IMEKO 24th TC3, 14th TC5, 6th TC16 and 5th TC22 International Conference
11 – 13 October 2022, Cavtat-Dubrovnik, Croatia

KIBBLE BALANCE FOR GRAM-LEVEL MASS MEASUREMENTS

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Abstract:

The redefinition of the kilogram in 2019 in terms of a fixed numerical value of a constant of nature, the Planck constant h, created the opportunity for the realisation of mass at any point on the scale and improvement in the uncertainty at sub-gram scales. In this work, part of the route to measurement of mass at small scales at the National Physical Laboratory (NPL) will be described. The aim is to miniaturise a system based on the planned NPL "Next Generation" Kibble balance to a simplified gram-level Kibble balance. This is a preparatory step for future work on a micro-electromechanical system (MEMS) scale Kibble balance.

Keywords: Kibble balance; kilogram; SI mass realisation; Planck constant

1. INTRODUCTION

Mass measurement is an essential part of daily life. Everything from taxation on commodities, to safety on aircraft, to the cost of groceries in supermarkets, to the dose of active ingredient in medicine, relies on accurate and precise mass measurements made with a high level of confidence.

The redefinition of the kilogram in terms of a fixed numerical value of a constant of nature, the Planck constant h [1], created the opportunity for an ongoing improvement in the confidence of mass measurement for decades to come. The new definition allows for the realisation of mass at any location and at any point on the scale, not just at 1 kg as was the case with the previous artefact-based definition.

To take full advantage of the benefits of the 2019 redefinition, new sensors and instruments that are linked to the Planck constant, either inherently or by calibration, need to be created. Several National Metrology Institutes (NMIs) are developing instruments capable of measuring small masses and forces, equivalent to 1 mg and below, using capacitive techniques [2]. At NPL, the aim is to use Kibble balance principles [3] to perform these measurements.

2. NPL DEMONSTRATION KIBBLE BALANCE

The NPL Demonstration Kibble Balance (Figure 1) was developed for the celebration of the redefinition of the kilogram in 2019. Its primary purpose was to serve as a tool for public engagement and, in the lead up to 20 May 2019, it was used to raise awareness of the significance of the redefinition of the kilogram by practical demonstration of the Kibble technique.

Figure 1: NPL Demonstration Kibble Balance showing the support frame (V), moving frame (Y), spherical mass (X), encoder read-head (Z), tare magnet and coil (T), weighing magnet and coil (W), and two disk flexures (U_1 and U_2)

The Demonstration Balance system was designed to be portable and comprise all the components of a working Kibble balance such as a magnet, coil, tare mechanism, guidance mechanism, current source, voltage measurement, velocity measurement, real-time computer, balance control software, and mass lift. The design was inspired by the planned NPL "Next Generation" Kibble balance

[3] and thus benefits from the advantages of reduced alignment errors [4].

The Demonstration Balance differs from the "Next Generation" balance in a few important ways. Firstly, it operates in air removing the need for a vacuum chamber. Secondly, voltage measurements are made by conventional electrical and resistance standards rather than requiring a Josephson Junction Array and Quantum Hall Resistance Standard held at the temperature of liquid helium (4.2 K). Thirdly, an encoder is used to make position and velocity measurements instead of a laser interferometer. Finally, spherical masses up to a nominal value of 50 g can be measured, five times less than specified for the "Next Generation" balance.

Post redefinition a secondary purpose for the Demonstration Balance has come to the fore. This system can act as a stepping stone to the measurement of smaller masses via the Kibble principles. The balance will be fitted with a bi-filar measurement coil, the control software will be updated to the same specification as the "Next Generation" balance, and a new simplified counter which takes continuous velocity measurements (to match the measurements taken by the voltmeter) has been developed. This approach to velocity measurement will be adopted to reduce the adverse effects of external vibration which is a common source of noise in Kibble balance measurement data [5].

The modified Demonstration Balance system has informed the design of a simplified, low cost, gram-level Kibble balance system aimed at measuring between 1 g and 10 g. The purpose of the gram-level Kibble Balance is to set the basis for future work on the development of a MEMS micro-Kibble balance with a target measurement range of 1 mg and below [6].

In this paper, the design of the gram-level Kibble Balance and its estimated performance based on an analytical model is presented.

3. NPL GRAM-LEVEL KIBBLE BALANCE

The gram-level Kibble balance is based closely on existing electromagnetic Kibble balances and therefore the established Kibble principles and equations can be used to analyse its expected performance.

3.1. Theory

Kibble balances operate in two modes, weighing and moving, and combine the results to calculate mass [4].

In weighing mode, the gravitational force acting on the mass M is balanced by an opposing electromagnetic force generated by current I in the weighing coil of length L in magnetic field B.

$$Mg = BLI \tag{1}$$

where g is local gravity.

In moving mode, a voltage V is induced in the weighing coil of length L by moving it through the magnetic field B at a known velocity u.

$$V = BLu \tag{2}$$

By assuming that the magnetic field B and the length of the weighing coil L are the same in both measurement modes, these quantities can be removed from the calculation of mass.

$$M = \frac{VI}{gu} \tag{3}$$

The gram-level Kibble balance operation will be implemented as described in [3].

3.2. Design

The aim of the gram-level Kibble balance is to be a simplified and low-cost system for realising mass in the range from 1 g to 10 g. To achieve these aims the Demonstration Balance system design was used as a baseline. Starting from a working system meant that the number of new parts to be designed and manufactured was significantly reduced. Some sub-systems and components were incorporated directly from the Demonstration Balance design including the real-time computer, software, electronics box, encoder, magnet, and magnet holder.

This paper will focus on the design of the components and sub-systems created specifically for the gram-level balance. These include the moving frame, support frame, guidance and counterbalance mechanism, and coils and are shown in Figure 2, Figure 3 and Figure 4.

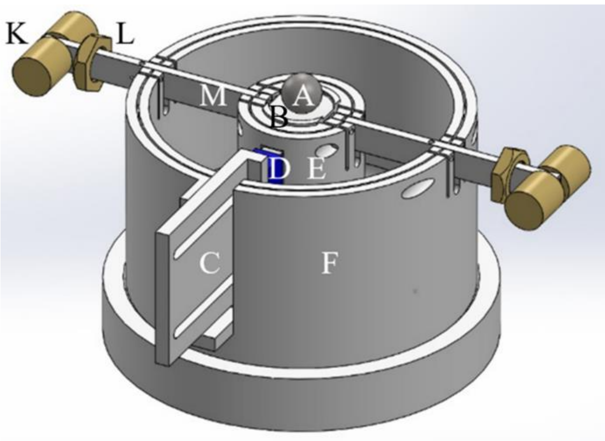

Figure 2: NPL gram-level Kibble Balance external view. The support frame (F), moving frame (E), balance pan (B), guidance rods (M), encoder mount (C), and a spherical test mass (A) are shown in grey. The encoder read-head (D) is shown in blue. The counterbalance tare masses (K), including fine adjustment nuts (L), are shown in yellow

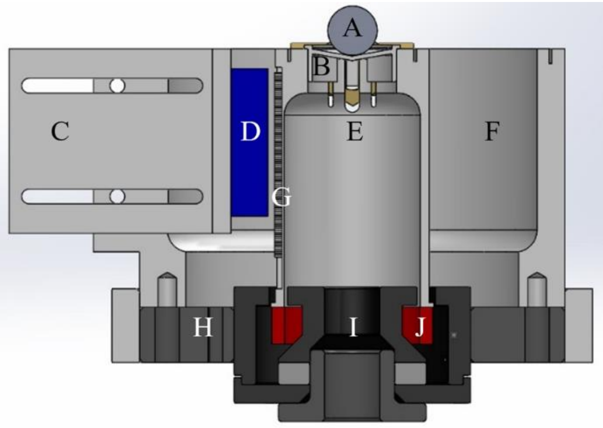

Figure 3: NPL gram-level Kibble Balance cross-section shown perpendicular to the guidance rods. The support frame (F), moving frame (E), balance pan (B), encoder mount (C), and spherical test mass (A) are shown in grey. The encoder read-head (D) is shown in blue and the encoder scale (G) is shown in black and white. The magnet (I) and magnet holder (H) are shown in dark grey and black respectively. The tare and weighing coils assembly (J) is shown in red

Magnet and Coils

The first simplifying design choice was to include only one magnet. The existing Demonstration Balance magnet was incorporated directly without any changes. Therefore, the new tare/drive coil and weighing coil were designed to fit inside the existing magnet gap and are wound one on top of the other forming a single 'two-coil' unit, shown in red in Figure 3 and Figure 4. Both coils are constructed from the same 0.26 mm diameter insulated copper wire however the drive/tare coil consists of a single strand of wire whilst the weighing coil is 'bi-filar' i.e., made from a twisted pair of wires. The two-coil unit is 10 mm in height and approximately 30 g in mass. It is 'self-supporting', with all wires secured together by an epoxy resin, and glued concentrically to the bottom face of the moving frame. The magnetic field B inside the gap of a typical Demonstration Balance magnet is approximately 0.17 T. The lengths of both coils were optimised to produce a BL product of approximately 1.5 T·m for the tare/drive coil and 2 T·m for the weighing coil.

Moving Frame

The moving frame is located inside the central structure (support frame) and supports the coils, encoder scale, and the mass pan. It is 3D-printed using generic PLA material with a fill factor of 15 % to keep the total mass of the moving frame assembly (coils, moving frame, encoder scale, mass pan) to approximately 80 g. The mass pan is conical to ensure spherical masses are positioned safely and centrally.

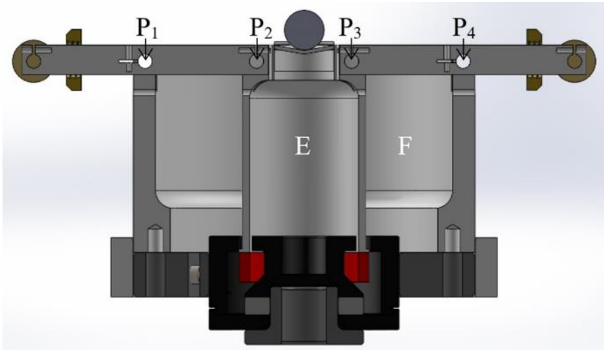

Figure 4: NPL gram-level Kibble Balance cross-section shown parallel to the guidance rods. The locations of four double-ended cross flexures are indicated by P_1 to P_4. The support frame (F) and moving frame (E) have symmetrical geometry, therefore the distance between P_4 and P_3 is equal to the distance between P_1 and P_2

Support Frame

The support frame is the outer support structure for the encoder read-head mount and the pivot points for the counterbalance mechanism. It is designed to fit on to the existing Demonstration Balance magnet holder which has the facility to adjust the relative position of the magnet with respect to the moving assembly. The support frame is also 3D-printed from generic PLA material.

Guidance and Counterbalance Mechanism

This system is based on lever arm principles and contains four double-ended cross flexures. There are two guidance rods that join the moving frame to the support frame and hold counterbalance masses, with a fine adjustment mass, outside the support frame. The support rods are 3D-printed using generic PLA material with a high fill factor for increased stiffness and rigidity. The counterbalance masses are brass cylinders mounted on a threaded rod and the fine adjustment masses are made from a sliced brass M12 nut. The flexures are inserted at the pivot points on the support frame and at the connections to the moving frame and are clamped into position using nylon bolts. This system will constrain the motion of the moving frame to the z-direction.

4. FEASIBILITY CALCULATIONS AND ESTIMATED PERFORMANCE

Equation (3) was adapted to produce a simplified analytical model.

$$M = \frac{V_\mathrm{w}\, V_\mathrm{m}\, \alpha}{R\, A\, g\, u} \quad (4)$$

where R is the resistance used to convert current to voltage, V_w is voltage measured during weighing mode, V_m is the induced voltage measured during moving mode, A is the gain required to amplify induced voltage V_m, α is an alignment factor due to any differences in the guidance mechanism performance between weighing and moving mode, g is local gravity and u is velocity of the weighing coil in moving mode.

This analytical model has been evaluated for masses in the range 1 g to 10 g based on the dimensions and characteristics of the gram-level Kibble balance design described in section 0.

4.1. Moving Mode

The tare/drive coil is used to generate the moving assembly motion during moving mode. If the weighing coil velocity u is chosen to be 1 mm s^{-1} then based on equation (2) a voltage V_m of approximately 2 mV will be induced in the weighing coil. To convert this voltage into the measurable range of the ADC (±10 V) an amplification A of 1000 is required, resulting in a measurable moving voltage V_m of approximately 2 V.

To ensure the weighing coil is kept inside the magnet during moving mode it is necessary to set its displacement range to ±2.5 mm from its zero, or weighing, position. Therefore, a maximum displacement of 5 mm results in a moving time of 5 s per direction.

4.2. Weighing Mode

The current I required to generate a force to oppose gravity acting on a mass in the range from 1 g to 10 g was calculated using equation (1) to be between 4.91 mA and 49.1 mA respectively. A resistance R of 100 Ω is required to convert these currents to a measurable voltage V_w in the range 0.49 V to 4.9 V.

The current carried in the weighing coil will result in power dissipated ranging from 1.87 mW at 10 g to 0.02 mW at 1 g.

4.3. Guidance and Counterbalance Mechanism

The torsional specification of the cross flexures selected for the guidance mechanism and distance between the guidance rod pivot point and its connection to the moving frame (indicated by P_1 to P_2 and P_4 to P_3 in Figure 4) have a significant impact on the performance of the balance.

$$\tau = -\kappa\,\theta \tag{5}$$

where τ is torque, κ is the total torsional spring constant of the flexures, and θ is the angular deflection of the rod.

To calculate the minimum detectable angle θ_{min} the small angle approximation can be assumed

$$\theta_{\mathrm{min}} \approx \sin\theta_{\mathrm{min}} \approx l_{\mathrm{e}}/l_r \tag{6}$$

where l_{e} is the encoder resolution and l_r is the distance between the pivot point on the support frame (P_1) and the connection on the moving frame (P_2).

As there are four cross flexures present in the system, κ can be assumed to be four times the specified torsional coefficient. The mass sensitivity of the balance design is determined by calculating the minimum force required to produce the torque generated by the minimum detectable deflection of the guidance rods.

$$m = \frac{\tau}{g\,l_r} = -\frac{4\,\kappa\,l_{\mathrm{e}}}{g\,l_r^2} \tag{7}$$

where m is the mass sensitivity and g is local gravity.

For the first prototype, the guidance mechanism has been designed with l_r = 50 mm and will include C-flex CD-20 double-ended cross flexures, each with a torsional coefficient of 38.84 mN·m·rad^{-1}. Combined with the encoder resolution l_{e} of 20 nm the resultant mass sensitivity m is 0.127 mg.

By combining equations (5) to (7) it is observed that the mass sensitivity m is directly proportional to torsional constant κ and inversely proportional to the square of the distance between the pivot point and the moving frame l_r^2. Changing the flexures to C-flex CD-10 with a torsional coefficient of 4.53 mN·m·rad^{-1} each is predicted to decrease m by a factor of 10, to 0.015 mg. Increasing l_r by 10 mm to 60 mm has a smaller impact on the mass sensitivity, decreasing m to 0.088 mg for CD-20 flexures and to 0.010 mg for CD-10 flexures.

During initial development work the CD-20 flexures will be used as their higher torsional coefficient suggests they are more robust and likely to withstand early testing.

4.4. Predicted Performance

The performance of the gram-level Kibble balance design has been predicted over the mass range 1 g to 10 g with the results plotted in Figure 5.

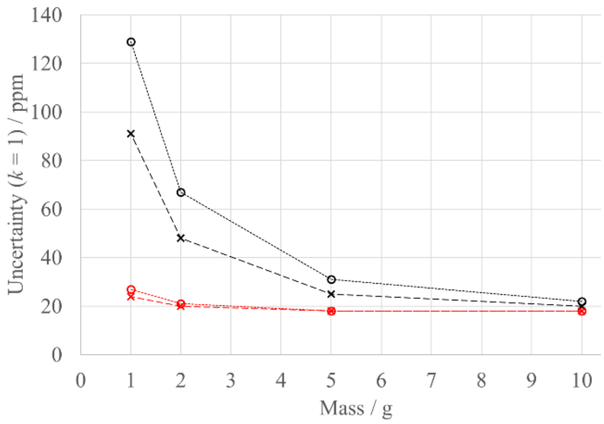

Figure 5: NPL gram-level Kibble balance predicted performance for different configurations of l_r and C-flex cross flexures. Black circles l_r = 50 mm and CD-20 flexures, black crosses l_r = 60 mm and CD-20 flexures, red circles l_r = 50 mm and CD-10 flexures, red crosses l_r = 60 mm and CD-10 flexures

The results show the relative advantages of adjusting the distance between pivot point and the moving frame l_r and upgrading the cross flexures to a higher sensitivity. For masses greater than 5 g there are minimal gains to be made by adjusting the properties of the guidance mechanism. However, for masses of less than 5 g, a large reduction in uncertainty can be achieved, for example an improvement by a factor of approximately 4.5 at 1 g, by upgrading the cross flexures to a higher sensitivity. For future work it is recommended to retain the dimensions of the guidance mechanism and upgrade the flexures to CD-10.

4.5. Uncertainty Budget

The results in Figure 5 show that the total uncertainty can be dominated by the torsional coefficient of the cross flexures. Once this contribution is reduced it appears that the total uncertainty is limited by the major contributions shown in Table 1. It has been assumed that local gravity will be calculated or modelled rather than measured directly. The total estimated uncertainty varies from 22 ppm at 1 g to 18 ppm at 10 g. Of the contributions listed in Table 1, weighing current is the most significant at 1 g with gravity, alignment, and moving voltage becoming the largest relative contributions at 10 g.

Table 1: Estimated uncertainty budget for NPL gram-level Kibble balance at 1 g including all major contributions except for mass sensitivity

Contribution	**Symbol**	**Uncertainty (k = 1) / ppm**
Local gravity	g	10
Alignment	α	10
Weighing current	$I = V_\mathrm{w}/R$	13
Moving voltage	$V = V_\mathrm{m}/A$	9
Velocity	u	5
	Total	**22**

Typical uncertainty for traditional E_1 mass calibration varies between 0.2 ppm at k = 1 for 1 g to 0.075 ppm at k = 1 for 10 g [7]. The estimated uncertainties for the gram-level Kibble balance show that this system, which utilises conventional electronics, cannot be competitive with existing traditional mass measurement in the range 1 g – 10 g. The intention of the gram-level Kibble prototype is to provide a route to the further miniaturisation of Kibble technology to the milligram level and below. At the milligram scale the relative uncertainty of currently available traditional mass measurement increases to a level which provides an opportunity for Kibble balances with conventional electronics [6].

5. SUMMARY

In this paper the design for a low-cost gram-level Kibble balance has been presented and evaluated analytically. The predicted results will inform the production and testing of a prototype at NPL. The analysis has also highlighted aspects of the design which will focus efforts on research and development to improve performance and guide further research on balances intended for measuring masses at the milligram level.

6. ACKNOWLEDGEMENTS

This work was funded by an Explorer Award from the National Physical Laboratory's Directors' Science and Engineering Fund and by the UK Government's Department for Business, Energy and Industrial Strategy (BEIS) through the UK's National Measurement System programmes.

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