Dependence of mutual inductance of a precise Rogowski coil on the primary conductor position

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Abstract – A Rogowski coil is well-known current-to-voltage transducer, and in order to use it for the high-accuracy measurement of AC current (at power supply frequency) all influencing quantities and their contribution should be recognized and analyzed. Therefore, in this paper the analysis of the partial influence due to the position of the primary conductor relatively to the secondary coil, with the nonhomogeneous density of secondary turns, is analyzed. The measured deviation of the mutual inductance in this case showed very good agreement with the theoretical prediction, which is promising toward the expected application of such sensor.

I. Introduction

The influencing quantities, which affect the current-to-voltage conversion by means of Rogowski coil (or sensor, Fig. 1, [1]), can be recognized as all imperfections that contribute to the deviations from the expected results. These contributions can be identified as mechanical, physical and electrical influences. The most obvious are mechanical (it is better to say geometrical) influences due to the imperfect preparation of the coils, as well as an isolating body on which the secondary coil has been wounded [2, 3]. The second, physical, contribution comes from the deviations from the presumptions on which the fundamental principle of that sensor is based: for instance, unhomogeneity of the magnetic flux generated by the primary conductor, the small (but finite) width of the wire used for secondary coil, etc. Further contributions come from the temperature influences and imperfections of the electronic devices used together with the sensor (for instance, electronic integrator).
The basic idea is to use a Rogowski coil for the high-accuracy measurement of AC current, primary for the frequency of power supply network (50 Hz or 60 Hz), and the first step in this direction is to recognize and analyze the influencing quantities, their contribution and importance, and to minimize these effects in the realization of a real sensor.
Therefore, in this paper it is analysed the influence of the position of primary conductor (i.e. shift of its axe) relatively to the secondary coil (i.e. its axial axe) for a real sensor with some discontinuity in the secondary windings. Theoretical consideration has been done with the presumption that the secondary coil with N turns is wounded on the toroidal body, made of nonmagnetic material with the rectangular cross-section. This theoretical approach for a real model is tested by the comparison with the realized model, and the results showed the deviations from the ideal model. This is important for the determination of the optimal mechanical construction of a sensor.

II. Mutual inductance of the geometrically non-ideal system

In the ideal situation, a primary conductor is set exactly on the longitudinal axe of the ideal toroidal Rogowski coil, with homogeneous density of secondary turns wounded by a wire with negligible cross-section (Fig. 1). The inner and outer radius of the coil are r_U and r_V, respectively, and the height of the secondary coil with rectangular cross-section is h. Induced electromagnetic force $e(t)$ in secondary turns depends on the mutual inductance M, and the derivate of the primary current $i(t)$:

$$e(t) = -M \cdot \frac{\mathrm{d}i(t)}{\mathrm{d}t} \qquad (1)$$

The magnetic flux in the coil is not homogeneous because it hyperbolically decrease [4, 5] with the distance from the axe of the primary conductor, as well as the length of the lines of magnetic flux is larger on the outside radius (r_V) than on the inner radius (r_U). Taking these into account, without any other influencing quantities, which in other words represents an *ideal model*, the mutual inductance of such system with N secondary turns is:

$$M = \frac{\Phi}{I} = \mu_0 \cdot \frac{N \cdot h}{2\pi} \cdot \ln \frac{r_V}{r_U} \quad (2)$$

Due to the imperfection of the realization of the first and last turn, even if all other turns are perfectly wounded, some discontinuity arise, which can be described as a sector of coil where there is no turn – this is marked with angle δ in Fig. 1. Detailed analysis of the influence of correlation between the discontinuity δ and the eccentricity of primary conductor (i.e. deviation from the central position), and associated influence on the calculation of mutual inductance, is presented in [6]. Furthermore, due to this discontinuity δ, the total flux in secondary coil is changed if the axe of the primary conductor does not conform to the axial axe of the coil, but exhibit an angle λ as it is shown in Fig. 1. However, in this situation the primary conductor is still in the centre of the secondary coil (point S), and there is no eccentricity (deviation) from the centre of toroidal coil, which is certainly easier case to analysed.

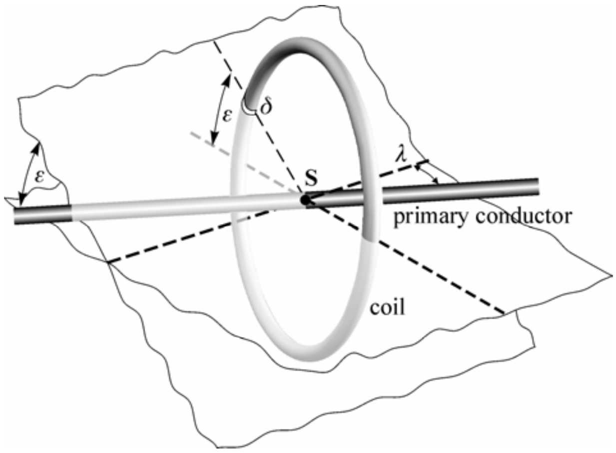

Fig. 1. Geometrical system of primary conductor and Rogowski coil

Since we want to examine the dependence of the mutual inductance (or enclosed flux) not only due to the angle λ but also on its position according to the discontinuity δ, we will fix the primary conductor in the plane of angle $\varepsilon = 0$ and the position of discontinuity δ toward the direction of this plane is defined by auxiliary angle ε.

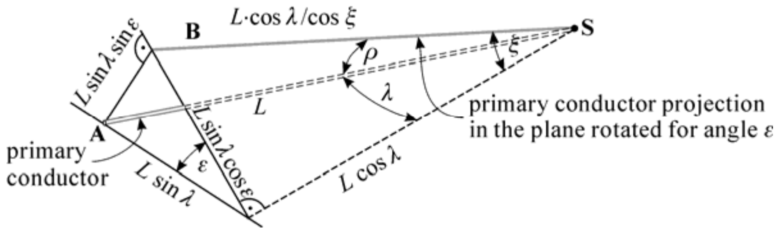

Fig. 2. Definition of the angles λ, ρ and ε

From the described situation, two relations can be written:

$$\xi = \mathrm{arctg}(\mathrm{tg}\lambda \cdot \cos\varepsilon) \quad (3)$$

$$\cos\rho = \sqrt{1 - (\sin\lambda \cdot \sin\varepsilon)^2} \quad (4)$$

Now we need to observe the projection of the primary conductor in the plane that is rotated for angle ε. This projection, according to Fig. 2 constitutes the angle ξ to the axial axe of the coil, which is presented in Fig. 3.

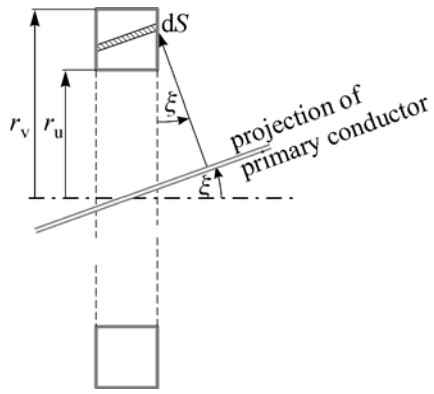

Fig. 3 Projection of primary conductor on the plane rotated for angle ε

Here we need to point out that, in the presented two-dimensional projection, the angle ρ, which is angle between the primary conductor and its projection in the presented plane, is not visible. However, in the calculation of the total flux we need to take into account the cosine of that angle, determined by (4). Furthermore, here is also obvious that the calculation of the magnetic flux for each turn (theoretically infinite thin) in the plane presented in Fig. 4 can be divided on three parts, i.e. three influencing areas: the first one (I), which is the closest to the primary conductor and forms triangle, the second one (II) that has a form of parallelogram, and the third one (III), which is triangle the most distanced from the primary conductor. These areas are marked on Fig. 4a and 4b, as well as the lower radial limit for the first area and the upper radial limit for the third area. In that manner the differential areas, necessary for the calculation of the total flux, can be taken into account.

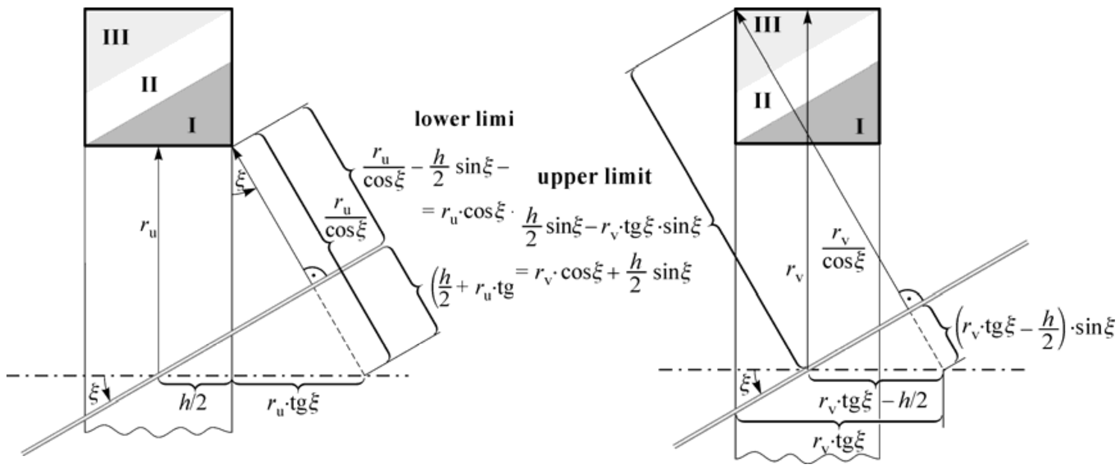

Fig. 4a. Lower limit for first area (I)

Fig. 4b. Upper limit for third area (III)

From further analysis of the geometrical relations follows the upper radial limit for the first area (I) and the lower radial limit for the third (III) area:

$$r_{\mathrm{gI}} = \frac{h}{2}\sin\xi + r_{\mathrm{U}}\cos\xi \tag{5}$$

$$r_{\mathrm{dIII}} = r_{\mathrm{V}}\cos\xi - \frac{h}{2}\sin\xi \tag{6}$$

The differential of angle ε the part of one turn is $(N\cdot \mathrm{d}\varepsilon)/(2\pi - \delta)$, and the inductance B on distance r from the primary conductor is equal to $\mu_0 \cdot I/(2\pi r)$. Thus, after calculations of the differential of fluxes in quoted areas and introducing the relations (3) to (6), the final expression for mutual conductance for a *real model* is

$$M_{\delta,\lambda} = \frac{\mu_0 \cdot N}{2\pi \cdot (2\pi - \delta)} \cdot (\mathrm{I}_1 + \mathrm{I}_2 + \mathrm{I}_3) \tag{7}$$

where

$$\mathrm{I}_1 = \int\limits_{\left(r_{\mathrm{U}}\cos\xi-\frac{h}{2}\sin\xi\right)}^{\left(r_{\mathrm{U}}\cos\xi+\frac{h}{2}\sin\xi\right)} \int\limits_{\left(\varepsilon+\frac{\delta}{2}\right)}^{\left(2\pi+\varepsilon-\frac{\delta}{2}\right)} \left[\frac{h}{2\cos\xi}-\frac{r_{\mathrm{U}}}{\sin\xi}+r\left(\mathrm{ctg}\xi+\mathrm{tg}\xi\right)\right]\cdot\frac{\sqrt{1-(\sin\lambda\cdot\sin\varepsilon)^2}}{r}\cdot \mathrm{d}r\mathrm{d}\varepsilon \tag{8}$$

$$\mathrm{I}_2 = \int\limits_{\left(r_{\mathrm{U}}\cos\xi+\frac{h}{2}\sin\xi\right)}^{\left(r_{\mathrm{V}}\cos\xi-\frac{h}{2}\sin\xi\right)} \int\limits_{\left(\varepsilon+\frac{\delta}{2}\right)}^{\left(2\pi+\varepsilon-\frac{\delta}{2}\right)} \frac{h}{r\cdot\cos\xi}\cdot\sqrt{1-\left(\sin\lambda\cdot\sin\varepsilon\right)^2}\cdot \mathrm{d}r\mathrm{d}\varepsilon \tag{9}$$

$$\mathrm{I}_3 = \int\limits_{\left(r_{\mathrm{V}}\cos\xi-\frac{h}{2}\sin\xi\right)}^{\left(r_{\mathrm{V}}\cos\xi+\frac{h}{2}\sin\xi\right)} \int\limits_{\left(\varepsilon+\frac{\delta}{2}\right)}^{\left(2\pi+\varepsilon-\frac{\delta}{2}\right)} \left[\frac{h}{2\cos\xi}+\frac{r_{\mathrm{V}}}{\sin\xi}-r\left(\mathrm{ctg}\xi+\mathrm{tg}\xi\right)\right]\cdot\frac{\sqrt{1-(\sin\lambda\cdot\sin\varepsilon)^2}}{r}\mathrm{d}r\mathrm{d}\varepsilon \tag{10}$$

To calculate the dependence of M on the angle λ, it is necessary to define a value of ξ according to (3), as well as the angles λ and δ of interest, and calculate the previous equations taking into account the change of auxiliary angle ε in the interval $[0, 2\pi]$. Since it is easier to measure the higher deviations of mutual inductance, for the test of a real model the discontinuity $\delta = \pi/6$ has been chosen, which is much greater value than can be expected in a normal set-up of a sensor. Therefore, on Fig. 5 are presented the results of such analysis, where $\Delta M/M$ is relative deviation of mutual inductance for a real model, defined by (7), to an ideal model, defined by (2), and calculated for values of angle $\lambda_1 = \pi/12$ and $\lambda_2 = \pi/6$. It is obvious the $\Delta M/M$ is higher when angle λ has greater value, as well as pretty good agreement between the theoretical predictions and measured values; these results are calculated for angle ε that starts from 0 (when the discontinuity is in the plane of projection) through the whole circle of 2π. As mentioned before, obtained $\Delta M/M$ is calculated for discontinuity δ which is much greater than in expected and normal set-up of a sensor, which means that in a real situation much lower deviations can be expected.

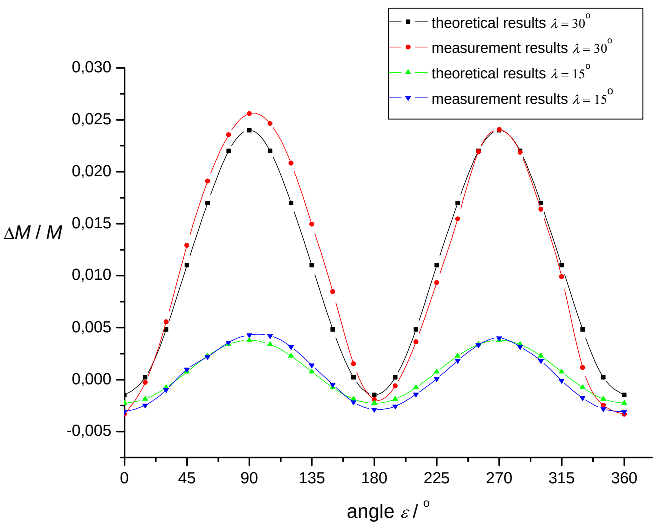

Fig. 5. Measured and calculated difference of the mutual inductance between the real model, defined by (7) and ideal case defined by (2), taking into account $\delta = 30°$ for two values of angle λ, 15° and 30°; the ratio $r_{\mathrm{V}} / r_{\mathrm{U}} = 2{,}5$

III. Conclusion

Due to its linearity and "indirect" method of current measurement, a Rogowski coil is promising choice for precise measurement of ac currents, not only at power frequencies, but also for much higher frequencies. Analysis presented in this paper shows that the positioning of the primary conductor relatively to the secondary coil could have important influence of the accuracy of this sensor, which leads to the proper set-up of in a real case.

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