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INFLUENCE OF COORDINATE SYSTEM ALIGNMENT ON THE CALIBRATION OF MULTI-COMPONENT FORCE AND MOMENT SENSORS

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Abstract: For the calibration of multi-component force and moment sensors no national or international standard exists. In different approaches for calibration procedures, a precise alignment of sensor and calibration reference is required. The influence of such an alignment, however, is not mentioned.

Based on a model of a calibration setup and an assumption of possible misalignments, an analysis of the systematic error, caused by such misalignment, is performed. Additionally, a strategy to identify the necessary coordinate transformation is given.

Keywords: force, moment, multi-component, calibration

1. INTRODUCTION

The measurement of forces and moments is a standard procedure in different fields of engineering and science. Typical sensors used are only sensitive to forces or moments acting in one direction. Multi-component sensors (MCS) extend the functionality of force or moment measurements by detecting those components in multiple directions at the same time.

A drawback of the vector characteristic of forces and moments is the mandatory description of the different components in an appropriate coordinate system. In a MCS, this sensor coordinate system is defined by the design and the electrical wiring of the sensing elements inside the sensor. Depending on that design and on the accuracy of the positioning of the sensing elements, the coordinate systems for forces and moments may be different from each other inside one single sensor.

As a result of the described vector characteristics, a transformation of coordinate systems is necessary if the values detected by the sensor are to be compared to values from external sources or evaluated in a test setup, for example [1].

Especially for the calibration process, which aims at identifying the characteristics of the sensor, the transformation between the sensor coordinate system and the coordinate system of the calibration device needs to be identified as accurately as possible.

At the Physikalisch-Technische Bundesanstalt (PTB), the National Metrological Institute of Germany, a calibration device for MCS is installed, which covers a measurement range for forces of up to 10 kN and for moments of up to 1 kN·m and reaches a relative measurement uncertainty of $\leq 2.2 \cdot 10^{-4}$ with certain exceptions [2]. To fully use the potential of the machine, the uncertainty introduced by the coordinate system alignment should be in the same order of magnitude.

2. INFLUENCE OF THE COORDINATE SYSTEM ALIGNMENT

To analyse the influence of the coordinate system alignment, a fictional 6 component sensor with identical sensing magnitudes for the different force and moment components is assumed ($F_x=F_y=F_z=F$; $M_x=M_y=M_z=M$). Both values, F and M, are observed independently. In **Fig. 1** two coordinate systems, one of a calibration device (a) and one of a sensor (b), are shown in a simplified, two-dimensional representation. The coordinate systems are displaced by Δx and Δy and rotated in xy-plane by an angle θ. The following analysis can easily be extended to the third dimension.

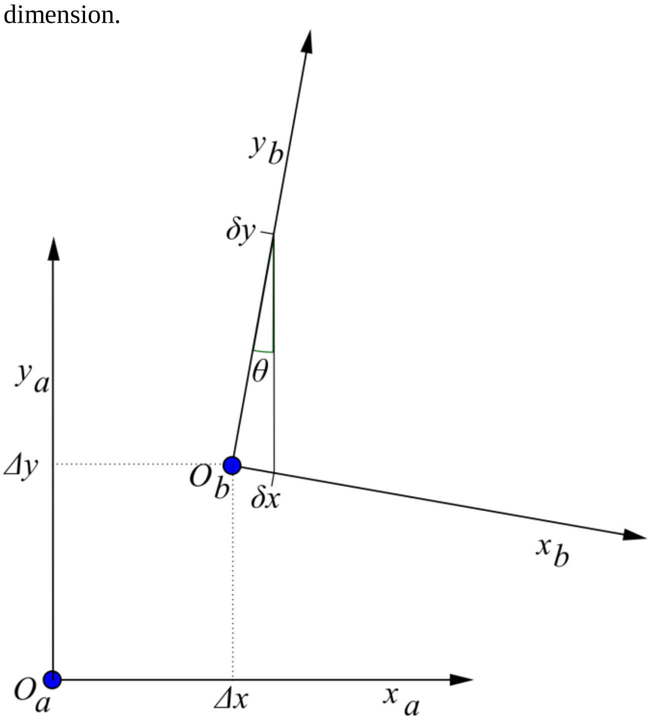

Fig. 1 coordinate systems of a calibration device (a) and a sensor (b)

Force analysis: The translation of the coordinate systems does not have an effect on the resulting amplitude or direction of the force vector. The rotation, however, needs to be identified for an accurate force analysis.

We assume a misalignment of θ, which represents a deviation of

$$\delta x / \delta y = \tan(\theta) . \qquad (1)$$

For a force $F_{ya} = F$ in the coordinate system of the calibration device, this misalignment results in a F_x component in the sensor coordinate system of

$$F_{xb} = \sin(\theta) \cdot F . \qquad (2)$$

Regarding the stated uncertainty of the calibration device of $\leq 2.2 \cdot 10^{-4}$, the uncertainty for the F_{xb} component, introduced by misalignment, should be in the same range. Hence, the rotation angle should be determined with an uncertainty of

$$\delta\theta \leq \arcsin(2.2 \cdot 10^{-4}) = 0.0126° . \qquad (3)$$

Moment analysis: The analysis performed for forces applies identically for moment vectors, as long as they are not based on a force-lever combination. A moment $M_{ya} = M$ in the reference coordinate system results in a component $M_{xb} = \sin(\theta) \cdot M$ in the sensor coordinate system.

If a force $F \neq 0$ is present in the system, the translation of the coordinate systems needs to be observed in addition to the rotation, as this translation represents a lever arm in one coordinate system. Depending on the construction type of the sensor, the nominal values for forces and moments may differ significantly. The parameter *moment-force-relation* describes this difference as a quotient of the nominal moment and the nominal force *M/F*. For typical commercially available MCS the moment-force-relation is in a range of $M/F = 0.1$ N·m/N to $M/F = 0.01$ N·m/N [3]. We assume the presence of only one force component $F_{ya} = F$ in the calibration coordinate system, applied to the origin O_a, a misalignment of $\Delta x = \Delta y$ and a moment-force-relation of $M/F = 0.1$ N·m/N. For this moment-force-relation, the misalignment has the lowest influence on the moment reading. From the deviation of Δx perpendicular to F, a moment M_{Ob} around the origin of the sensor coordinate system results:

$$M_{Ob} = F \cdot \Delta x . \qquad (4)$$

Again, the uncertainty for the M_{Ob} component, introduced by misalignment, should be in the same range as the stated uncertainty of the reference system. Including the moment-force-relation, the deviation Δx needs to be identified with an accuracy of

$$\delta x \leq 2.2 \cdot 10^{-4} \cdot M / F = 2.2 \cdot 10^{-5} m . \qquad (5)$$

If the sensor to be calibrated shows a moment-force-relation of $M/F = 0.01$ N·m/N, the influence of the misalignment increases by a factor of 10.

3. DEVIATION OF INTERNAL COORDINATE SYSTEMS

In the analysis of the influence of the coordinate system alignment, force and moment coordinates were observed independently. When using a MCS, it would be expected for those coordinate systems to be identical. However, based on the production process, it may happen that the sensing elements for forces and moments differ slightly in position or orientation which leads to different positions and orientations of the coordinate systems.

We can distinguish between the coordinate system on the basis of the sensing elements *U* and the calibrated coordinate system *V*. The coordinates described in *U* contain the sensor output signals U_{Fs} and U_{Ms} (typically in mV/V) while the coordinates in V describe the calibrated force and moment values V_{Fs} and V_{Ms} (in N and N·m). To transform the sensor output signals to the calibrated force signals, a cross talk compensation matrix *C* is frequently used:

$$\begin{bmatrix} V_{Fs} \\ V_{Ms} \end{bmatrix} = C \cdot \begin{bmatrix} U_{Fs} \\ U_{Ms} \end{bmatrix} . \qquad (6)$$

For a six component sensor, the matrix *C* consists of 6 × 6 parameters, including the sensitivity of the main components.

The matrix *C* can be split up into four matrices C_{FF}, C_{FM}, C_{MF}, C_{MM} of 3 × 3 parameters each:

$$C = \begin{bmatrix} C_{FF} & C_{FM} \\ C_{MF} & C_{MM} \end{bmatrix} . \qquad (7)$$

Each of the sub-matrices can be interpreted as a coordinate transformation of input values to output values. This transformation can include scaling *S* and rotation *R* among additional factors *A*:

$$C_{xy} = S \cdot R \cdot A . \qquad (8)$$

For the matrices C_{FF} and C_{MM}, *S* represents the sensitivity of the main components, *R* contains the rotation between the sensor coordinate system and the calibrated coordinate system and *A* includes additional factors like cross talk. In the matrices C_{FM} and C_{MF}, *R* compensates a rotation between the force and moment coordinates in *U* while *S* and *A* contain the crosstalk influence of force loads on moment readings and vice versa.

This shows, that the cross talk compensation matrix contains information about the rotational alignment between the different coordinate systems of the sensor (C_{FM}, C_{MF}), as well as the rotation between the sensor coordinate system and the calibration coordinate system (C_{FF}, C_{MM}).

As a result of that fact, the procedure to determine the parameters of the compensation matrix has an influence on the resulting accuracy of the sensor. If a systematic error, i.e. based on secondary components, is included in the calibration process, it may seem as if the sensor shows a systematic error which will be compensated by the parameters in the cross talk compensation matrix. A possible source for such systematic errors may be the separate calibration of force and moment parameters in different calibration devices. During dismounting and remounting of the sensor, the alignment may change which would be interpreted as misalignment of the internal coordinate systems.

This fact was already mentioned by Schwind [3] for the calibration of MCS in dead weight standard machines. But it also appears when calibrating MCS using a reference standard with misaligned coordinate systems.

4. PROPOSED ALIGNMENT STRATEGY

As mentioned in the introduction, the identification of the position of the coordinate systems using geometry parameters can be difficult. The accuracy of the given coordinate system of each device depends on the production inaccuracies of the device, the alignment of the sensor in the calibration setup depends on production inaccuracies of different mounting surfaces and adapting elements. The coordinate systems themselves may not be contacted by a measurement device, as they do not necessarily have a physical implementation, which means that the point of origin, as well as the different axis orientations, can be anywhere inside or outside the sensor.

The coordinate systems, however, are inherent in the output values of the different components of the sensor and the calibration device. It seems reasonable to use that information to identify the necessary coordinate transformation from the calibration coordinate system to the sensor coordinate system.

The relationship between the sensor output values and the reference measurement values can be described by the following equation:

$$\begin{bmatrix} V_{Fs} \\ V_{Ms} \end{bmatrix} = \kappa \cdot \begin{bmatrix} V_{FR} \\ V_{MR} \end{bmatrix} \qquad (9)$$

where V_{Fs}, V_{Ms} represents the sensor output values, κ a transformation matrix and V_{FR}, V_{MR} the reference values.

The transformation matrix κ describes the coordinate system transformation between sensor coordinate system and reference coordinate system and consists of a rotation matrix R and a translation matrix T:

$$\kappa = R \cdot (I + T) \,. \qquad (10)$$

In this equation I represents a 6×6 identity matrix, R describes the consecutive rotation around z (R_θ), y (R_ψ) and x (R_φ):

$$R = \begin{bmatrix} R_\varphi \cdot R_\psi \cdot R_\theta & 0 \\ 0 & R_\varphi \cdot R_\psi \cdot R_\theta \end{bmatrix} \qquad (11)$$

with

$$R_\varphi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix}, \quad R_\psi = \begin{bmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{bmatrix}, \quad R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \qquad (12)$$

T represents the force component influence on the moment readings, based on three translations δx, δy and δz:

$$T = \begin{bmatrix} 0 & 0 \\ T_t & 0 \end{bmatrix} \qquad (13)$$

with

$$T_t = \begin{bmatrix} 0 & -\delta z & \delta y \\ \delta z & 0 & -\delta x \\ -\delta y & \delta x & 0 \end{bmatrix}. \qquad (14)$$

Including (6) and (10) in (9), the equation describing the relationship between sensor output U and reference values changes to:

$$C \cdot \begin{bmatrix} U_{Fs} \\ U_{Ms} \end{bmatrix} = R \cdot (I + T) \cdot \begin{bmatrix} V_{FR} \\ V_{MR} \end{bmatrix} \qquad (15)$$

Assuming the existence of the inverse of C, (15) can be rewritten:

$$\begin{bmatrix} U_{Fs} \\ U_{Ms} \end{bmatrix} = C^{-1} \cdot R \cdot (I + T) \cdot \begin{bmatrix} V_{FR} \\ V_{MR} \end{bmatrix}. \qquad (16)$$

If C is known, the six transformation parameters φ, ψ, θ, δx, δy and δz can be calculated from one measurement of all six components by iterative optimization, as the functions are non-linear based on the sine and cosine terms of the rotation. First results of a measurement using a commercially available MCS show good convergence for R and T, the relative deviation between reference and measured values after the calculated coordinate transformation is $< 1 \cdot 10^{-3}$.

On the other hand, if the rotation R and the translation T are known, the matrix C can be calculated from a set of six independent measurements either by matrix multiplication directly from the measurement:

$$C = R \cdot (I + T) \cdot V \cdot U^{-1} \qquad (17)$$

or by iterative optimization of the parameters of C to solve the equation

$$R \cdot (I + T) \cdot V - C \cdot U = 0\,. \qquad (18)$$

Experimental results using the aforementioned MCS are in good agreement for both methods with relative deviations of $< 1 \cdot 10^{-5}$ for C.

In the typical case of a calibration of a MCS, the values of the different elements of the cross talk compensation matrix C are to be identified. Hence, C is not known and can therefore not be used to determine the coordinate transformation if it is unknown.

In (16), the term $C^{-1} \cdot R \cdot (I + T)$ contains all 42 unknown parameters (36 parameters of C, 3 of R, 3 of T) in a 6×6 matrix. Those parameters are not independent and the equation system cannot be solved.

To overcome this issue, the orientation of the sensor can be changed which results in a second set of measurements where R and T will be changed while C stays constant as its inherent to the sensor:

$$\begin{bmatrix} U_{Fs2} \\ U_{Ms2} \end{bmatrix} = C^{-1} \cdot R_2 \cdot (I + T_2) \cdot \begin{bmatrix} V_{FR2} \\ V_{MR2} \end{bmatrix}. \qquad (19)$$

From this second set of six independent measurements, 36 additional equations are obtained while only six additional parameters need to be solved. This results in an overall equation system of 72 equations and 48 parameters, which can be solved by iterative optimization.

A verification of the proposed alignment strategy is currently being performed. If the proposed strategy proves to be aplicable, an analysis of sensitivity will be added to calculate the uncertainty introduced by this approach.

5. SUMMARY

For the calibration of multi-component sensors, the precise identification of the coordinate transformation from the calibration facility coordinate system to the sensor coordinate system plays an important role. The influence of external and internal misalignment was observed and resulting deviations on the calibration result were estimated.

From different possible sources for such misalignments and the difficulties to identify them, an alignment strategy was developed. It makes use of the output values of the sensor and the reference system to fit the necessary parameters which describe the coordinate transformation.

Drawbacks of the proposed strategy are discussed and further investigation subjects are identified in order to perform a complete calibration of a sensor.

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