INSTANTANEOUS FREQUENCY ESTIMATION OF MULTICOMPONENT SIGNALS CHARACTERIZED BY INTERFERING LINEAR TRAJECTORIES

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Abstract **-** The paper deals with signals that include nonstationary components. For these signals, the estimation of the instantaneous frequency of each component is especially difficult in time intervals where its trajectory interferes with another one, for example by crossing it. The attention is mainly paid to linear trajectories, seeing that any trajectory can be generally approximated as linear in the nearby of the crossing point.

The authors propose a method based on the use of a two dimensional subspace of the chirplet transform, known as bowtie subspace, that allows accurate estimates of the instantaneous frequency even in the presence of crossing trajectories. The use of a restricted subspace allows a reduced computational burden and less time consumption with respect to several approaches based on advanced multiparametric transforms, typically exploited when multicomponent nonstationary signals are analyzed. The performance granted by the proposed method is assessed through a number of tests performed on simulated, emulated, and real signals.

Keywords **-** Multicomponent signal, instantaneous frequency estimation, time-frequency representation, chirplet transform, bowtie subspace.

I. INTRODUCTION

In many real-life applications technicians have to deal with nonstationary signals, i.e., signals with a time-varying spectral contents; many significant examples include speech, radar, sonar and biomedical applications. If the signal consists of one or more narrow-band components, then it is usually described in terms of the instantaneous frequency (IF) of each component [1].

In the presence of nonstationary multicomponent signals IF estimations are performed by means of time-frequency (TF) analysis tools. In particular, to this end the results offered by TF tools, which are represented by a function of time and frequency that highlights how the energy of the multicomponent signal is distributed in the frequency domain at any time instant, are analyzed by means of peak location algorithms [2]-[8]. Since every component is associated to a local peak of the TF results, the use of peak location algorithms for detecting relative peaks allows the estimation of the instantaneous frequency of all components.

Among several different TF tools, the spectrogram is usually the first choice [3]. But, it is based on the short-time Fourier transform (STFT), thus it suffers from an inherent poor resolution in the TF domain [2]. As a consequence, the spectrogram based algorithms for the IF estimation barely meet typical measurement accuracy requirements.

The Wigner-Ville distribution (WVD) [4]-[6] is often adopted in place of the spectrogram. It is optimal for the analysis of monocomponent signals characterized by linear frequency trajectories (chirps). Unfortunately, WVD suffers from cross-terms when multicomponent signals are analyzed [2]. To overcome the drawback of cross-terms, two modified versions of the WVD, namely the pseudo Wigner-Ville distribution (PWVD) and its smoothed version (SPWVD) have been introduced; but, they attain the cancellation of the cross-terms at the expense of a reduced resolution in the TF domain [7].

Alternatively, methods based on the chirplet transform (CT) have been proposed for the IF estimation [9], [10]. CT is an advanced TF tool that maps a one-dimensional signal into a five-dimensional function. However, its results are still represented in the TF domain when constant values are assigned to the other parameters, namely scaling factor, chirping in time and chirping in frequency. Also, in [11] a method based on a modified version of the CT that introduces a so-called curvature parameter is proposed. Thanks to the curvature parameter, the method can offer a new degree of freedom in the analysis of nonstationary signals with nonlinear frequency trajectories.

For signals characterized by IF trajectories with a periodic evolution versus time, a specific method based on the warblet transform is proposed in [12].

However, in the presence of multicomponent signals characterized by IF trajectories that can interfere, none of the aforementioned methods provide satisfying results in the interference region. Such signals can be observed in several systems including digital wireless communication systems operating in license-free bands, such as the ISM band, space-based communication systems, navigation systems, power systems fed through switched-mode converters, etc.

In order to attain accurate IF estimations for multicomponent signals characterized by interfering frequency trajectories, an original approach based on the use of a two-dimensional subspace of the CT, which is known as bowtie subspace (BS), is herein proposed. In particular, a simple algorithm for the IF estimation of linear frequency trajectories that cross each other is presented. The attention is mainly paid to linear trajectories, seeing that any trajectory can be generally approximated as linear in the nearby of the crossing point. The algorithm works on the BS by using a suitably chosen binary mask to separate different linear FM components. It requires the computation of a two-dimensional function for every time instant, therefore, it has a reduced computational burden with respect to that required by a five-dimensional CT computation.

The paper is organized as follows. A brief introduction to the traditional CT is given in Section II. The proposed method is described in Section III. In Section IV the performance of the proposed method is evaluated by means of several tests carried out on simulated, emulated and real multicomponent signals.

II. THEORETICAL BACKGROUND

The CT is based on the use of chirplets, which represent windowed versions of chirps. The CT projects an input signal, which is a function of time, onto a set of chirplets, all obtained by modifying an original window function $g(t)$ known as mother chirplet [9]. Beside time and frequency shifting, peculiar to the STFT, and scaling, peculiar to the wavelet transform (WT), the CT introduces two more parameters, namely chirping in time and chirping in frequency, which provide, respectively, rotation and shearing [11]. These parameters offer additional degrees of freedom and allow the CT to optimize the analysis of trajectories characterized by different features.

In this paper, a Gaussian window function expressed by:

$$g(t) = \frac{1}{\sqrt{\sqrt{\pi}a}} e^{-\frac{1}{2}\left(\frac{t}{a}\right)^2} \tag{1}$$

where a is a time-scale parameter, is used as the mother chirplet [9]. This choice provides the best resolution and makes shearing operations unnecessary. In fact, for the Gaussian window shearing would provide no new degrees of freedom that cannot be obtained by suitably combining chirping in time and scaling operations.

The analytical expression of the CT of the signal $s(t)$ is given by:

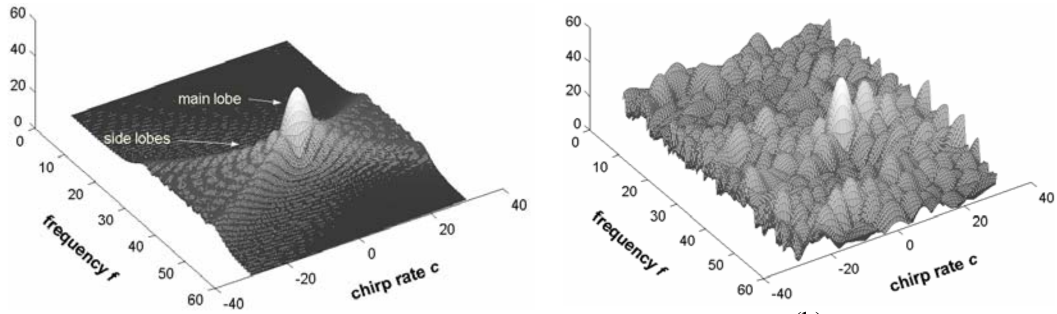
$$CT_s(t, f, a, c) = \int s(\tau) h^*(\tau - t, f, a, c) d\tau \tag{2}$$

where t represents the time-shift of the kernel $h(\tau, f, a, c)$ expressed by:

$$h(\tau, f, a, c) = g(\tau) e^{j2\pi(f + c\tau)\tau} \tag{3}$$

in which j is the imaginary unit, f the frequency shift and c is the chirp rate.

The 2-D slice that takes into account only the parameters f and c of the 4-D space used by the transform given in (2) is called bowtie subspace (BS) [9]. It is named after a 3-D graph of the chirp in this subspace, which is represented by a peak with bowtie shaped contours, as shown in Fig.1(a). The coordinates of the peak in the BS provide the complete information about the observed chirp, because they allow estimations of both the chirp rate and IF of the chirp for a given time instant [9]. The fact that this peak is not a perfectly sharp spike is a consequence of the resolution limit of the transform. In the following, the highest peak will be referred to as main-lobe, and minor peaks as side-lobes.



(a) (b)

Fig.1. 3-D graph of the BS representation of: (a) chirp, (b) chirp corrupted by additive Gaussian noise. The clearest gray tones of the color-map are associated to the highest values of the BS representation.

However, when the signal under test is corrupted by noise, the detection and estimation of its IF trajectory can be difficult. As an example, the BS representation of the chirp corrupted by an additive white Gaussian noise (AWGN) is shown in Fig.1(b). In this example, the chirp under test has unitary amplitude, and the AWGN is characterized by a variance equal to 1.7. Several peaks produced by the AWGN can be recognized around the main peak associated to the chirp. In the presence of high noise levels, a simple interpretation of the coordinates of the highest peak as the chirp rate and frequency of the chirp could produce false results for some time instants.

III. PROPOSED METHOD

In this section, a method for reliable and accurate IF estimations of multicomponent signals with interfering trajectories is proposed. The method can achieve IF estimations even in critical conditions, i.e., in the presence of low SNR values.

A. Introductive notes

Due to its random nature, the AWGN can produce the highest peak in any area of the BS at a considered time instant; on the contrary, any component characterized by a linear frequency trajectory always produces the peak in the same stripe of the BS. Specifically, in the analysis of components characterized by linear frequency trajectories, one of the coordinates of the BS, namely the chirp rate *c*, doesn't vary but remains constant, whereas, the average frequencies of the linear trajectory within two successive time intervals (i.e., two BSs that differ for close by time values) are very correlated to one another and are characterized by very similar values. The search for the peak value can be, thus, reduced to a limited area of the whole BS. Precisely, for the given time instant, the peak can be searched for within a rectangle centered at the point whose coordinates are the chirp rate and average IF obtained at the previous instant. This naturally holds only if the chirp rate and average frequency are correctly determined at the initial time instant.

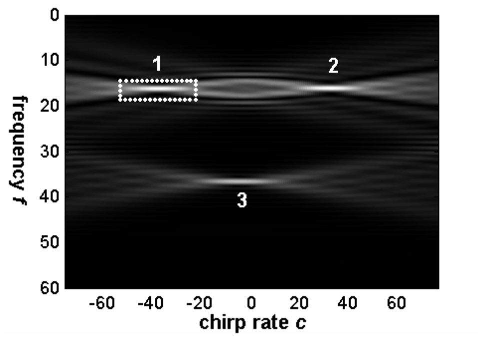


Fig.2. 2-D image related to the BS representation of a multicomponent signal with three different components characterized by linear frequency trajectories. The clearest gray tones are associated to the highest values of the BS representation.

In order to clarify the aforementioned procedure, Fig.2 shows the graph of the BS representation of a multicomponent signal that exhibits three different components at time instant *t*. At this instant, one component is detected in the rectangle delimited by a dotted line. The center of this rectangle has the same coordinates as the detected peak. The adopted procedure defines the borders of this rectangle so as to include only the main-lobe of the chirp representation. At the next time instant the peak is searched for only within this rectangle in the new BS representation. When the coordinates of the peak in the new BS representation are gained, the center of the rectangle is set to this new position, and it is then used for detecting the peak in the successive BS representation. In this way, by eliminating the need for searching the entire BS it is also eliminated the possibility to come across an occasional peak higher than the desired peak, produced by the AWGN.

The BS representation offers a major advantage in IF estimations of multicomponent signals that are characterized by interfering trajectories. With reference to the components denoted by numbers 1 and 2 in Fig.2, it can be noted that at the considered time instant, these two components are characterized by the same *f* coordinate, i.e., they have the same frequency value. Therefore, these components interfere at the considered instant. In the BS they can be distinguished since they have different chirp rates *c*. When multicomponent signals are analyzed it is necessary to employ one rectangle for the analysis of each component in order to estimate its IF trajectory.

B. Implementation guidelines

The rectangle used in the aforementioned analysis is implemented as a binary mask. More precisely, it is defined as a two-dimensional function of *f* and *c* whose values are 1 for the points inside the rectangle and 0 for the points outside. The size of the rectangle depends on the size of the main-lobe of the analyzed component, which in turn depends on three factors, namely the adopted time-scale factor *a*, the resolution in the frequency domain, and the resolution in the chirp rate domain. The size of the rectangle can be practically determined on the basis of a rough approximation of the spectrum of the observed signal. From now on, the product of the binary mask and BS will be referred to as masked bowtie subspace (MBS).

Concerning the estimation of the chirp rate and average frequency of each component in the BS at the initial time instant, a parallel estimation and comparative analysis of the results offered by the FFT analysis and BS representation is proposed. If the coordinates of the components at three subsequent time instants coincide, or are not different from each other within 3%, the initial IF estimate is taken as a reliable one.

IV. PERFORMANCE ASSESSMENT

The performance offered by the proposed method has been appraised through a number of tests carried out on simulated, emulated and real multicomponent signals.

A. Tests on Simulated Signals

A1. Monocomponent signals

For monocomponent signals, a simple comparison between IF estimations achieved through peak location algorithms applied to the results offered by: a) the spectrogram (SPEC), b) the entire bowtie subspace (EBS) and c) the masked bowtie subspace (MBS) has been drawn.

To perform this analysis, the following monocomponent signal $s(t)$ is considered:

$$s(t) = \cos\left(2\pi\left(57.1 - 27.4t\right)t\right), \quad 0 < t < 2s. \tag{4}$$

The signal has also been corrupted by an AWGN characterized by variance equal to 1.7. The number of samples within the considered time interval has been 512. The performance of each method against noise is evidenced by a root mean square error (RMSE) evaluated according to [12]:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left[20\log_{10}\left(\frac{f_{es}(n)}{f_{nom}(n)}\right)\right]^2} \tag{5}$$

where $f_{es}(n)$ and $f_{nom}(n)$ are the estimated and nominal frequency trajectories, respectively, and n stands for the discrete time variable. The spectrogram has been computed using a Hanning window characterized by 128 samples, while the Gaussian chirplet characterized by 128 samples and the time-scale parameter $a = 1$ has been used for the BS analysis.

The averaged RMSE values over 200 runs for the three IF estimations related to three compared methods have been: $\text{RMSE}_{\text{SPEC}}$ = 3.36dB, RMSE_{EBS} = 0.58dB and RMSE_{MBS} = 0.16dB.

A2. Multicomponent signals

The proposed method has been applied to the signal $s(t)$ obtained by summing different components. In particular, three components have been considered, as follows:

$$s(t) = \cos\left(2\pi\left(69.1 - 37.9t\right)t\right) + 0.98\cos\left(2\pi\left(3 + 30.4t\right)t\right) + 1.02\cos\left(2\pi\left(79.1 - 5.8t\right)t\right), \quad 0 < t < 2s. \tag{6}$$

The signal has also been corrupted by an AWGN characterized by variance equal to 0.8. The number of acquired samples within the considered time interval has been 512. The BS of the signal $s(t)$, without the AWGN, at the time instant at which the IFs of the first two components coincide is shown in Fig.2.

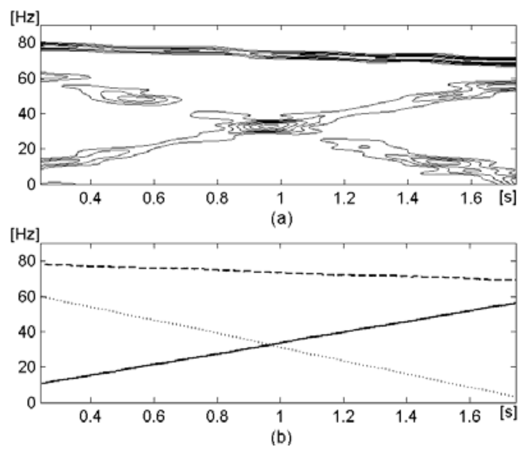


Fig.3. (a) Contour plot of the spectrogram of the simulated signal. (b) IF trajectories estimated by the proposed method.

Fig. 3(a) depicts the contour plot of the spectrogram of the corrupted simulated signal. Due to the presence of the AWGN, the automatic estimation by means of peak location algorithms produces very rough results. On the contrary, the proposed method accurately estimates the IF trajectories that are given in Fig.3(b).

B. Tests on Emulated Signals

In order to test the method on emulated signals, a suitable measurement station has been set up. It consists of a processing and control unit, namely a personal computer, an arbitrary waveform/function generator (14-bit resolution, 20 MHz maximum output frequency), and a digital scope (8-bit vertical resolution, 1 GHz analog bandwidth, 4 GS/s maximum sample rate). They all are interconnected by means of an IEEE-488 interface bus.

In particular, using the modulation capabilities of the adopted function generator, the multicomponent signal has been generated and successively acquired by means of the digital scope. As an example, the signal $s(t)$ made up of four different components, i.e.,

$$s(t)=\cos\left(2\pi\left(50+125t\right)t\right)+\cos\left(2\pi\left(280-125t\right)t\right)+\cos\left(2\pi\left(420-45t\right)t\right)+\cos\left(2\pi\left(460+15t\right)t\right) \qquad (7)$$

is considered. The number of acquired samples has been 1000, the sample rate 1 kS/s. The signal is also corrupted by an AWGN with variance equal to 0.6. The contour plot of the spectrogram of the corrupted signal is shown in Fig.4(a), and the IF trajectories estimated by means of the proposed method are shown in Fig.4(b). The spectrogram has been computed using the Hanning window with 128 samples, while the Gaussian chirplet with 128 samples and a = 1 has been used for the BS analysis.

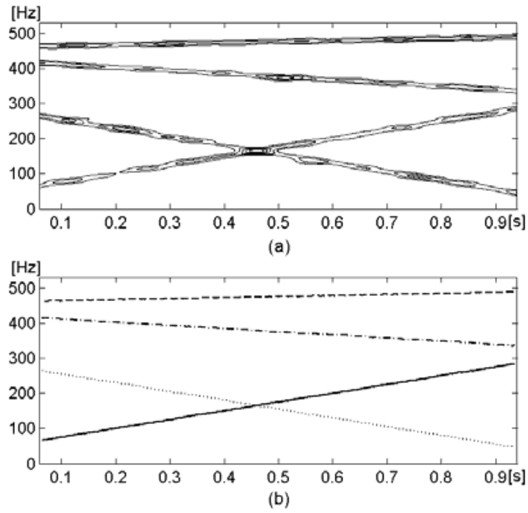


Fig.4. (a) Contour plot of the spectrogram of the emulated signal. (b) IF trajectories estimated by the proposed method.

C. Tests on Real Signals

The proposed method has also been applied to real multicomponent signals. In particular, some preliminary results related to the signal that has been picked up by means of a B-filed probe in the vicinity of an electrical cable are shown.

The cable supplies a large motor with a deformed current produced by an electronic adjustable speed drive. The detected signal represents the radiated emissions during a deceleration test. It is characterized by the sample rate equal to 2 kS/s. It exhibits two dominant components, namely a downchirp, i.e., the component characterized by a negative chirp rate, and a stationary component at 100 Hz.

Fig.5(a) shows the spectrogram of the whole signal. In Fig.5(a) an area of interest is pointed out by a bold rectangle. In this area, the aforementioned dominant components interfere with each other. The time domain evolution of the portion of the signal related to the aforementioned area, which is within 27 - 41 s, is shown in Fig.5(b). This portion of the signal is normalized in the amplitude and counts only 1280 samples. Moreover, Fig.5(c) is the contour-plot on the TF plane of the spectrogram of the considered portion; the spectrogram has been evaluated using the Hanning window with 512 samples. Finally, Fig.5(d) shows the estimated IF of the detected components. The chirplet characterized by 512 samples and a = 4 has been used to perform the BS analysis.

V. CONCLUSION

An original approach based on the use of a two-dimensional subspace of the chirplet transform, namely bowtie subspace (BS), for accurate IF estimations of multicomponent signals characterized by linear interfering frequency trajectories has been proposed. The method overcomes the limits of several alternative proposals that cannot provide satisfying results in the interference region. The main drawback of the method is its poor capability of achieving IF estimations of multicomponent signals that have close IF trajectories and at the same time are considerably different in power. This is due to the presence of side-lobes from high-power chirps that completely hide the low-power ones in the BS.

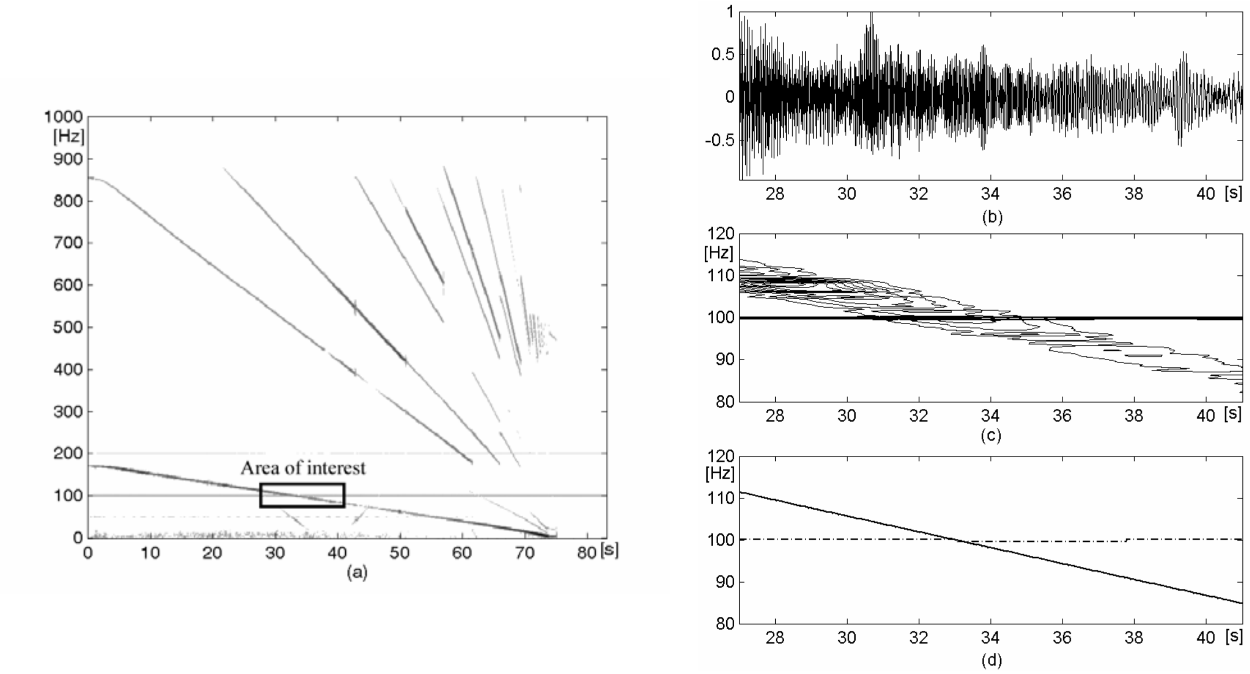


Fig.5. (a) Spectrogram of the whole signal. (b) Normalized portion of the observed signal. (c) Contour plot of the spectrogram of the analyzed portion; (d) IF trajectories of the downchirp and stationary component at 100Hz.

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