Methods of upgrading the uncertainty of type A evaluation. Elimination the influence of unknown drift and harmonic components

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Abstract- A new approach to upgrading the type A uncertainty evaluation by investigation the trend and periodical systematic components in the regularly in time performed observations is presented in the paper. This is a part of best practice, which authors recommend as upgrading the routine procedures in uncertainty evaluation according to ISO GUM recommendation. The cleaning of raw data set by elimination the systematic components and the influence of these cleanings on standard uncertainty is presented and disused. Two numerical examples are discussed.

Key words: uncertainty type A, unknown systematic components, trend, harmonic disturbances

1. Introduction

The measuring process could be presented in the graphical form as the model shown in Fig. 1. The unknown true value of measured quantity, is at the input of the model, processed in measurement chain and is mathematically treated to obtained parameters which characterised the measurement result. The measured quantity is influenced due to internal and external fluctuation of influencing environment and due to ageing process of elements constituting measurement chain.

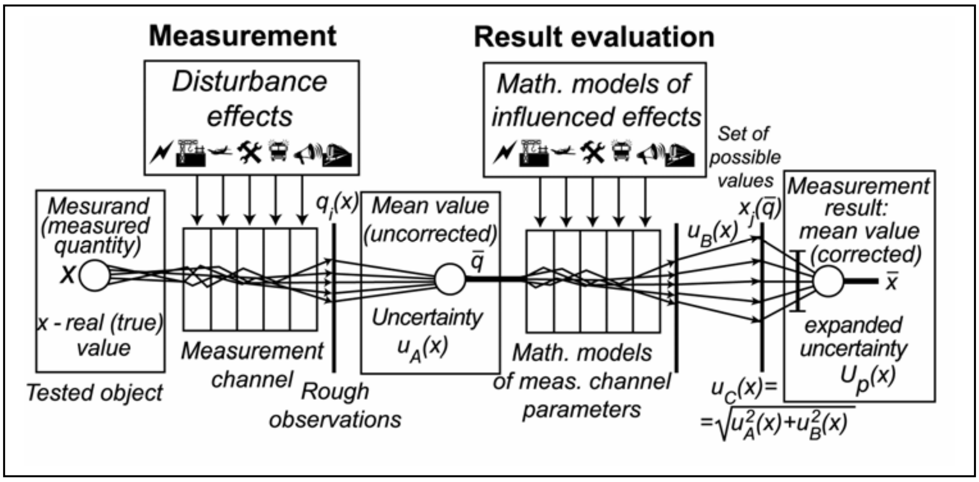


Fig. 1 Process of measurement and the result $\bar{x}$ and uncertainties evaluation of the measurand *X*.

The ISO international guide GUM [1] recommends the evaluation of uncertainty as measure of the measurement accuracy using model of Fig 1. The data processing procedure refers to calculation of the result of measurements as the mean value $\bar{x} = \bar{q}$ of n observation q_i of the measured X and its uncertainty $U_P(x)$ as probability interval (earlier expanded uncertainty) at p level of confidence. To obtain the last one the combined uncertainty $u_c(x)$ is calculated as geometrical sum of two components $u_A(x)$ and $u_B(x)$, i.e.: the type A uncertainty and type B uncertainty. Procedure of the uncertainty $u_A(x)$ evaluation according to ISO GUM [1] is summarised in Table 1. It is calculated by the statistical method. The standard deviation of the mean value of observations $s(\bar{q})$ is recognized as standard uncertainty $u_A(x)$.

Table 1 Evaluation of the $u_A(x)$ by GUM method A

Corrected values of observations: $q_1, q_2, q_3, ..., q_n$ where:- n – number of observations	
Mean value:	$\bar{x} \equiv \bar{q} = \frac{1}{n}\sum_{i=1}^{n} q_i$
Variance of the sample:	$s^2(q_i) = \frac{1}{n-1}\sum_{i=1}^{n}(q_i - \bar{q})^2$
Experimental standard deviation:	$s(q_i) = \sqrt{s^2(q_i)}$
Standard uncertainty of type A:	$u_A(x) = s(\bar{q}) = \frac{s(q_i)}{\sqrt{n}} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(q_i - \bar{q})^2}$

The GUM recommendation for type A uncertainties calculation consists of :

1. Elimination from the sample of observations q_i all known systematic components.
2. Assumption that set of observations are statistically independent (uncorrelated) and all values are equally weighted.
3. Assumption that all values of observations are randomly distributed, and distribution parameters calculated from the sample are equal to the parameters of the population.

The literature, which is dealing with measurement accuracy, [3-10], is now based mainly on GUM approach of uncertainty evaluation [6-10]. The considerations and examples refers mainly to limited numbers of independent observations. Several software procedures were developed as the procedures presented in [9] and also Monte Carlo method, MCM, now included in Supplement to GUM [8].

There are some areas in science, research and technology for which the actual GUM recommendations cannot be directly applied due to its limitations.

GUM have several serious limitations in method A uncertainty evaluation, so its universalism is limited as: such limitations are as follows:

- set of rough observations is not the sample of the pure random population,
- does not refers to other then Normal distribution, for which the other then standard uncertainty u_A is more adequate,
- do not take into consideration the order and relations between trails. It means that recomadation GUM do not refers to calculation of uncertainties in the cases in which the parameters of object under test are time variable, are influencing be external ambient conditions, as such situations require the stochastic stationary and non stationary models,
- do not refers to evaluation of uncertainties of dynamic parameters and Digital Signal Processing for different algorithms.

Even after removal all a priori known systematic components from raw data observations, their corrected values still may be not the sample of the pure random and normal population. Many of regular (systematic) components in observations are still unknown, but they should be also eliminated if it may be possible. It could be named here as "cleaning of the rough observations" or of input signals. If an additional information is known, e.g. procedure: how observations of the measured constant quantity X are collected, i.e. regularly sampled as series in time or space or by other known way, then some of undesirable components as trend or harmonics in relation to the length of the sample, could be eliminated. Only partly it can be done by the input filtration, more - by special digital algorithms after their identification. We show two simply methods on some specially simulated numerical examples of regularly sampling observations.

Furthermore even enough properly "cleaned" observations are not always statistically independent. They could be autocorrelated, especially if observations have been taken with high density. Also the best distribution for the real corrected observations could be different that of the normal one. The mean value of the sample of such observations is not always the most likelihood parameter of their distribution and other unbiased estimators should be used, as the midrange of rectangular distributions and MAD (median) of Laplace double-exponential ones. Last two problems are discussed concisely in the next - part 2 of this paper [12] including proposals of solving them. Methods of upgrading estimations of the uncertainty $u_B(x)$ and expanded uncertainty $U_P(x)$ are presented in [11].

II. Raw measurement data cleaning

The data presented in the Example I (type A uncertainties) and Example II (type B uncertainties) were processed at the really data handling by investigation linear and periodical components from collected data. The Least Square Method was applied.

Example 1

The 121 measurement trials of uniformly sampled unknown value were collected with a certain time period. The te recorded values are presented in Tab.1.

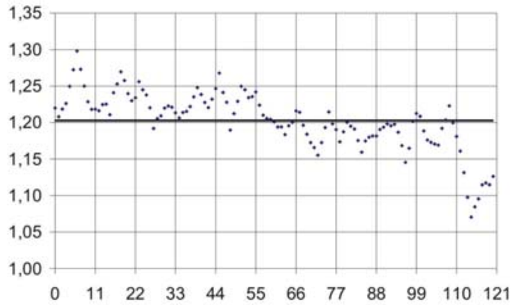
Table 2. Raw data

1.2200	1.2080	1.2186	1.2263	1.2497	1.2725	1.2981	1.2731	1.2500	1.2286	1.2181
1.2183	1.2162	1.2247	1.2253	.2108	1.2409	1.2529	1.2696	1.2577	1.2397	1.2300
1.2341	1.2562	1.2449	1.2378	1.2203	1.1920	1.2056	1.2092	1.2198	1.2227	1.2210
1.2134	1.2064	1.2138	1.2154	1.2220	1.2352	1.2479	1.2385	1.2277	1.2206	1.2320
1.2466	1.2679	1.2412	1.2279	1.1897	1.2123	1.2291	1.2498	1.2450	1.2343	1.2356
1.2420	1.2239	1.2101	1.2057	1.2044	1.2011	1.1940	1.1941	1.1836	1.1956	1.2002
1.2159	1.2142	1.1963	1.1840	1.1726	1.1657	1.1553	1.1726	1.1932	1.2146	1.1983
1.1904	1.1736	1.1874	1.2003	1.1950	1.1911	1.1754	1.1594	1.1748	1.1799	1.1817
1.1816	1.1907	1.1937	1.1982	1.1956	1.1977	1.1868	1.1684	1.1455	1.1648	1.2019
1.2126	1.2086	1.1885	1.1760	1.1729	1.1706	1.1692	1.1921	1.2036	1.2229	1.1996
1.1810	1.1609	1.1314	1.0975	1.0704	1.0845	1.0954	1.1146	1.1172	1.1148	1.1263

Solution:

The raw data in order of grabbing from the object and appearance in collected data set are presented in graphical form in Fig. 2a and its histogram in Fig. 2b.

The Fig. 3a presents the same raw data and trend line, the Fig. 3b presents the histogram after removing from collected data the trend line.



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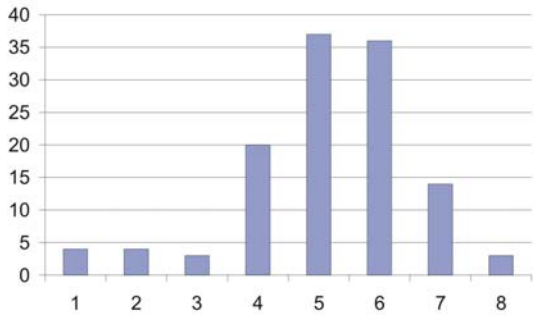

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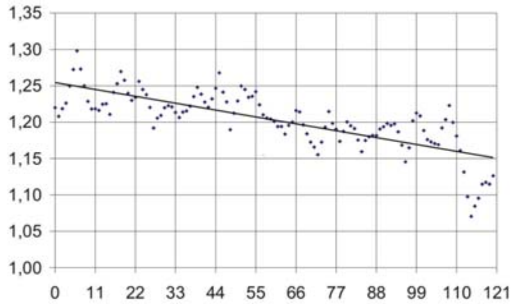
Fig. 2 Set of collected data, a) the raw data in order of registration and the mean value b) histogram of the raw data



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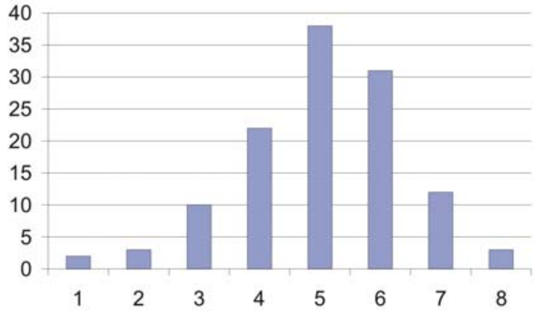


b)

Fig. 3. Set of collected data, a) the raw data in order of registration and the line of the trend b) histogram of the raw data after elimination of the trend

Fig. 4 presents the dispersion of data around mean value of colleced dara after removing trend and optimal periodical component discovered in data set. The Fig. 4b presents the histogram of data set as in Fig. 4a.

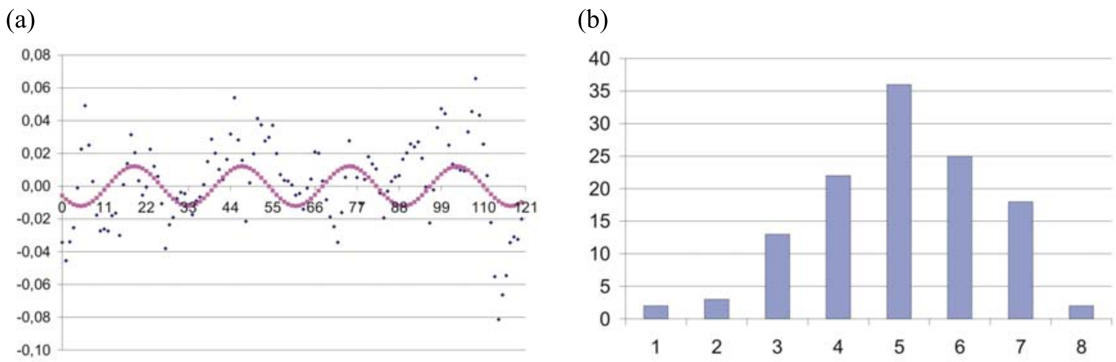


Fig. 4. Dispersion of collected data a) the raw data in order of registration and the line of the trend b) histogram of the raw data after elimination of the trend

The calculated parameters, which characterise the raw set of the data and after elimination systematic components are collected in Tab. 3.

Table 3 The parameters of colleted measurement data sample of correlated observations

	Raw measurement data	**Cleaned measurement data** (trend removed)
Criterion χ^2 for normal distribution	$\chi^2 = 34{,}36 > \chi^2_{5,\,0{,}05} = 11{,}1$ negative result	$\chi^2 = 4{,}888 < \chi^2_{5,\,0{,}05} = 11{,}1$ positive result
Mean value	$\bar{v} = \frac{1}{121}\sum_{i=1}^{121} v_i \approx 1{,}2027$	$\bar{V} = \bar{q} = \frac{1}{121}\sum_{i=1}^{121} q_i \approx 1{,}2027$
Standard deviation (no correlation analysed)	$s(v_i) = \sqrt{\frac{1}{121-1}\sum_{i=1}^{121}\left(v_i - \bar{v}\right)^2} = 0{,}0395$	$s(q_i) = \sqrt{\frac{1}{121-1}\sum_{i=1}^{121}\left(q_i - \bar{q}\right)^2} = 0{,}0264$
Ratio of standard deviations	$s(v_i)/s(q_1) = 0{,}0395/0{,}0257 = 1{,}54$	

Conclusion: The standard deviation was lowered up to 35 % after elimination of linear trend from raw measurement data.
The further investigations of periodical component were worthless. The spectrum of harmonics is presented in Fig. 5.

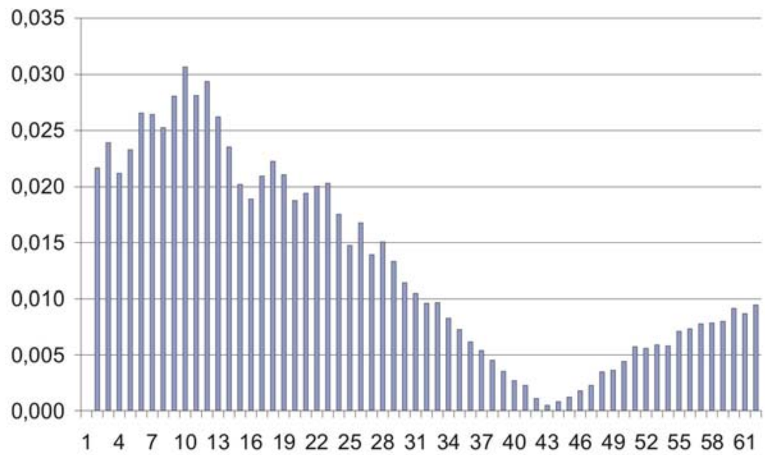


Fig. 5 Spectrum of harmonics of desertion of measurement data given in Example 1. Relative value of amplitude of each harmonic component to the RMS value of all components vs. order of harmonic.

Example 2

The set of measurement results contaminated by known periodical signal are as follows:

Table 4. Raw data of the example 2

6,822	2,699	5,044	1,816	-0,161	-0,546	1,935	4,881	2,419	-0,939	-0,298	6,267
5,456	2,478	1,651	-1,473	-1,642	-0,459	-0,121	5,459	2,909	5,266	-1,158	-0,123
-0,041	6,291	1,052	-0,016	0,094	8,007	6,843	6,622	7,135	0,491	2,224	6,554
4,375	7,798	4,114	7,142	6,845	4,101	7,056	7,158	6,413	8,447	4,648	7,645

8,899	11,732	7,704	10,874	7,318	4,332	7,215	7,382	12,499	9,169	12,006	11,745
11,177	7,582	13,859	8,480	7,433	9,193	10,012	7,291	8,923	5,934	13,750	8,464
9,873	9,430	9,783	8,307	5,442	7,183	10,296	10,020	11,525	10,785	12,501	5,314
12,002	5,604	4,885	4,755	6,672	9,822	4,861	4,237	3,979	8,964	6,566	8,831
1,829	8,438	3,358	1,417	8,454	4,869	7,054	3,330	8,075	6,166	3,312	1,908
1,258	5,028	7,681	4,551	7,377	1,164	5,820	3,133	3,222	2,954	3,066	4,290
5,562	8,535	1,452	4,571	3,260	9,625	8,141	5,622	7,375	9,423	5,955	8,816
11,407	11,274	7,272	6,626	8,935	4,091	9,406	5,393	7,987	5,159	7,365	10,211

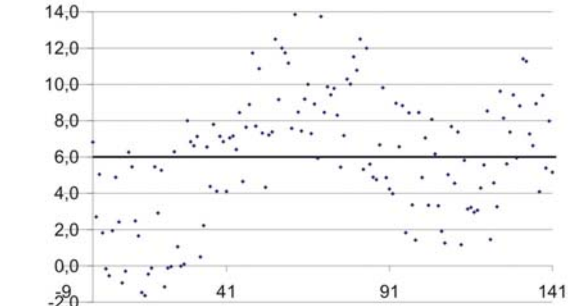
The raw data are presented in graphical form in Fig. 6.

Solution:

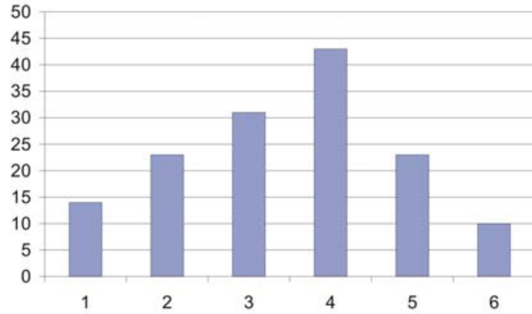
The raw data in order of appearance in time in collected data set are presented in graphical form in Fig. 6a and its histogram in Fig. 6b.

The Fig. 7a presents the same raw data and trend line, the Fig. 7b presents the histogram after removing from collected dat the trend line.

The Fig. 8a presents the dispersion of data around mean value of collected data after removing the trend and the periodical component discovered in data set. The Fig 8b presents the histogram of data set of Fig. 8a.

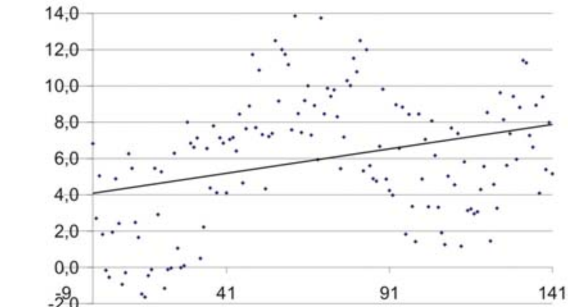


(a)



(b)

Fig. 6 Set of collected data, a) raw data in order of registration and the mean value b) histogram of raw data.



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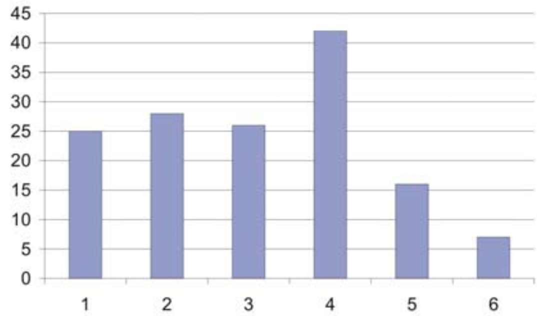
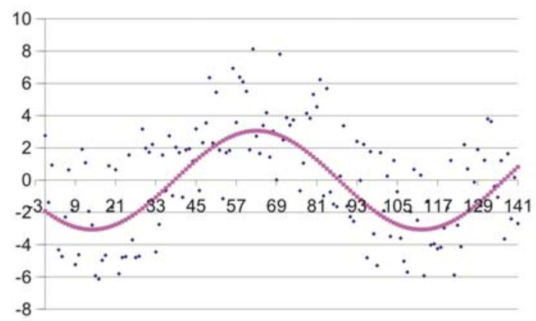
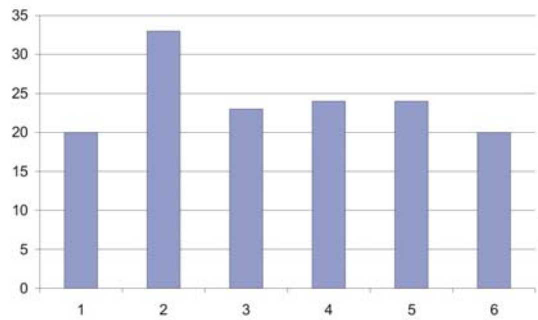


(b)

Fig.7 Set of collected data, a) the raw data in order of registration and the line of the trend b) histogram of the raw data after elimination of the trend

(a)





b)

Fig 8 Set of dispersion of data a) data after removing the mean value and the trend b) histogram of dispersions presented in 4a, after elimination of the periodical component from the data

III. Final conclusion

For evaluation of measurement uncertainties by the type A method there is proposed the upgrading of the procedure recommended by ISO GUM guide. It is "cleaning" of the raw regularly sampled data measurements based on identification and removing from them the trends of linear and periodical characters.

The rule after data clearing is such that the standard uncertainty lowers. It was proved by many elaborated examples of which two are quoted in this paper.

The further proposals of modification of the procedure of the uncertainty type A calculations are presented in part 2 [12], while improvement procedures of the uncertainty type B and the overall uncertainty UC is in [11].

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