to-information converters”, Acta IMEKO, vol.4, No.1, pp.111–120, 2015. DOI: http://dx.doi.org/10.21014/acta_imeko.v4i1.175

[13] P.Daponte, L.De Vito, G.Iadarola, M.Iovini, S.Rapuano, “Experimental comparison of two mathematical models for Analog-to-Information Converter”, Proceedings of the 19th International Workshop on ADC Modelling and Testing, Budapest, 2016.

[14] M.P.Li, “Jitter, Noise, and Signal Integrity at High-Speed”, Prentice Hall, 2008.