

# The PUMA method applied to the measures carried out by using a PC-based measurement instrument

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**Abstract-** The paper deals with the uncertainty management in the measurements performed by using a generic PC-based measurement instrument. To accomplish the task, the uncertainty estimation is performed by using the PUMA (Procedure for Uncertainty Management) method, which is an iterative technique completely based on the “Guide to the Expression of Uncertainty in Measurement” rules, but which offers a more engineering methodology. With this approach, it is possible to optimize the cost of the measurements versus the uncertainty target, avoiding the use of inadequate or, on the contrary, too expensive instrumentation. According to the PUMA method, in the paper different approaches to evaluate the uncertainties are presented, starting from the more precise methodologies and keeping on with the coarser, but quicker and cheaper ones.

## I. Introduction

The measurement instruments based on analog-to-digital conversion of acquired signals and their successive processing are widely used in each sector of the measurement field. Besides the traditional stand-alone instruments, these days the measurement instruments based on a data acquisition board connected to a personal computer, are more and more frequently utilized. The main reason of the spread of these systems, henceforth called PC based measurement instruments (PCBMIs), is that they are quite less expensive and more flexible if compared with the traditional stand-alone instrumentation. The working principle of a PCBMI is very easy to describe: the physical quantity is transduced in electric signal which is conditioned to be adapted to the successive circuits; the signal is sampled at a frequency at least twice his bandwidth and converted in numerical codes; the acquired samples are processed by the suitable measurement digital signal processing block, usually developed by the user, to get the measurement results which are displayed in a virtual panel of the PC monitor.

However, given that the PCBMIs are usually designed, assembled and programmed by the users themselves, the difficulties in a correct evaluation of the uncertainties have limited their spread on the industrial environment and in the test and calibration laboratories. In fact, for a correct employment in a quality management system, it is essential to characterize all the employed measurement instruments and to estimate the measurement quality [1,2]. According to the ISO – “Guide to the Expression of Uncertainty in Measurement” (GUM) [3], the index which quantifies this quality is the standard uncertainty associated with the measurement results.

We already handled the topic and in order to characterise a generic PCBMI we proposed a numerical method which, by means of an ad hoc developed software tool, estimates the uncertainties using the Monte Carlo approach [4].

In this paper, starting from the acquired experience, we deal with the uncertainty management, which is the discipline of optimising the uncertainty target and the cost of the measurements. Without a systematic approach to the uncertainty evaluation, the performed measurements could be either not adequate for the target uncertainty or too expensive.

To achieve the objective, we perform the uncertainty estimation by means of the PUMA (Procedure for Uncertainty Management) approach. This iterative method is completely based on the GUM rules but provides a more engineering methodology. The PUMA method is described in the ISO/TS 14253-2 standard [5] that was developed by the Technical Committee ISO/TC 213. In spite this standard deals with the Geometrical Product Specifications (GPS), and therefore with the geometrical and mechanical measurements, its extension to electric measurements is natural and could result useful.

Although the PUMA approach is established upon on the GUM rules, the basic inspiration is completely different: the GUM prescribes that, in the uncertainty evaluation, it is necessary to avoid overestimations, since an overestimation debases the measurement quality; however, when it is necessary to know if an already available instrumentation and an already defined measurement environment are appropriate for a stated target uncertainty, the uncertainty overestimate can be tolerated, especially when an accurate uncertainty estimation involves high expenses.

## II. The PUMA method

The PUMA method (graphically presented in fig.1) is based on a sequence of uncertainty over-estimations, starting from the coarser but quicker ones and carrying on with more accurate but more expensive and more time-consuming techniques. By applying only the first iteration of the method, it is very probable to prove that the uncertainty of the considered measurement process is minor than the target uncertainty and, consequently, that the measurement process is adequate. Just in few practical cases, there is the need of using sophisticated mathematics and statistics.

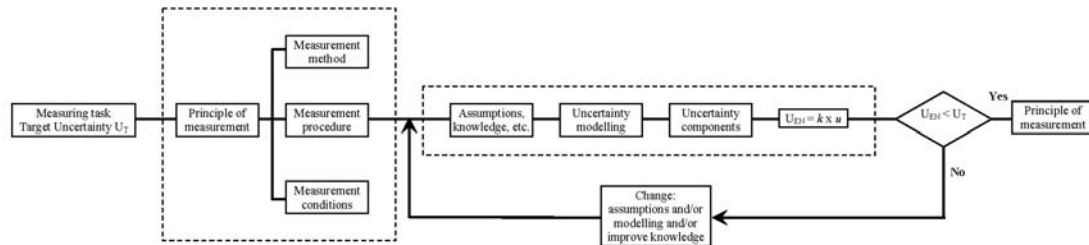


Fig.1 . Graphical representation of the PUMA method

The uncertainty management is carried out starting from the description of the measurement purpose, the measurement instrumentation, the measurement conditions and from the definition of the uncertainty target value  $U_T$ . On this basis, from the first iteration of the PUMA approach, a first estimate  $U_{E1}$  of the measurement uncertainty is obtained. If  $U_{E1} < U_T$ , than the chosen measurement procedure is adequate to the measurement purpose. If  $U_{E1} \ll U_T$ , the chosen measurement procedure is obviously still technically adequate, but there is the chance to change the measurement instrumentation and/or conditions to make the measurement less expensive. If  $U_{E1} > U_T$ , it is necessary to perform another iteration obtaining a more accurate and lower estimate  $U_{E2}$  of the uncertainty. By comparing this new value with the uncertainty target, it is possible to verify that the measurement procedure is adequate or that it is necessary to perform another iteration. After having used all the chances to get the more accurate uncertainty estimation, if  $U_{EN} < U_T$ , than the chosen measurement procedure is really inadequate to the measurement.

The aim of the paper is the extension of the PUMA method to the PC-based measurements, defining various less and less coarse methods; obviously it is necessary starting from a rigorous and precise evaluation methodology which can be used as reference. For this purpose we use the already mentioned numerical approach which was validated in [4] and which is described in the next chapter.

## III. Numerical approach for the uncertainty assessment of PC based measurement

For the characterisation of the whole measurement chain of a PCBMI, according to [3], the first steps to perform are: the identification of the error sources which give a contribution to the uncertainty of the measurement result during the transduction of the quantities, the conditioning of the signals, the A/D conversion and the digital signal processing; the quantification of the standard uncertainties associated with each error source.

Without any lose of generality we do not consider the errors generated by transducers and conditioning accessories. Even if these errors are often predominant compared to the errors generated in the A/D conversion, the transducers and conditioning accessories variety is so wide, that it is necessary to analyse separately each particular situation. On the contrary, it is possible to carry out a general treatment in the case of the A/D conversion process. In any case the proposed method can be extended to each particular transducer and/or conditioning accessory, by identifying all the error sources, evaluating the associated standard uncertainties and analyzing how these uncertainties influence the uncertainty of each acquired sample. We do not consider either the errors generated during the digital signal processing since these errors are usually negligible if the software block of the PCBMI is correctly designed.

So in the following we consider only the main error sources generated during the A/D conversion, which are: offset, gain, integral non-linearity (INL), noise, cross-talk, settling time, timing jitter, quantization and differential non-linearity (DNL) [6].

The following step to perform is the quantification of the uncertainties associated with these error sources. It can be carried out by means of statistical methods with a Type A evaluation according to the GUM, (but in order to estimate the uncertainties associated with all the sources it is necessary to test a

statistically sufficient number of instruments of the same kind), or it is also possible to turn to manufacturers' specifications (Type B evaluation). Of course the second way is less expensive and less time consuming, since it does not require any kind of test from the user. However evaluating the standard uncertainties starting from the manufacturers' specifications is not a very effortless task, since each manufacturer furnishes the specifications in an arbitrary way, sometimes inventing some new parameter. In any case it is necessary to formulate some arbitrary hypothesis on the kind of the distributions.

For the offset, gain, temperature drift and long-term stability errors, the manufacturers declare an interval  $\pm a$  where the error surely lies. According to the GUM, provided that there is no contradictory information, each input quantity deviation is to be considered equally probable to lie anywhere within the interval given by specification, that is modeled by a rectangular probability distribution. As for the non-linearity errors, the worst-case values of INL and DNL are usually reported in the specifications. The standard uncertainty related to noise can be directly obtained from the technical specifications, since it is usually expressed as rms value. The cross-talk errors are produced by the interference in the multi-channel acquisition. Its related uncertainty is expressed as minimum ratio between the signal rms value and the interference signal rms value. The settling time is the amount of time required for a signal that is amplified to reach a stated accuracy and stay within the specified range of accuracy. The manufacturer declares this range for the maximum sampling rate and for the full-scale step, but the errors on the measured signal depend on the actual sampling rate and on the actual step. Impact of timing jitter uncertainty is transformed on amplitude uncertainty as a function of signal derivatives. The manufacturer declares the aperture jitter value, typically expressed as rms value.

After the identification of the uncertainty sources and the evaluation of the associated standard uncertainties, to assess the combined standard uncertainty of the measurement results, other two steps have to be carried out: composition of these standard uncertainties to obtain the combined standard uncertainty of each acquired sample; study of how the uncertainties of each acquired sample combine and propagate during the digital signal processing. To perform simultaneously these tasks we proposed a numerical approach based on the Monte Carlo method, developing a software tool which simulates a true A/D conversion and takes into account all the uncertainty sources. The tool is placed between a measurement input signal simulator and the software block of the PCBMI and simulates a set of measurements carried out by different realizations of the same instrument. In the following, its working principle is described.

The input signal simulator generates  $N$  samples as if they were obtained from an ideal sampling process of the signal and  $N$  samples are sent to the A/D simulation tool. The core of the tool is a FOR loop executed  $M$  times. The  $N$  samples vector, inside the loop, is modified in order to simulate the errors generated during the A/D conversion process.

To simulate the offset, a constant value is added to each sample of the signal. This value is a random number within the range declared by the manufacturer. For each simulated measurement, the generated random number changes so that it lies in the specification range according to a rectangular distribution. In the same way, gain errors are simulated. In this case each sample of the signal is multiplied by a constant value. A white noise is added to simulate the thermal noise, and to simulate the crosstalk interference, another signal is added. The INL errors are simulated distorting the transfer function with components of second, third, fourth and fifth order and with other two spurious components, so that the maximum deviation from a linear transfer function is always equal to the maximum INL value declared in the specifications. As for the settling time errors, the software tool calculates the errors for the actual sampling rate and the actual step between each two contiguous samples, starting from the settling time accuracy at the maximum sampling rate declared by the manufacturer. The timing jitter errors are simulated by multiplying a random number, within the range of aperture jitter declared in the specifications, by the derivative of the signal; the so obtained values, which are the amplitude errors caused by the sampling time errors, are added to each sample. At last, the simulation of the quantization process, which takes into account the DNL errors, is performed.

The so modified  $N$  samples are sent to the software block of the instrument, which calculates the measurement result. The  $M$  measures are collected outside the loop and the standard deviation of the measurements results, that is the combined standard uncertainty, is calculated.

The main advantage of this method is that it intrinsically takes into account every possible correlation between each quantity. However, it is obvious that the effectiveness of the described approach is strictly depending on how the A/D conversion process and the introduction of the errors are simulated. So with the aim of validating the approach, we applied the numerical method on various DSP basic blocks, which are typical of a measurement chain. The obtained results have been compared with the ones obtained by means of experimental tests and the comparison, as described in detail in [4], has positively validated the numerical method.

#### IV. The PUMA method applied to PCBMI

The described numerical approach leads to an accurate estimation of the uncertainties of a PC-based measurement; however, it is necessary to truthfully simulate a real A/D conversion process. This entails a large usage of resources and time both for the software developing and for performing the M simulations. In order to save resources, according the PUMA methodology, it is necessary to find a coarser, but faster, approach which leads to an overestimate of the uncertainty. For this purpose, it is possible to use a simplified version of the numerical approach. The offset, gain and INL errors are simulated as previously described; as for noise, cross-talk, settling time, timing jitter, quantization and DNL, the corresponding rms value is calculated for each error source starting from the worst case manufacturer specifications; provided that with good approximation these uncertainty sources can be considered not correlated, the root sum square of the rms values is calculated, obtaining an equivalent random noise which is added to the input signal by the software tool. The advantage of the simplified version of the numerical approach is a huge reduction of the software tool complexity and of its execution time; in fact by using this version, the simulation of settling time, timing jitter and quantization (plus DNL) is performed just adding an equivalent noise to the input signal; obviously the usage of the worst cases specifications to simulate settling time, timing jitter, quantization and DNL leads to an overestimate of the uncertainties.

Another approach to the uncertainty estimation is the use of a theoretical method applying the uncertainty propagation law of the GUM. When a measurand estimate  $y$  is determined from  $N$  other samples  $x_1, x_2, \dots, x_N$ , through a functional relation  $y = f(x_1, x_2, \dots, x_N)$ , the combined standard uncertainty estimate  $u_c(y)$  of the measurement result is the positive square root of the estimated variance  $u_c^2(y)$ , obtained from:

$$u_c^2(y) = \sum_{i=1}^N \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=1, j \neq i}^N \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} r(x_i, x_j) u(x_i) u(x_j)$$

where  $u(x_i)$  is the estimated standard uncertainty associated with the sample estimate  $x_i$  and  $r(x_i, x_j)$  is the estimated correlation coefficient associated with the samples  $x_i$  and  $x_j$ .

The evaluation of the correlation coefficients is a very hard task, also because they are strictly depending on the input signal. On the other hand to ignore the correlations causes a heavy underestimate of the uncertainties. So the theoretical approach is actually inapplicable because of difficulties in the exact identification of correlation coefficients. But if we consider separately each uncertainty source, we can observe that as for the offset and gain, the correlation coefficients are approximately equal to 1, while as regards the other uncertainty source the correlation coefficients can be supposed equal to 0. Moreover, in case of errors due to gain, the relative standard uncertainty  $u_r(x) = u(x)/|x|$  has to be considered constant on each input sample. In all other cases it is the absolute standard uncertainty  $u$ , to be considered constant on each input sample. Therefore, all the uncertainty sources can be divided approximately in three classes:

- I. completely correlated input quantities and  $u_i = \text{const}$ ;
- II. completely correlated input quantities and  $u_{rII} = \text{const}$  ( $u_r$  = relative uncertainty);
- III. not correlated input quantities and  $u_{III} = \text{const}$ .

In this way, it is possible to overcome the difficulties of the exact evaluation coefficients, since, by means of the proposed classification, the uncertainty sources are divided in three classes with supposed correlation coefficient exactly equal to 1 or 0. Moreover the uncertainty propagation law becomes easier to apply, that is respectively for the three classes:

$$u_{cI}(y) = u_I \sum_{i=1}^N \left( \frac{\partial f}{\partial x_i} \right) \quad u_{cII}(y) = u_{rII} \sum_{i=1}^N \left( \frac{\partial f}{\partial x_i} x_i \right) \quad u_{cIII}(y) = u_{III} \sqrt{\sum_{i=1}^N \left( \frac{\partial f}{\partial x_i} \right)^2}$$

Starting from the above considerations, the idea of an approximated theoretical method has arisen. It is based on the following steps:

- to subdivide the uncertainty sources in the three classes;
- to carry out the root sum square of the uncertainties of each class, obtaining three values ( $u_{cI}$ ,  $u_{cII}$  and  $u_{cIII}$ ) of uncertainty for each acquired sample;
- to apply the propagation law separately for each source class, getting three standard uncertainty values  $u_{cI}$ ,  $u_{cII}$ ,  $u_{cIII}$ .
- to carry out the root sum square of these three values obtaining the combined standard uncertainty of the measurement result.

There are some approximations in this method: the first one consists of combining the uncertainties after they are propagated, whereas actually the uncertainties first are combined in each acquired sample and then propagate through the software block; the other approximation is the subdivision of the uncertainty sources in the three classes with supposed correlation coefficient exactly equal to 1 or 0. These approximations lead to uncertainty overestimations slightly heavier than the ones obtained by using the simplified numerical approach. An interesting advantage of the theoretical approach is that it can be applied during the instrument design stage when the software block of the instrument is not already developed.

The last, faster and coarser approach is an even simpler version of the theoretical method; in this case the correlation coefficients of all uncertainty sources are set equal to 1; as for the gain errors, it is always considered the worst case that is setting  $u_{gain} = u_{rgain} \times Full\ Range$ ; in the propagation law, the absolute values of the partial derivatives are considered:

$$u_c(y) = u \left| \sum_{i=1}^N \left( \frac{\partial f}{\partial x_i} \right) \right| .$$

Therefore the application of the theoretical method becomes quite fast and simple, since it is enough just performing the following steps:

- to carry out the root sum square of all the uncertainties, obtaining a single value ( $u$ ) of uncertainty for each acquired sample;
- to apply the simplified propagation law, getting the combined standard uncertainty of the measurement result.

Obviously to consider the uncertainty of each acquired sample totally correlated and to consider the worst case of the gain error lead to a quite heavy overestimate of the measurement uncertainties.

## V. A practical case

With the aim of verifying the application of the PUMA method, we applied all the proposed uncertainty estimation approaches on various DSP basic blocks that are typical of a measurement chain. As example in the following, the procedure for the measurement of the mean and the rms values of 100 samples is reported. The PCBMI is constituted of the National Instruments™ AT-MIO-16E10 data acquisition board (16 single-ended or 8 differential channels, successive approximation 12 bit ADC, 100 kS/s max sampling rate,  $\pm 10$  V maximum input signal range) and a PC with an INTEL™ 870 MHz processor; the LabView™ 7.0 is the programming language used to drive the acquisition board, to process the acquired samples and to realize the user interface. The used sampling rate is 10 KS/s. The input signal is a 2 KHz sinusoid with a 4 V peak value plus a 5 V DC signal. We consider a Type B evaluation of standard uncertainties, based on manufacturer's specifications, assume rectangular distributions and suppose to operate within  $\pm 1$  K of the data acquisition board self-calibration temperature, within  $\pm 10$  K of factory calibration temperature, after one year of the factory calibration and to set the gain equal to 0.5.

In Table I the considered uncertainty sources, the manufacturer specification and the standard uncertainty values are reported:

Table I  
Uncertainty sources

Uncertainty source	offset	gain	INL	DNL	quantization	noise	settling time for full scale step	time jitter	cross talk
Manufacturer specification	$\pm 1500$ $\mu\text{V}$	0.06 %	$\pm 1$ LSB	$\pm 0.5$ LSB	$\pm 0.5$ LSB	0.07 LSB rms	$\pm 0.1$ LSB in 100 $\mu\text{s}$	$\pm 5$ ps	- 80 dB
Standard uncertainty values	866 $\mu\text{V}$	346 ppm	2819 $\mu\text{V}$	1410 $\mu\text{V}$	1410 $\mu\text{V}$	342 $\mu\text{V}$	282 $\mu\text{V}$	140 $\mu\text{V}$	707 $\mu\text{V}$

Starting from these values, by the application of the four uncertainty estimation approaches, the result of table II are obtained:

Table II  
Uncertainty values [ $\mu\text{V}$ ]

Measurand	Simplified theoretical method	Theoretical method	Simplified numerical method	Numerical method
Mean value	5000	2000	1800	1300
Rms value	4400	2200	2100	1400

Analyzing the results, it is possible to notice that in some cases the coarser approach lead to an uncertainty estimation roughly four times greater than the one obtained by using the more precise methodology. However, it must be underlined that the application of the simplified theoretical approach requires a negligible amount of time and of resources, if compared with the time and the resources involved by the numerical method.

We obtained similar results considering other data acquisition boards and other measurement algorithms.

## VI. Conclusion

In the paper, four approaches for the uncertainty assessment of a measurement carried out by using a generic PCBMI were presented. The approaches imply different levels of the estimate accuracy, but at the same time different amount of required resources. Therefore, the four methodologies are perfectly adequate for the implementation of the PUMA method and for a correct management of the uncertainty budget. Often, in fact, it is not necessary to obtain a very accurate uncertainty evaluation, but it is enough to know if an already available instrumentation and an already defined measurement environment are appropriate for a stated target uncertainty.

All the proposed uncertainty estimation methods can be extended to transducers and signal conditioning accessories. After the identification of the error sources that arise during the quantities transduction and the signals conditioning and after the evaluation of the related standard uncertainties, the obtained values have to be divided in the three classes in order to carry out the theoretical method. As for the numerical methods, the software tool has to be modified in order to simulate, beside the A/D conversion, the whole measurement process.

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