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MEASURING THE INDUCED AC VOLTAGE IN THE PLANCK-BALANCE WITH AN AC QUANTUM VOLTMETER

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Abstract:

The Planck-Balance 1 (PB1) is a compact Kibble balance with the aim to calibrate weights of high quality. In contrast to most other Kibble balances PB1 employs a sinusoidal trajectory in the velocity mode, rather than a linear one. Thus, the induced voltage is an AC signal of 4 Hz. At PTB we implemented a commercial AC Quantum Voltmeter in order to ensure traceability to highest level. Here, we show the working principle along with some preliminary results.

Keywords: Planck-Balance; Kibble balance; AC Quantum Voltmeter; PJVS

1. INTRODUCTION

The Kibble balance is an experimental setup for realizing the SI unit of mass, the kilogram [1]. To this end the mechanical force due to a mass m under test is counterbalanced by means of a voice coil actuator with a magnetic flux density B of its permanent magnet and a coil length L. For a local gravitational acceleration g an equilibrium of the balance is reached when

$$m \times g = (B \times L) \times I \tag{1}$$

is satisfied, i.e. the electrical current I through the coil has to be adjusted, according to the weight. The product BL is termed *force factor* (or also *geometric factor*). In order to obtain an accurate value of m from equation (1) the force factor must be known to high accuracy. Following the idea of Bryan Kibble [2], it is to be determined in the so-called *velocity mode*. In this mode the voice coil system acts like a microphone, where the coil is moved through the magnetic field of the permanent magnet. This produces an induced voltage U across the coil ends, which is proportional to the relative velocity v between the magnet and the coil, according to Faraday's law, as

$$U = (B \times L) \times v. \tag{2}$$

When this force factor has been determined, then, in the so-called *force mode*, the value of a mass under test can be measured in a usual weighing procedure from equation (1). This is done by measuring the current with the mass on the weighing pan, and then without mass. The differential current is proportional to the mass.

In most Kibble balances the trajectory of the coil is linear, so that the induced voltage is a constant. This voltage signal is traceable to a Programmable Josephson Voltage Standard (PJVS). A linear trajectory, however, requires a long travel range of some millimetres to centimetres. In the set-up of Planck-Balance 1 [3] we have only less than 80 µm of total travel. This is why we have chosen to drive the coil with a sinusoidal trajectory. The idea of an 'oscillating Kibble balance' goes back to Chris Sutton, at Measurements Standard Laboratory (MSL), in New Zealand [4]. A similar method is also applied by the Turkish national metrology institute TÜBİTAK UME [5]. The advantage of a harmonic oscillation is that we can take continuous data over long-time span and use all data along the travel range. The disadvantage though is that the traceability of the determined induced voltage is more complicated. During the last years, however, much work has been done to accurately calibrate AC signals. The most accurate way is the use of an AC Quantum Voltmeter (ACQVM) [8]. The data analysis also is slightly different for an AC Kibble balance. Assuming a constant force factor, equation (2) becomes

$$\begin{aligned} U_0 \times \sin(2\pi \times f \times t) \\ = (B \times L) \times v_0 \\ \times \sin(2\,\pi \times f \times t) \end{aligned} \tag{3}$$

as the induced voltage, as well as the trajectory, becomes sinusoidal.

Neglecting higher harmonics in the signal, which arise mainly due to a non-constant force factor, the force factor is determined from the amplitudes of the induced voltage and the velocity. Those amplitudes are obtained by a three-parameter sine fit to the sampled data, where the frequency f is known very accurately, since the oscillation is driven with a stable frequency generator. A more detailed description of the analysis can be found in [6].

In this contribution we report on how we implemented such a voltage standard in PB1, and show some preliminary measurement results.

2. EXPERIMENTAL SETUP

When measuring a DC voltage signal to highest accuracy with a PJVS then the voltage output of the PJVS is adjusted as close as possible to the DC voltage under test. The voltage difference then is measured with a common digital voltmeter (DVM), e.g. a 3458A or nanovoltmeter.

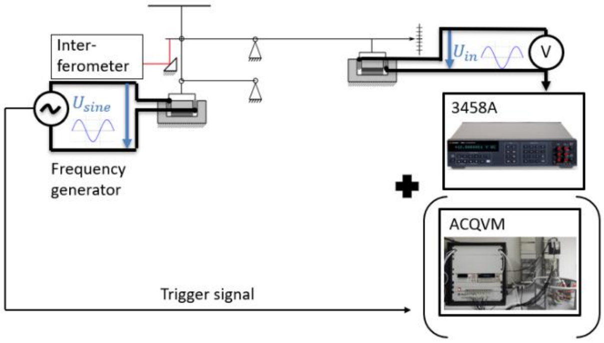

Figure 1: Measurement setup. The actuator coil is driven with a frequency generator, which also gives a trigger signal for synchronising the ACQVM

For an AC voltage signal the voltage level is continuously changing, so that the PJVS output voltage must be adapted with time, in order to keep the difference between the PJVS output and the voltage under test minimum. It is then possible to measure the difference voltage signal with a DVM. Such a continuously changing voltage level is realised in an ACQVM (see e.g. [7] and [8]). In our setup we employ a commercial AC Quantum Voltmeter from the company Supracon [9]. Its specified relative calibration uncertainty is 0.02 µV/V. As long as the ACQVM maintains the respective voltage level, the DVM samples the voltage difference and finally provides a mean value for this sampling interval. As the voltage is changing sinusoidally during this sampling time, the measured voltage amplitude U_{meas} will be lower than the true input voltage amplitude. This can be understood as the averaging acts like a filter. The induced bias can be analytically corrected by

$$U_{\text{corr}} = U_{\text{meas}} \times \frac{(\pi \times f \times t_{\text{int}})}{\sin(\pi \times f \times t_{\text{int}})} \qquad (4)$$

where t_{int} denotes the integration, or aperture time of the DVM.

The frequency f of the input signal in PB1 lies between 0.5 Hz and 10 Hz (but usually, and especially in this work, is 4 Hz). Such low frequencies can be easily handled by our ACQVM, which allows frequencies up to 2 kHz [10]. In order to get lowest differences between the output voltage of the ACQVM and the input voltage signal, the ACQVM needs to be synchronised with the input signal, regarding frequency, phase and amplitude.

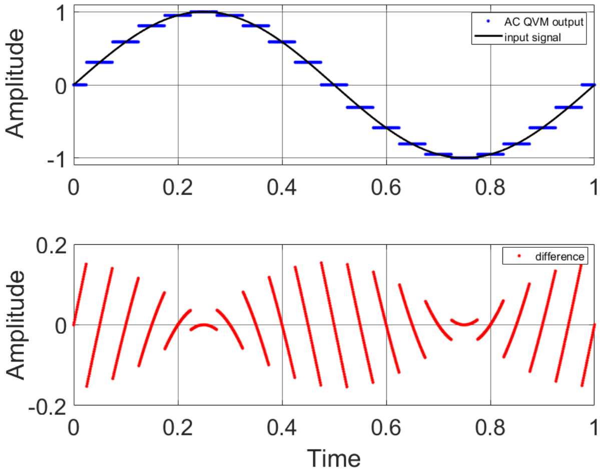

Figure 2: Sampling scheme with an ACQVM. The voltage level output must be synchronized with the input signal to optimise accuracy. The voltage output of the ACQVM is adjusted to the mean voltage of the specific range

In PB1 this is realised by employing a frequency generator (NI PXI 5412) that is locked to a stable 10 MHz reference standard (see Figure 1). This frequency generator acts as a voltage source which sets the coil into harmonic oscillation with an amplitude of about 23 µm. Simultaneously, the frequency generator sends trigger signals to the DVM, ACQVM, and the laser interferometer.

For the sake of performance comparability, measurements were performed with directly sampling the AC voltage signal, and later by including the ACQVM.

When measuring without an ACQVM, the induced voltage is sampled directly with a 3458A in DCV mode with a sampling frequency of 4 kHz. For this sampling frequency the aperture time is set to 2.27×10^{-4} s. The sampling lasts 10 s for each measurement cycle, thus including 40 oscillation cycles. By means of a three-parameter sine fitting we obtain the voltage amplitude of the coil [6]. The model function contains the first four higher harmonics of the fundamental oscillation frequency. The amplitude of the fundamental note is taken as induced voltage U of equation (2).

Simultaneously, a laser interferometer measures the coil position as a function of time, with a sampling frequency of 4 kHz. In a similar manner, the motion amplitude is obtained by a three-parameter sine fitting, including the same higher harmonics. The coil velocity v then is obtained by multiplication of the position amplitude of the fundamental note by the factor $2\,\pi\,f$.

When including the ACQVM, the number of voltage steps per cycle is set to 127. Over each voltage step a 3458A is employed to take data and return the mean voltage difference value for this respective voltage level. The data acquisition is triggered by the ACQVM, which automatically sets the aperture time of the 3458A to 3/5 of the timer interval. That means that 1/5 at the beginning and the end of the timer interval are skipped. To be more specific, if the signal frequency is 4 Hz, and the number of voltage steps of the ACQVM is set to 250, then the timer interval becomes 1 ms, and the aperture time 600 µs. 30 such oscillation cycles were measured in each measurement. Sampling frequency is the same as for a measurement without ACQVM (see above). The deletion of certain points at the beginning and end of each step is necessary to avoid systematic errors due to parasitic ringing [10].

For all measurements the oscillation of the coil is in steady state, and the first few cycles are omitted in order to avoid transients in the data. Each measurement campaign lasted at least six hours. For a better comparability, all data were corrected to 23 °C with a common temperature coefficient of -0.066 236 T·m·K^{-1} of the SmCo-magnet of the voice coil actuator.

3. RESULTS

The study has to be considered preliminary, and mainly focuses on a performance test of the ACQVM implementation. As mentioned above, in these investigations we compared two sorts of measurement. The first one consisted of a direct measurement of the input voltage by means of a DVM of type 3458A (from Keysight), that has been calibrated at the desired voltage range of 1 V against the PJVS. The voltage is sampled in DCV mode.

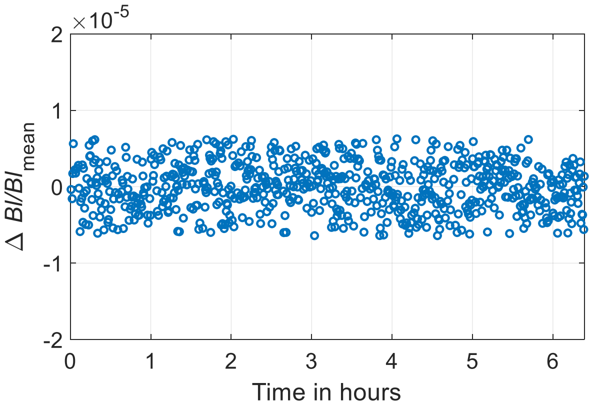

Figure 3: Example of measurement campaign for a direct sampling with 3458A. The data have been corrected for temperature changes

In the second measurement campaign a similar DVM is taken to measure the difference signal between the input voltage and the synchronized voltage steps provided by the ACQVM. Also, this DVM was calibrated against the PJVS beforehand. The use of different DVMs had to be done for convenience.

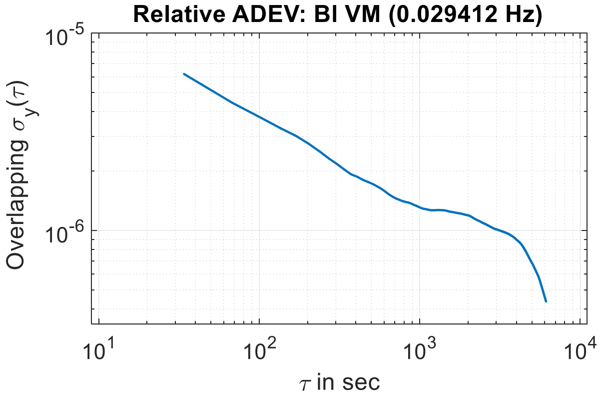

Figure 4: Overlapping relative Allan deviation for the measurement of Figure 3. Sampling with 3458A in DCV mode

Figure 3 shows an example measurement of a direct sampling with a 3458A, after temperature correction. The calculated relative standard deviation is 3.11 × 10^{-6}. The time span between two measurements is about 34 s. Figure 4 shows the relative Allan deviation for the measurement in Figure 3. After one hour of measurement a relative Allan deviation of better than 1 × 10^{-6} can be reached.

For the measurement including the ACQVM the standard deviation is bigger. In the example, depicted in Figure 5 and Figure 6, the relative standard deviation is 6.14 × 10^{-6}, with a relative Allan deviation after one hour of measurement that is slightly bigger than 1 × 10^{-6}. Also, the time between two measurements is by about 10 % higher than when directly sampling with 3458A.

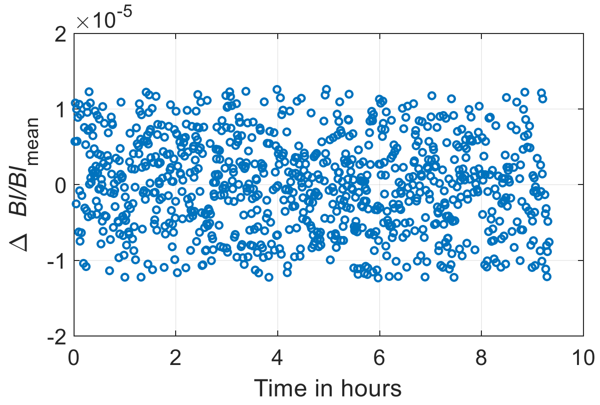

Figure 5: Example of measurement campaign for a differential sampling where the test signal was compensated by the ACQVM. The data have been corrected for temperature changes

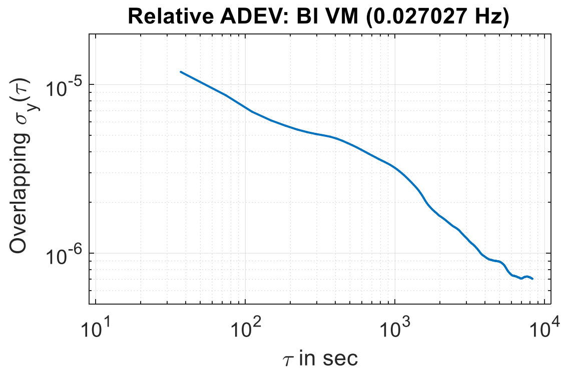

Figure 6: Overlapping relative Allan deviation for the measurement of Figure 5

For both cases a linear three parameter sine fitting algorithm was applied to the sampled voltage values, resulting in an estimate for the signal amplitude. In both cases a correction, as shown in equation (4), was applied after sine fitting procedure. The frequency taken in the sine fitting is the same as the set frequency of the signal generator.

We always used the induced voltage in the velocity mode with PB1 at a signal frequency of 4 Hz. The voltage level was of about 68 mV, and the signal had a total harmonic distortion (THD) of about 1.44×10^{-3}, for the direct measurement with 3458A, and 9.13×10^{-4} for the measurement with the ACQVM.

The quantity to be compared in this investigation is the force factor BL, rather than the pure voltage amplitude. Figure 7 shows results of the measurements, 5 of them (in red) with a direct measurement and 4 (in blue) integrating the ACQVM. Each measurement took at least 6 hours. For the calculation of BL also the interferometer data were corrected for the change in the refractive index, as the measurement took place in air, and all values were corrected to a nominal temperature of 23 °C. This is necessary, since the relative temperature coefficient of the magnet is about $3 \times 10^{-4}\,\mathrm{K}^{-1}$ and the laboratory is not actively temperature stabilised.

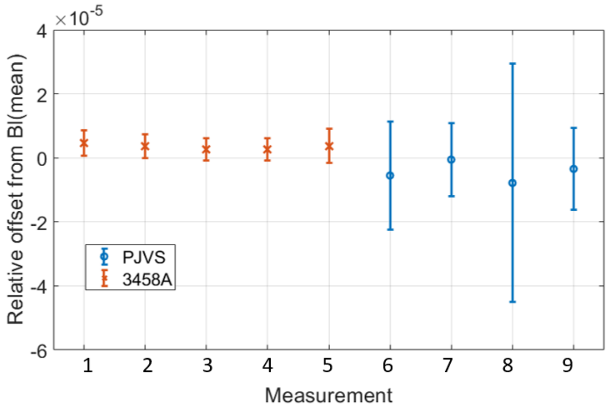

Figure 7: Measurement results (not in chronological order). Error bars show standard deviations

In Figure 7 it can be observed that the measurements with ACQVM show a trend to be lower than direct measurements with the 3458A. The reason is not clear and further investigations are under way to clarify this inconsistency. Table 1 shows a summary of the main measurement parameters.

Table 1: Comparison of direct sampling with 3458A and with ACQVM included

	3458A	**ACQVM**
Relative standard deviation	3.11×10^{-6}	6.14×10^{-6}
Time between measurements	34 s	37 s
Relative Allan deviation after 1 h	9×10^{-7}	1×10^{-6}
THD	1.44×10^{-3}	9.13×10^{-4}
Induced voltage amplitude	68 mV	68 mV
Amplitude of coil trajectory	12 µm	12 µm

4. DISCUSSION

The implementation of the commercial ACQVM was straight forward. However, additional software was necessary from the company with additional cost. Nevertheless, it was relatively easy to integrate in our control code, which is written in NI LabVIEW language.

It was noted that the measurement with the ACQVM shows higher standard deviation. This is attributed to the lower number of sampling points per cycle. After a measurement time of an hour, however, both measurement methods have shown the same Allan deviation, so that there is effectively no loss in performance. The metrological gain, on the other side, is enormous. The AC measurement is now directly traceable to the voltage standard, providing lowest measurement uncertainties, even in AC mode.

Currently we are evaluating the measurement uncertainty of the AC voltage measurement in the Planck-Balance. Therefore, we are running various experiments with different frequencies, amplitudes and voltage steps per cycle. As a source we take a highly stable frequency generator with low THD. Here, we investigate only the voltage measurement, without running the PB1, and without determination of the force factor. It was also noticed that the step number of 127 was not ideal. Ideally, the number of steps per cycle should be an integer multiple of the internal clock frequency of the AWG1104 bias source, which is typically 200 MHz [10]. This might be a reason for the systematic offset between ACQVM and 3458A measurements.

In a next step the velocity measurements will be combined with the force mode measurements, where both modes employ the Josephson standard.

5. SUMMARY

It was shown that an ACQVM was successfully implemented in the velocity mode of the Planck-Balance. This allows for a traceability of the voltage measurement on highest accuracy level. Although showing a higher standard uncertainty, a measurement with an ACQVM does not provide a loss in performance, when compared the Allan deviation after one hour of measurement time.

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