

UNCERTAINTY REDUCTION IN MEASUREMENT SYSTEMS BY STATISTICAL PARAMETER DESIGN

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Abstract - *In measurement system design, uncertainty is usually reduced by a cost increase: more accurate components are selected and narrower variations to influence parameters are imposed. In this paper, an alternative cost-saving approach is proposed: suitable values of design parameters capable of minimizing the measurement uncertainty due both to component's uncertainty and to influence parameters are identified without any loss in other metrological performances. With this aim, a general method based on statistical parameter design is proposed. The method effectiveness is highlighted by an experimental case study related to the design of a pass-band passive filter.*

Keywords - Uncertainty reduction, Parameter design, Optimization, Experiment design, Pass-band filter.

1. INTRODUCTION

In the design of engineering systems, the variance of the quality characteristic is usually reduced by a cost increase (*tolerance design*) [1]: higher-quality components are selected, and narrower variations to influence parameters are imposed. A quite different approach has been proposed in control system design (*robust control*) [2], in electronic circuit design (*tolerance and sensitivity analysis*) [3], or in advanced quality engineering (*parameter design* [4] and *robust design* [5-7]): the system output is made as much insensitive as possible both to influence parameters and to component tolerances by selecting a suitable system configuration. This is usually carried out through analysis techniques of system theory [2], range methods [8], evolutionary algorithms [9], or statistical experiment design [10]. All these techniques adopt a different strategy for the same aim of identifying different nominal values of system components or, more in general, of design parameters, and, thus, do not involve any cost increase in the system.

This approach has been applied also to the design of measurement systems [11]: the configuration capable of minimizing the difference between actual and ideal output, and, simultaneously, maximizing the sensitivity of the measurement system, is selected through parametric optimization techniques by a deterministic approach of control theory. However, the main problem in measurement system design is related to the random variability of the output, i.e. to the measurement uncertainty. Thus, in the

design of a measurement system, a deterministic approach turns out to be misleading.

In this paper, an alternative statistical approach to uncertainty reduction, as well as to other metrological performance improvement, in measurement system design is proposed. At first, the idea of statistical parameter design of measurement systems is advanced. On this basis, a procedure of uncertainty reduction as well as of optimization of metrological characteristics in the whole measurand range is proposed. Finally, the proposed approach is highlighted by an experimental case study of a pass-band passive filter.

2. THE BASIC IDEA

The designer of a measurement system attains the desired relation between the output y (*measurement result*) and the input m (*measurand*) by acting on suitable *design parameters* $\underline{x}^T = (x_1, \dots, x_n)$ [12]. They are all those parameters controlled by the designer, both in the system architecture and in the measurement process (e.g. nominal value of passive components, power supply voltage level, and so on). Uncertainty in measurement results arises both from uncertainty of the design parameters (e.g. component tolerance ranges, instability of references, and so on), and from the action of *influence parameters* $\underline{u}^T = (u_1, \dots, u_k)$. These are all the parameters of the measurement system, the environment, and the measurand system which can not be modeled by the designer (e.g. climatic factors, electromagnetic fields, and so on), and thus, act in uncontrolled way on the project. Usually, suitable limits to their maximum variations during the measurement process are imposed.

In the design of a measurement system, a relation between the design parameters x_i ($i = 1, 2, \dots, n$) and the measurement result y is defined (*design characteristic*): $y = f(x_1, x_2, \dots, x_n)$. Usually, parameters x_i are modeled as random variables with a mean value estimate \tilde{x}_i and a standard deviation estimate $s(x_i)$. The measurement result \tilde{y} is computed as: $\tilde{y} = f(\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)$. If the variables x_i are not mutually correlated and the higher-order moments of their distributions are negligible, the output standard deviation is estimated in \tilde{y} as [13]:

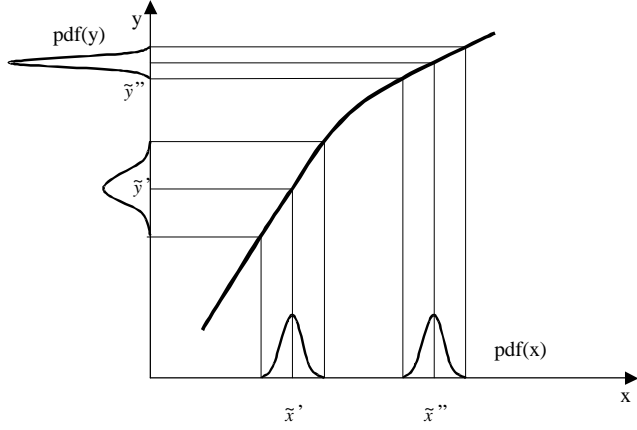


Fig. 1. Uncertainty of the measurement y for a given uncertainty of the design parameter x .

$$s(\tilde{y}) = \sqrt{\sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \right)^2 s^2(\tilde{x}_i)}, \quad (1)$$

where the partial derivatives are computed in the means $(\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)$ of the quantities x_i . Consequently, if n mean values $(\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)$ of the quantities (x_1, x_2, \dots, x_n) are

found such that the weights $\left(\frac{\partial f}{\partial x_i} \bigg|_{x=\tilde{x}_i} \right)^2$, $(i=1 \dots n)$, in (1)

are minimized, the measurement uncertainty on y will be reduced without a significant cost increase.

This minimization process is based on the hypothesis that the effect of a design parameter x_i on the system output y is quite *non linear*. This concept is highlighted in Fig.1 for the design of a measurement system by means of a single parameter x . A random variation of x from \tilde{x}' due to its own uncertainty with a given probability density function (*pdf*) gives rise to a particular *pdf* of y and, therefore, to a given uncertainty of the measurement system output. In the traditional approach to the uncertainty reduction, more expensive components of the measurement system with a lower uncertainty are suitably selected. In this way, the spread of x is reduced and, as a consequence, also the spread of y . Unfortunately, also the project cost is increased correspondingly. Conversely, in the proposed approach, a different nominal value \tilde{x}'' of the parameter x is identified around x' , such that a new output y'' is achieved. The value y'' is suitably selected so that the design characteristic $y(x)$ will have a smaller slope. Owing to the smaller slope, the same spread of x will give rise to a *pdf* of y with a quite smaller spread (Fig. 1). Therefore, in this case, with the same components, i.e. the same uncertainty level and thus same costs, the system will exhibit a smaller uncertainty.

Analogous considerations can be carried out also for the influence parameters: n mean values $(\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)$ of the design parameters can be identified such that the uncertainty due both to design parameter uncertainty and to influence parameters is minimized.

3. THE PROPOSED PROCEDURE

The procedure is aimed at searching the minimum of the uncertainty of the measurement system by varying the design parameters without causing significant losses in other metrological characteristics (e.g. linearity, sensitivity, and so on).

Analogously as in previous works [14-16], the proposed procedure consists of 8 steps, grouped in 3 main phases: (i) *problem definition*, (ii) *model identification*, and (iii) *optimum prediction and result verification*.

3.1 Problem Definition

Step 1: Problem description. The problem is defined by identifying the architecture of the measurement system, the environment, the measurand, and their mechanisms of interaction.

Step 2: Uncertainty source identification. The most significant uncertainty sources are identified: they can derive from the uncontrolled variation of the influence parameters (outer uncertainty), or from the uncertainty of the design parameters (inner uncertainty). If necessary, preliminary experiments for assessing the magnitude of their effects and suitable limits to be imposed to the variations of the influence parameters are carried out.

Step 3: Definition of an objective function. During the search of the parameter design values corresponding to the minimum of the uncertainty of the measurement system, losses in other metrological characteristics have to be avoided. Therefore, during the search these characteristics have to be constrained to their nominal values. The problem turns out to be a multi-dimensional nonlinear constrained minimization, not easy to be solved. In the practice, it is usually simplified by defining a suitable objective function [1], [14]: its maximization involves automatically the reduction of the output uncertainty and the satisfaction of the constraints on other metrological characteristics. In the practice, an usual case consists of not only minimizing the output uncertainty over all the input range, but also simultaneously maximizing the sensitivity and the linearity of the system. In this case, the measurement system has as output:

$$y = f(\underline{x}, m) = B_0(\underline{x}) + B_1(\underline{x})m, \quad (2)$$

and the objective function will be:

$$\eta = 10 \cdot \log \left(\frac{\vartheta_{\beta} \beta^2}{\vartheta_{\sigma} \sigma_o^2 + \vartheta_{NL} \sigma_{NL}^2} \right), \quad (3)$$

where:

\mathbf{b} is the estimate of the coefficient B_1 in (2), obtained by a linear regression in the m domain of the function $f(\underline{x})$,

- σ_o^2 is the overall system variance, computed as the mean of the function $\sigma_*^2(m)$:

$$\sigma_o^2 = E[\mathbf{s}_*^2(\underline{x}, m)] \quad (4)$$

- σ_{NL}^2 is the variance of the linear regression error,

- and \mathbf{q}_b , \mathbf{q}_s , and \mathbf{q}_{NL} are multiplying coefficients belonging to the interval $[0,1]$, aimed at eventually privileging \mathbf{b} , \mathbf{s}_o^2 , or σ_{NL}^2 in the optimization.

The logarithm in (3) acts as a *linearizer*, for the selection of a linear model of \mathbf{h} during the successive phase. The (3) shows that by maximizing \mathbf{h} , the sensitivity (expressed by \mathbf{b}) will increase, while simultaneously the uncertainty \mathbf{s}_o and the nonlinearity \mathbf{s}_{NL} will decrease, according to the values imposed to \mathbf{q}_b , \mathbf{q}_s , and \mathbf{q}_{NL} .

Step 4: Identification of the design parameters. The design parameters are selected capable of acting significantly on the output variability, and statistically independent, in order to use a linear model for the objective function. The variation intervals for searching the maximum of \mathbf{h} are arranged on three levels (in the initial, the medium, and the final part of the range), in order to appreciate at least the curvature of \mathbf{h} .

3.2. Model Identification

Step 5: Experiment planning and execution. The whole experimentation of the combinatorial space of the design parameter levels could be burdensome. The hypothesis of design parameter independence and the linearizing action of the logarithm in the objective function allow experiment plans Resolution III to be used [1], [10], [16-17]. Among them, in the practice the plans of Genichi Taguchi are used [1]. Once the matrix for the design parameters (design matrix) has been selected, the experiments are carried out according to the defined protocol. In each combination of the parameter design defined by a row of the design matrix, the effects of the inner and outer uncertainty sources have to be experimented. Analogously as for the design parameters, these can be explored systematically by varying the influence parameters inside their variation intervals and the design parameters inside their uncertainty bands through a further experimental plan (uncertainty matrix). The uncertainty matrix is “nested“ into the design matrix, i.e. is executed for each row of the design matrix.

Step 6: Result analysis: ANOM e ANOVA. The hypothesis of design parameter independence and the linearizing action of the logarithm in (3) allow a discrete linear superimposed-effect model to be used:

$$\mathbf{h} = \mathbf{m} + \sum_{i=1}^n \mathbf{d}_{iq} + \mathbf{e} \quad (5)$$

where \mathbf{m} is the average of \mathbf{h} in all the experiments of the design matrix, \mathbf{d}_{iq} is the effect of x_i at the level q , n is the number of the design parameters, and \mathbf{e} is the model error. The effects \mathbf{d}_{iq} are estimated by means of the analysis of mean (ANOM). The error \mathbf{e} is assessed by means of analysis of variance (ANOVA).

In the ANOM, the mean \mathbf{m} is estimated by the N results of the design matrix:

$$\mathbf{m} = \frac{1}{N} \sum_{j=1}^N \mathbf{h}_j \quad (6)$$

The effects \mathbf{d}_{iq} are estimated as: $d_{iq} = m_{iq} - \mathbf{m}$, where m_{iq} are the averages of the results of the n_{iq} matrix experiments where the i -th design parameter occurs at the q -th level:

$$m_{iq} = \frac{1}{n_{iq}} \sum_{j=1}^{n_{iq}} \mathbf{h}_j \quad (7)$$

The model (3) of \mathbf{h} and the properties of the matrix imply that d_{iq} estimates δ_{iq} :

$$d_{iq} = \mathbf{d}_{iq} + \frac{1}{n_{iq}} \sum_{j=1}^{n_{iq}} \mathbf{e}_j \quad (8)$$

where \mathbf{e}_j is the error on \mathbf{h} in the j -th experiment. The \mathbf{d}_{iq} are derived by an average, thus the matrix not only reduces the experiment burden, but also the estimation uncertainty by $1/\sqrt{n_{iq}}$.

In the ANOVA, the statistical significance of the incidence of each design parameter on the objective functions is assessed: the variance of \mathbf{h} caused by the parameter is weighted to the uncertainty of the model (5). This is carried out by a Fisher test on the variance ratio:

$$F_i = \frac{\sigma_i^2}{\sigma_\epsilon^2} \quad (9)$$

where:

$$\sigma_i^2 = \frac{\sum_{q=1}^{n_w} n_{iq} \cdot d_{iq}^2}{\gamma_i} \quad (10)$$

$\mathbf{g} = n_w - 1$ denotes the degrees of freedom of the i -th parameters, \mathbf{s}_ϵ^2 is the error variance:

$$\sigma_\epsilon^2 = \frac{\sum_{j=1}^N \epsilon_j^2}{\gamma_\epsilon} \quad (11)$$

and \mathbf{g}_ϵ are the degrees of freedom of the error:

$$\gamma_\epsilon = \gamma_t - \sum_{i=1}^n \gamma_i \quad (12)$$

with \mathbf{g} the degrees of freedom of the total sum of the deviation squares. The error variance \mathbf{s}_ϵ^2 is derived from the *theorem of deviance decomposition* as:

$$\sum_{j=1}^N \mathbf{e}_j^2 = \sum_{j=1}^N \mathbf{h}_j^2 - N \cdot \mathbf{m}^2 - \sum_{i=1}^n \sum_{q=1}^{n_w} (n_{iq} \cdot d_{iq}^2) \quad (13)$$

where n_w is the level number of each design parameter, n_{iq} is the experiment number in the design matrix where the i -th parameter occurs at the q -th level. Given a confidence level, the parameters having negligible values of F_i will be neglected in the optimization.

3.3. Optimum Prediction and Result Verification

Step 7: Prediction of the optimum configuration. The optimum configuration is predicted through the model (5) by means of the design parameter effects maximizing the objective function (\mathbf{h}_{opt}).

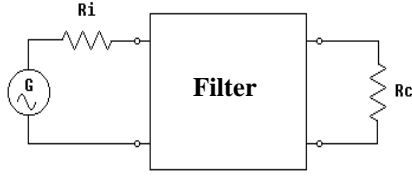


Fig. 2. The case study of the filtering circuit.

Step 8: Experimental verification. The prediction is verified by measuring the objective function value \mathbf{h}_{opt}^* in the optimum configuration in order to compute the prediction error: $\epsilon_p = |\eta_{opt} - \eta_{opt}^*|$. Then, ϵ_p is tested to be inside a confidence interval with amplitude related to the required optimization accuracy: $\epsilon_p \leq |z \cdot \sigma_p|$, where σ_p is the standard deviation of ϵ_p . The uncertainty \mathbf{s}_p has two independent sources: the limited number n_r of replication of the verification experiments and the uncertainty on the estimate of \mathbf{h}_{opt} :

$$\sigma_p^2 = \sigma_\epsilon^2 \cdot \left(\frac{1}{n_r} + \frac{1}{n_0} \right) \quad (14)$$

where n_0 is the size of the equivalent sample [14]:

$$\frac{1}{n_0} = \frac{1}{N} + \sum_{i=1}^{n_k} \left(\frac{1}{n_{iopt}} - \frac{1}{N} \right) \quad (15)$$

with n_k the number of significant parameters, and n_{iopt} the number of levels where the i -th parameter occurs at the optimum level in the matrix.

5. EXPERIMENTAL CASE STUDY

The proposed procedure is highlighted by referring to a pass-band filter Philips PM 6410, with a frequency range from 200 Hz to 2000 Hz, a -3 dB bandwidth with cut-off frequencies 800 Hz and 1500 Hz, and a center band frequency of 1000 Hz.

Step 1: Problem description. From the end-user point of view, the most important problem is the loss in the filter performance due to the actual behavior of the external circuit. The actual behavior has two main causes (Fig.2): (i) the actual working of the input circuit, described by an input level E and an equivalent resistance R_i , and (ii) the actual load working, described by the equivalent load resistance R_c . In an industrial application of the filter, these parameters derive from actual components of mass production, and, thus, they are affected by uncertainty (inner uncertainty). The user is interested in making the main dynamic metrological characteristic of the filter, namely the frequency response, as much independent as possible from this uncertainty. In particular, both the deterministic and the random deviations of the frequency response have to be reduced. Thus the response uncertainty has to be reduced by constraining also simultaneously the mean response as much close as possible to its nominal value. With this aim, a suitable combination of

the values of the above parameters has to be found according to the proposed basic idea.

Step 2: Uncertainty source identification. Owing to the use of low-cost components, let to be assumed that the uncertainty affecting the above parameters of the input and output equivalent circuits E , R_i , and R_c , is $\pm 5\%$ of their experimented values. Therefore, in the test protocol, their variation inside this band will be discretized on 3 levels depending on the corresponding value of the design parameters in the current experiment: (i) value -5% , (ii) value, (iii) value $+5\%$. In the experiments, this variation will be obtained through reference generator and variable resistors. The combinatorial variations of all of these levels are made systematic by an experimental plan. The parameters are assumed as statistically independent, thus a plan Resolution III L9 is considered for their variation inside the uncertainty bands (uncertainty matrix) [1]. It allows up-to 4 3-level independent parameters to be studied. The effects of possible influence parameters (outer uncertainty) are not relevant in the analysis. They are filtered in the experiments by carrying out averaged measurements of the filter frequency response.

Step 3: Definition of an objective function. The structure of the objective function is determined by the above mentioned necessity of minimizing the variance and the mean deviation from its nominal value of the frequency response over the interest bandwidth:

$$\eta = 10 \cdot \log \left(\frac{1}{\vartheta_\sigma \sigma_o^2 + \vartheta_\Delta \sigma_\Delta^2} \right) \quad (16)$$

where σ_o^2 is the mean of the attenuation variance

$$\sigma_j^2 = \frac{1}{n_u - 1} \sum_{i=1}^{n_u} (y_i - \tilde{y})^2 \quad (n_u=9 \text{ in the uncertainty matrix}),$$

observed in the j -th of the $n_m=10$ frequency points. The shift of the mean attenuation \tilde{y} from its nominal value \tilde{y}_{nom} is to be constrained according to a logic related to the filter use. Thus, it must weight more in the -3 dB band than in the remainder of the frequency range:

$$\sigma_\Delta^2 = \frac{\frac{1}{n_m} \sum_{j=1}^{n_m} P_j \cdot (y_{jnom} - \tilde{y}_j)^2}{\sum_{j=1}^{n_m} P_j} \quad (17)$$

where \tilde{y}_j is the mean of the actual attenuation samples at the j -th frequency, y_{jnom} the nominal value of the attenuation at the j -th frequency, and $P_j = y_{jnom} / y_{1kHz}$ is the weight adopted in the j -th frequency subrange, with y_{1kHz} the nominal attenuation at the centre-band frequency of 1.0 kHz. The multiplying coefficients in \mathbf{h} were assumed as $\mathbf{q}_\Delta = 0.75$ and $\mathbf{q}_0 = 1.00$, in order to privilege the uncertainty reduction.

Step 4: Identification of the design parameters. The design parameters are the external circuit parameters E , R_i , and R_c . As a matter of fact, the user will select their values capable of

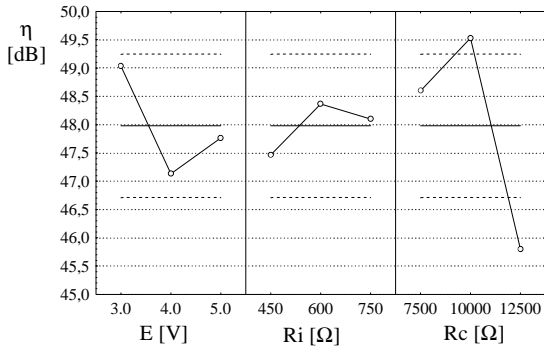


Fig. 3. The parameter effects on the objective function.

maximizing h . With this aim, a suitable range of values was selected for each of them according to the specifications of the filter manufacturer. In particular, analogously as for the uncertainty parameters, these ranges were discretized on three levels (Tab. I): (i) for E , 3.0, 4.0, and 5.0 V, (ii) for R_i , 450, 600, and 750 Ω , and (iii) for R_c , 7500, 10000, and 12500 Ω .

Step 5: Experiment planning and execution. The hypothesis of statistical independence of the design parameters leads again to the selection of a Resolution III plan. In particular, the need for experimenting 3 3-level parameters suggests an L9 plan (design matrix) also for the design parameters (Tab.I). The matrix has 4 columns and 9 rows, therefore for 3 parameters the last column will remain empty ("e"). This produces an excess of experimental information increasing the accuracy of the model identification. For each design matrix row, the values of the filtering circuit parameters are determined according to the level reported in the corresponding column. Then, 9 experiments are carried out by configuring correspondingly the uncertainty matrix. From their results, the values of the objective function reported in each row of Tab.I are computed according to (5).

Step 6: Result analysis: ANOM e ANOVA. The model (5) was identified according to the ANOM procedure by obtaining the parameter effects reported in Fig.3. Then, the estimate error was evaluated according to the ANOVA procedure. In Fig.3 the error limits on the effects are reported for a confidence level of 99.97%. Results of the ANOVA are reported in Tab.II. A Fisher test on the obtained values of F_i was performed. A prevailing impact of the load resistance R_c was surveyed, while the other two parameters turned out to be negligible. The variances of the negligible parameters

Tab. I – Experiment matrix L9 (e: empty column).

Exp. N°				η [dB]
	E [V]	Ri [Ω]	Rc [Ω]	
1	3.0	450	7500	48.5
2	3.0	600	10000	51.6
3	3.0	750	12500	47.0
4	4.0	450	10000	48.2
5	4.0	600	12500	44.7
6	4.0	750	7500	48.5
7	5.0	450	12500	45.7
8	5.0	600	7500	48.8
9	5.0	750	10000	48.8

Tab. II – ANOVA Results.

Param.	Freedom Degrees	Variance [dB ²]	F _i
E	2	2.8	2.6
R _i	2	0.7	0.6
R _c	2	11.3	10.3
Error	2	1.1	

were "pooled" in order to maximize the error [1], [14].

Step 7: Prediction of the optimum configuration. From Fig.3 the optimum configuration is argued easily: $E_1 = 3.0$ V, $R_{i2} = 600$ Ω , and $R_{c2} = 10000$ Ω . The optimum value $\eta_{opt} = 50.9$ dB was predicted by the model (5).

Step 8: Experimental verification. The optimum configuration was verified by using the uncertainty matrix to generate experimentally the uncertainty. Analogously as in the design matrix, the experiment was repeated 3 times according to the replication number of the array L9. The following results were obtained: $s_0^2 = 0.00000239$, $s_p^2 = 0.00001812$, and $\eta_{opt}^* = 48.0$ dB. The corresponding prediction error e_p is 2.9 dB. The verification test gave positive result: $2.9 \text{ dB} < 3s_p = 4.2 \text{ dB}$.

Consequently: (i) the parameter covariance is negligible and the linear model (5) is adequate, (ii) the filtering uncertainty is strongly influenced by the load resistance R_c , and (iii) the selection of the value of $R_c = 10000$ Ω , will minimize the impact of the actual working of the external circuit on the filter performance.

5. CONCLUSIONS

An application-independent cost-saving procedure for reducing the uncertainty of a measurement system has been proposed. The combination of the design parameters minimizing the uncertainty effects of both the influence parameters and the uncertainty of the design parameters is searched. The other metrological performances are simultaneously constrained to their nominal values by a suitable objective function. Experimental design techniques allows the experiment burden to be reduced.

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