Single-Mode Piezoelectric Ultrasonic Transducer Equivalent Circuit Parameter Calculations and Optimization Using Experimental Data

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Abstract- The impedance characteristics of piezoelectric ultrasonic transducers can be described with the help of an electrical equivalent circuit. The circuit permits transducer analysis. Therefore, it is desirable to have a way of determining the equivalent circuit parameters of a given transducer. We propose a method which can basically be divided in two parts: 1) Approximate calculation of the circuit parameters and 2) Increase the accuracy of the parameters by optimization techniques. For its availability and relatively simple implementation, the optimization toolbox of MATLAB was used to perform the second part. The results obtained show a good match between the measured and the optimized data.

I. Introduction

Understanding the characteristics of ultrasonic transducers enables one to optimize an ultrasonic system's performance. It is not uncommon to find transducers that do not exactly meet their specifications, or others with degrading performance over time, therefore, there is a need for transducer characterization.

The impedance characteristics of piezoelectric ultrasonic transducers, around their fundamental resonances (series and parallel), can be described with the help of an electrical equivalent circuit. The circuit permits the analysis of a transducer, namely, it is useful to investigate about the electrical signal to be applied, the design of electrical impedance matching networks [1] to obtain maximum power transfer, and the design of acoustic impedance matching layers. Its parameters can be used to obtain some properties of the piezoelectric materials, such as mass, stiffness, capacitance, inductance and damping, over a specific frequency range [2][3]. Thus, it is desirable to have a way of determining the electrical equivalent circuit parameters of a given transducer.

Some impedance analysers can calculate the parameters of the equivalent circuit, based on its measurements. The problem of this approach is that some of these analysers cannot measure impedances over the desired frequency range. Our approach is therefore an alternative to the latter analysers and to the ones that do not have the equivalent circuit function at all.

II. Equivalent Circuit Parameters Calculation

We have used the Butterworth-Van Dyke equivalent circuit, which has been extensively used in the literature [4]-[7]. This circuit is shown in Figure 1. The resistance R_1 represents the mechanical dissipations, the inductance L_1 the mass, the capacitance C_1 the compliance (flexibility), and C_0 the static capacitance of the transducer [1].

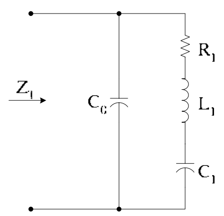

Figure 1: Single-mode unloaded piezoelectric ultrasonic transducer electrical equivalent circuit.

The total impedance of this circuit is given by:

$$Z_T = \frac{(L_1C_1\omega^2 - 1) - j(R_1C_1\omega)}{(R_1C_0C_1\omega^2) + j[L_1C_0C_1\omega^3 - \omega(C_0 + C_1)]} \tag{1}$$

Although, there are other approaches to obtain the circuit parameters [4], we have derived the exact equations from the magnitude of equation (1) at its series resonant frequency ω_s and at its parallel resonant frequency ω_p (antiresonance), which are given by:

$$\omega_s = \frac{1}{\sqrt{L_1 C_1}} \tag{2}$$

$$\omega_p = \frac{1}{\sqrt{L_1 C_{eq}}} \tag{3}$$

where $C_{eq} = (C_0 C_1)/(C_0 + C_1)$. The resulting equations for the circuit parameters are the following:

$$C_0 = \sqrt{\frac{(Z_{\omega s})^2(\omega_p^{\ 2} - \omega_s^{\ 2}) + \sqrt{(2\omega_p^{\ 2} Z_{\omega s} Z_{\omega p})^2 + (Z_{\varpi s})^4 (\omega_p^{\ 2} - \omega_s^{\ 2})^2}}{2(\omega_p^{\ 2} Z_{\omega p} Z_{\omega s})^2}} \tag{4}$$

$$R_1 = \sqrt{\frac{(Z_{\omega s})^2}{1 - (C_0 \omega_s Z_{\omega s})^2}} \tag{5}$$

$$C_1 = C_0 \left[\left(\frac{\omega_p}{\omega_s} \right)^2 - 1 \right] \tag{6}$$

$$L_1 = \frac{1}{C_1 \omega_s^{\ 2}} \tag{7}$$

where $Z_{\omega s}$ and $Z_{\omega p}$ are the values of the measured impedance magnitude at ω_s and ω_p, respectively.

In practice, the parameters obtained using these equations are only approximations to the true parameters because the exact values of the resonant and the antiresonant frequencies, and therefore, the corresponding impedance magnitude values are not known. In our approach, the values of ω_s and ω_p are assumed to be the values of ω at which the measured impedance magnitude is at its minimum and at its maximum, respectively.

III. Equivalent Circuit Parameters Optimization

In order to increase the accuracy of the calculated values, we have used optimization algorithms to fit the magnitude of equation (1) to the measured impedance magnitude of a given transducer. These algorithms need initial values for the circuit parameters and they are very sensitive to them. The reason for the latter is that the function to be minimised usually has many local minima besides the global one. The parameters obtained from our equations are used as the initial values in the algorithms, preventing convergence to a local minimum, as they are close enough to the true parameters.

The parameter optimization was performed using the optimization toolbox of MATLAB [8]. Specifically, we used the function called *lsqcurvefit*. This function solves nonlinear curve-fitting problems in the least-squares sense. It permits the use of the following algorithms: Gauss-Newton (GN), Levenberg-Marquardt (LM) and Trust-Region (TR) [8].

It was found that the optimization algorithms did not converge if the parameters obtained from our equations were directly used as initial values. This happens because of the very low numeric value of some of the parameters (capacitances). To get around this situation, we forced the algorithms to search for percentages of the calculated parameters. For instance, we used $C_{1initial} = P \cdot C_1$, (P being the variable changed by the algorithm, while C_1 is kept constant) instead of $C_{1initial} = C_1$. In this way, all the initial values (value of P associated with each parameter; set to unity in the first iteration) were in the same order of magnitude and the algorithms converged to the global minimum.

IV. Summary of the Approach to Obtain the Circuit Parameters

Basically, our approach to obtain the values of the parameters is the following:

1. Measure the impedance magnitude and phase (we used the HP4192A LF impedance analyser) of the transducer to be modelled over a frequency range around the manufacturer specified resonant frequency (single-mode modelling);
2. From the impedance magnitude curve get approximations to the values of ω_s and ω_p (minimum and maximum, respectively);
3. Read the corresponding values for $Z_{\omega s}$ and $Z_{\omega p}$;
4. Calculate the parameters using equations (4) to (7);
5. Use optimization toolbox of MATLAB to optimize the calculated parameters.

These steps are summarised in Figure 2:

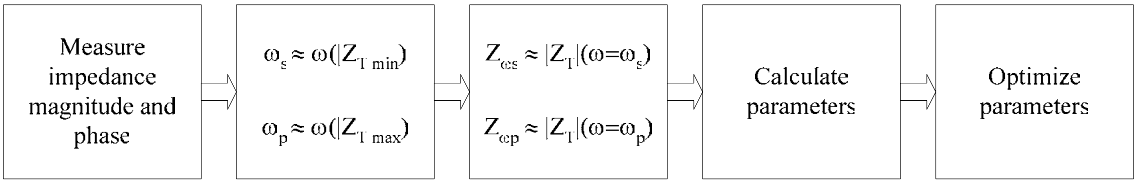

Figure 2: Block diagram summarising the procedure to obtain the circuit parameters

V. Simulation and Experimental Results

For a simulated situation, in which we exactly know the values of the circuit parameters, the error between the true and calculated values can be calculated, as shown on Table 1.

Table 1: An example of the error obtained using equations (4) to (7).

Parameters		True	Calculated	True/Calculated	|Error| (%)
R_1	(Ω)	1600	1358.4	1.18	15.1
L_1	(mH)	115	66.85	1.72	41.87
C_1	(pF)	215	382.69	0.56	78
C_0	(nF)	2.3	2.288	1.01	0.52
f_s	(Hz)	32008	31458	1.02	1.72
f_p	(Hz)	33470	33987	0.98	1.55

These error results are just an example to check the accuracy of the equations we have derived. One should note that as the errors of ω_s and ω_p tend to zero, the errors of the other parameters also tend to zero. For instance, these results show that there is a difference of 15.1% between the calculated and the true value of R_1. Optimizing the calculated parameters, in a simulated situation, reduces the error to virtually zero. This happens because the fitting function (magnitude of (1)) is, in this case, an exact model of the data. An example of the performance of the optimization algorithms mentioned in section III is shown on Table 2.

Table 2: Gauss-Newton (GN), Levenberg-Marquardt (LM) and Trust-Region (TR) performance example

Optimization Information	P = 0.9			P = 1			P = 1.15		
	GN	LM	TR	GN	LM	TR	GN	LM	TR
Iterations	68	16	243	12	16	153	7	18	278
Function Evaluations	558	130	1220	98	130	770	54	146	1395
Squared 2-norm residual	1.35×10^8	0.054	0.054	0.054	0.054	0.054	0.054	0.054	0.054

From Table 2 one can see that if the algorithms start at the exact calculated values (P=1), the best performance is obtained by the GN algorithm. One can also see that for P=0.9 the GN algorithm does not converge to the global minimum (residual is very high). Therefore, the GN algorithm is preferred when the residual is approximately zero at the global minimum. Although the LM algorithm is less efficient than the GN, it is the most robust of the three, as it can find the optimum solution when the GN fails. The Trust-Region algorithm is the less efficient in this specific situation.

Figure 3 shows the measured impedance magnitude and phase of an unloaded 32 kHz piezoelectric ultrasonic transducer. As one can note, there is a significant error between the experimental (crosses) and calculated (dashes) values. After optimization, this error was significantly reduced. For this specific ultrasonic transducer, the values of the circuit parameters obtained were the following: R_1=2091.9 Ω, L_1=286.95 mH, C_1=84.9 pF, C_0=2.34 nF.

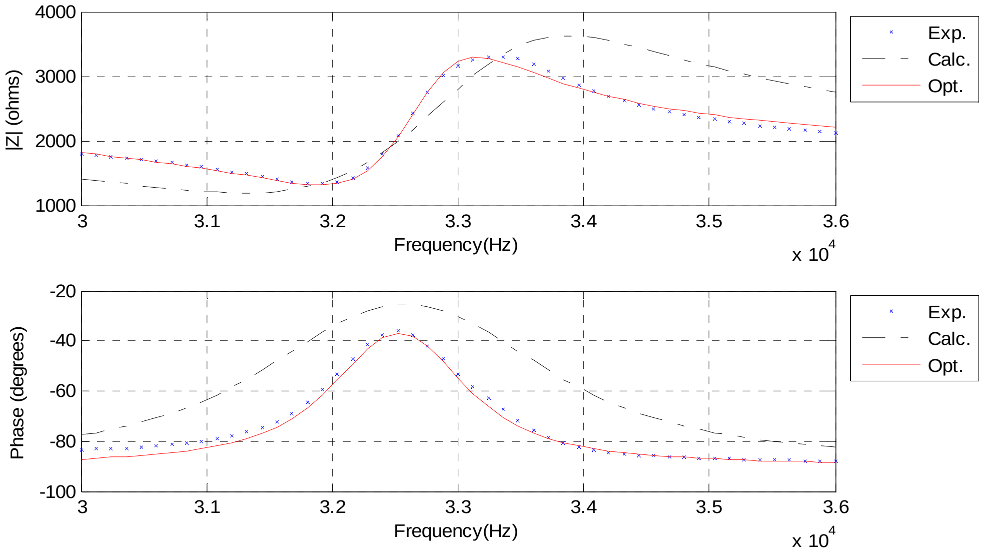

Figure 3: Experimental, calculated and optimized curves of electrical impedance magnitude and phase of an unloaded 32 kHz piezoelectric ultrasonic transducer.

VI. Conclusions

From the work presented in this paper, one can arrive at the following conclusions:

- The assumption that the resonant (ω_s) and the antiresonant (ω_p) frequencies are equal to the frequencies at which the magnitude of Z_T is minimum and maximum, respectively, introduces error in the calculated parameters. The accuracy of calculated parameters depends on this error;
- The calculated parameters are close enough to the true parameters, enabling convergence to the global minimum;
- The employed optimization algorithms are very sensitive to low numeric values;
- The parameters obtained are only valid for the chosen frequency range;
- The impedance (magnitude and phase) curves obtained using the parameters found with our approach agree well with experimental curves, indicating good parameter accuracy.

References

[1] Khmelev, V., Savin, I., Barsukov, R. and Tsyganok, S., "Problems of Electrical Matching of Electronic Ultrasound Frequency Generators and Electroacoustical Transducers for Ultrasound Technological Installations", *5th International Siberian Workshop on Electron Devices and Materials*, pp. 211-215, 2004

[2] Tilmans, H., "Equivalent Circuit Representation of Electromechanical Transducers: I. Lumped-Parameter Systems", *Journal of Micromechanics and Microengineering*, vol. 6, no. 1, pp. 157-176, 1996

[3] Hong Du, X., Ming Wang, Q. and Uchino, K., "Accurate Determination of Complex Materials Coefficients of Piezoelectric Resonators", *IEEE Transactions Ultrasonics, Ferroelectrics and Frequency Control*, vol. 50, no. 3, pp. 312-320, 2003

[4] Coates, R., "Multiple-Mode Acoustic Transducer Calculations", *IEEE Transactions Ultrasonics, Ferroelectrics and Frequency Control*, vol. 36, no. 4, pp. 471-473, 1989

[5] Prokic, M., *Piezoelectric Transducers Modelling and Characterization*, MP Interconsulting, Le Locle, Switzerland, 2004

[6] Sherrit, S., Wiederick, H. and Mukherjee, B., "Accurate Equivalent Circuits for Unloaded Piezoelectric Resonators", *IEEE Ultrasonics Symposium*, vol. 2, pp. 931-935, 1997

[7] Rosenbaum, J., *Bulk Wave Theory and Devices*, Artech House Inc., Boston, 1988

[8] *Optimization Toolbox User's Guide*, version 3, The MathWorks, 2004