IMEKO 22nd TC3, 15th TC5 and 3rd TC22 International Conferences
3 to 5 February, 2014, Cape Town, Republic of South Africa

MODEL PARAMETER IDENTIFICATION FROM MEASUREMENT DATA FOR DYNAMIC TORQUE CALIBRATION

Leonard Klaus[1], Barbora Arendacká[2], Michael Kobusch[1], Thomas Bruns[1]

[1]Physikalisch-Technische Bundesanstalt (PTB), Bundesallee 100, 38116 Braunschweig, Germany
[2]Physikalisch-Technische Bundesanstalt (PTB), Abbestraße 2-12, 10587 Berlin, Germany
Email: leonard.klaus@ptb.de

Abstract: For the dynamic calibration of torque transducers, a model of the transducer and an extended model of the mounted transducer with the measuring device have been developed. The dynamic behaviour of a torque transducer is described by model parameters. This paper describes the model with the known and unknown parameters and how the calibration measurements are going to be carried out. The principle for the identification of the transducer's model parameters from measurement data is described using a least squares approach.

Keywords: model parameter identification, dynamic torque calibration, dynamic measurement, mechanical model.

1. INTRODUCTION

Research in the field of the dynamic calibration of torque transducers is being carried out in the context of the European Metrology Research Programme (EMRP) Joint Research Project IND09 "Traceable Dynamic Measurement of Mechanical Quantities" [1]. An existing prototype measuring device [2] was modernised and extended, and a model-based description of the dynamic behaviour of torque transducers was developed [3]. For future dynamic torque calibrations, it will be necessary to be able to identify those model parameters of a transducer to be calibrated from measurement data.

2. MODEL

The model of the torque transducer is linear and time invariant (LTI) and consists of two mass moment of inertia elements connected by a torsional spring and a damper in parallel.

Torque transducers are always coupled on both ends to their mechanical environment, which may have influence on the transducer's dynamic behaviour. To be able to include these effects in the model-based description of the dynamic behaviour of the transducer, it was necessary to extend the model of the transducer to a model of the mounted transducer with the dynamic torque measuring device (i.e. the mechanical environment in case of a calibration).

This extended model represents the physical components of the measuring device and of the transducer under test and assumes LTI behaviour as well (see Fig. 1). It consists of elements for the mass moment of inertia (MMOI), the torsional spring and the torsional damper. The equation of motion is described as an inhomogeneous system of ordinary differential equations:

$$\boldsymbol{J}\,\ddot{\boldsymbol{\varphi}} + \boldsymbol{D}\,\dot{\boldsymbol{\varphi}} + \boldsymbol{C}\,\boldsymbol{\varphi} = \boldsymbol{M}\,. \tag{1}$$

In this equation $\boldsymbol{J}$ denotes the mass moment of inertia matrix, $\boldsymbol{D}$ denotes the damping matrix, $\boldsymbol{C}$ the stiffness matrix and $\boldsymbol{\varphi}$ the angle vector and its derivative vectors ($\ddot{\boldsymbol{\varphi}}$, $\dot{\boldsymbol{\varphi}}$), respectively. The forced excitation is described by $\boldsymbol{M}$.

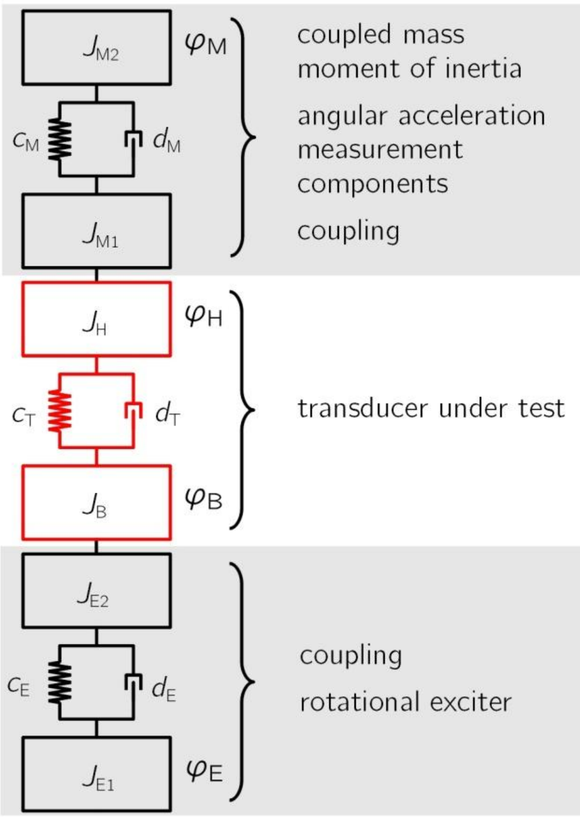

Figure 1: Model of the dynamic torque calibration device (marked in black) including the transducer under test (marked in red).

For the described model as depicted in Fig. 1, the model approach leads to a mass moment of inertia matrix J of

$$\boldsymbol{J} = \begin{bmatrix} J_{\mathrm{M2}} & 0 & 0 & 0 \\ 0 & (J_{\mathrm{M1}} + J_{\mathrm{H}}) & 0 & 0 \\ 0 & 0 & (J_{\mathrm{B}} + J_{\mathrm{E2}}) & 0 \\ 0 & 0 & 0 & J_{\mathrm{E1}} \end{bmatrix} , \tag{2a}$$

a damping matrix D of

$$\boldsymbol{D} = \begin{bmatrix} d_{\mathrm{M}} & -d_{\mathrm{M}} & 0 & 0 \\ -d_{\mathrm{M}} & (d_{\mathrm{M}} + d_{\mathrm{T}}) & -d_{\mathrm{T}} & 0 \\ 0 & -d_{\mathrm{T}} & (d_{\mathrm{T}} + d_{\mathrm{E}}) & -d_{\mathrm{E}} \\ 0 & 0 & -d_{\mathrm{E}} & d_{\mathrm{E}} \end{bmatrix} , \tag{2b}$$

and the corresponding stiffness matrix C

$$\boldsymbol{C} = \begin{bmatrix} c_{\mathrm{M}} & -c_{\mathrm{M}} & 0 & 0 \\ -c_{\mathrm{M}} & (c_{\mathrm{M}} + c_{\mathrm{T}}) & -c_{\mathrm{T}} & 0 \\ 0 & -c_{\mathrm{T}} & (c_{\mathrm{T}} + c_{\mathrm{E}}) & -c_{\mathrm{E}} \\ 0 & 0 & -c_{\mathrm{E}} & c_{\mathrm{E}} \end{bmatrix} . \tag{2c}$$

The angle vector and its derivative vectors $\boldsymbol{\varphi}$, $\dot{\boldsymbol{\varphi}}$, $\ddot{\boldsymbol{\varphi}}$ are given by

$$\boldsymbol{\varphi} = \begin{bmatrix} \varphi_{\mathrm{M}} \\ \varphi_{\mathrm{H}} \\ \varphi_{\mathrm{B}} \\ \varphi_{\mathrm{E}} \end{bmatrix}, \quad \dot{\boldsymbol{\varphi}} = \begin{bmatrix} \dot{\varphi}_{\mathrm{M}} \\ \dot{\varphi}_{\mathrm{H}} \\ \dot{\varphi}_{\mathrm{B}} \\ \dot{\varphi}_{\mathrm{E}} \end{bmatrix}, \quad \ddot{\boldsymbol{\varphi}} = \begin{bmatrix} \ddot{\varphi}_{\mathrm{M}} \\ \ddot{\varphi}_{\mathrm{H}} \\ \ddot{\varphi}_{\mathrm{B}} \\ \ddot{\varphi}_{\mathrm{E}} \end{bmatrix}. \qquad \begin{matrix} (2d) \\ (2e) \\ (2f) \end{matrix}$$

The forced excitation of the rotational exciter is given by

$$\boldsymbol{M} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ M \end{bmatrix}. \tag{2g}$$

3. KNOWN AND UNKNOWN MODEL PARAMETERS

To be able to identify the unknown model parameters of the torque transducer, it was necessary to identify the measuring device's model parameters first. To this end, dedicated auxiliary measuring set-ups for the determination of the mass moment of inertia, of torsional stiffness [3] and of torsional damping [4] were developed. Based on the measurement results from these set-ups, the previously unknown model parameters of the dynamic torque calibration device have been determined.

The extended model of the measuring device now consists of a set of known model parameters, which represents the components of the measuring device, and of a set of unknown model parameters representing the transducer under test (see Table 1), which need to be identified.

Table 1: Model parameters of the measuring device and of the device under test.

	Known parameters of the measuring device	Unknown parameters of the DUT
MMOI	$J_{\mathrm{M2}}, J_{\mathrm{M1}}, J_{\mathrm{E2}}$	$J_{\mathrm{H}}, J_{\mathrm{B}}$
Torsional stiffness	$c_{\mathrm{M}}, c_{\mathrm{E}}$	c_{T}
Damping	$d_{\mathrm{M}}, d_{\mathrm{E}}$	d_{T}

4. CALIBRATION MEASUREMENTS

For the calibration measurement with a transducer under test, periodic sinusoidal excitation is generated by the rotational exciter with given frequencies. The transfer function of the measuring device depends on the device under test (see Fig. 2). The control of the excitation frequency and magnitude, including abort conditions and a predetermination of the frequency response of each set-up, is carried out by means of a closed-loop vibration controller.

The chosen excitation frequencies are based on the 1/3 octave series (for frequencies up to 125 Hz) or the 1/12 octave series (above 125 Hz), respectively. The frequency range of excitation starts at 12.4 Hz and ends at 1 kHz.

For the calibration, the angle of excitation at the top $\varphi_{\mathrm{M}}(t)$, the rotational acceleration at the bottom $\ddot{\varphi}_{\mathrm{E}}(t)$ and the output of the transducer $u_{\mathrm{DUT}}(t)$ are acquired. A discrete Fourier transform (DFT) is calculated for the determination of the frequency, the magnitude and the phase of each signal.

These values are input start values for the determination of the frequency f and the magnitude A by approximating a monofrequent sine function like

$$y(t) = A \cdot \sin(2\pi f\, t + \varphi) + B \cdot t + C \tag{3}$$

including a magnitude offset C, drift B and phase offset φ.

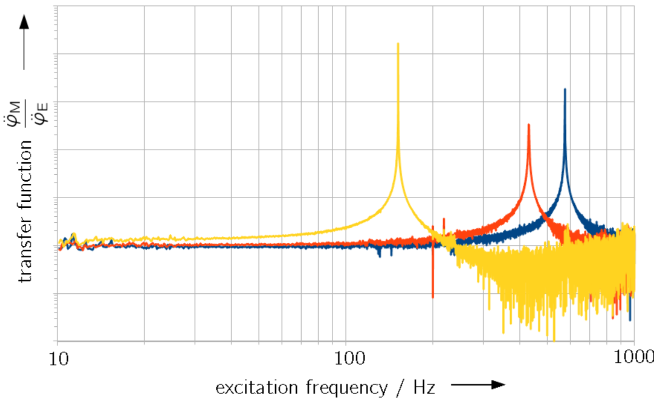

Figure 2: Frequency response of the measuring device with DUTs of different torsional stiffness and MMOI measured with random noise excitation.

5. PARAMETER IDENTIFICATION

Mounting different devices under test with different properties (MMOI, torsional stiffness, or damping) will influence the frequency response of the measuring device (see Fig. 2). From this variability in the frequency response of the measuring device, the properties of the DUT will be identified. The output of the transducer u_{DUT} is assumed to be proportional to the difference Δ_{HB}, of the torsion angles at the top (φ_{H}) and at the bottom (φ_{B}) of the transducer as

$$u_{\mathrm{DUT}}(t) = \rho \cdot \left(\varphi_{\mathrm{H}}(t) - \varphi_{\mathrm{B}}(t)\right) = \rho \cdot \Delta_{\mathrm{HB}}(t) \ , \tag{4}$$

because of the measurement principle (strain gauges) and the known linear behaviour. As the transducer under test is measuring the torque and not the angle difference, a proportionality factor ρ is introduced. Influences of the complementary conditioning amplifier will be compensated. Therefore, the amplifier will be calibrated dynamically first.

For the parameter identification, all signals acquired are assumed to be harmonic. In this case we can assume the following relationship for the angle φ, the angular velocity $\dot{\varphi}$ and the angular acceleration $\ddot{\varphi}$:

$$\begin{aligned} \varphi(t) &= \hat{\varphi} \cdot \mathrm{e}^{\mathrm{i}\omega t} \\ \dot{\varphi}(t) &= \mathrm{i}\omega\, \hat{\varphi} \cdot \mathrm{e}^{\mathrm{i}\omega t} = \mathrm{i}\omega\, \varphi(t) \\ \ddot{\varphi}(t) &= -\omega^2\, \hat{\varphi} \cdot \mathrm{e}^{\mathrm{i}\omega t} = -\omega^2\, \varphi(t)\ . \end{aligned} \tag{5}$$

Here, i denotes the imaginary number $\mathrm{i} = \sqrt{-1}$ and ω is the angular frequency $\omega = 2\pi f$.

The parameters of the DUT will be identified by analysing the output of the transducer and the mechanical input, which is measured as the angular accelerations $\ddot{\varphi}_{\mathrm{M}}$ and $\ddot{\varphi}_{\mathrm{E}}$ at the top and at the bottom of the coupling elements (see Fig. 1). This leads to the following frequency response equations:

$$\begin{aligned} H_{\mathrm{top}}(\mathrm{i}\omega) &= \frac{\rho \cdot \Delta_{\mathrm{HB}}(\mathrm{i}\omega)}{\ddot{\varphi}_{\mathrm{M}}(\mathrm{i}\omega)} \\ H_{\mathrm{bott}}(\mathrm{i}\omega) &= \frac{\rho \cdot \Delta_{\mathrm{HB}}(\mathrm{i}\omega)}{\ddot{\varphi}_{\mathrm{E}}(\mathrm{i}\omega)}\ . \end{aligned} \tag{6}$$

These equations are based on the ordinary equation system of the model (Eqs. (1), (2a), (2b), (2d), (2e), (2f), (2g)) and contain the known model parameters of the measuring device, as well as the still unknown parameters of the DUT.

For the input of the frequency response functions of Eq. (6), the results from Eq. (5) lead to

$$\Delta_{\mathrm{HB}} = \frac{-\omega^2 J_{\mathrm{M2}}\, \varphi_{\mathrm{M}}}{(\mathrm{i}\omega d_{\mathrm{T}} + c_{\mathrm{T}})} - \left(\frac{\omega^2 (J_{\mathrm{M1}} + J_{\mathrm{H}})}{(\mathrm{i}\omega d_{\mathrm{T}} + c_{\mathrm{T}})} \cdot (\omega^2 J_{\mathrm{M2}} + \mathrm{i}\omega d_{\mathrm{M}} + c_{\mathrm{M}})\, \varphi_{\mathrm{M}} \right). \tag{7}$$

For $H_{\mathrm{top}}(\mathrm{i}\omega)$ then follows

$$H_{\mathrm{top}}(\mathrm{i}\omega) = -\rho \cdot \frac{J_{\mathrm{M2}} + (J_{\mathrm{M1}} + J_{\mathrm{H}}) \cdot t(\mathrm{i}\omega)}{\mathrm{i}\omega d_{\mathrm{T}} + c_{\mathrm{T}}}\ , \tag{8}$$

with

$$t(\mathrm{i}\omega) = \frac{\varphi_{\mathrm{H}}}{\varphi_{\mathrm{B}}} = \frac{-\omega^2 J_{\mathrm{M2}} + \mathrm{i}\omega d_{\mathrm{M}} + c_{\mathrm{M}}}{\mathrm{i}\omega d_{\mathrm{M}} + c_{\mathrm{M}}} \tag{9}$$

consisting only of the known model parameters of the measuring device. The expression for $H_{\mathrm{bott}}(i\omega)$ is more complex, additionally to Eq. (9) we denote

$$b(\mathrm{i}\omega) = \frac{-\omega^2 J_{\mathrm{E2}} + \mathrm{i}\omega d_{\mathrm{E}} + c_{\mathrm{E}}}{\mathrm{i}\omega d_{\mathrm{E}} + c_{\mathrm{E}}}\ , \tag{10}$$

which is again not dependent on the DUTs parameters. With $b(\mathrm{i}\omega)$ and $t(\mathrm{i}\omega)$ we finally obtain

$$H_{\mathrm{bott}}(i\omega) = \frac{H_{\mathrm{top}}(\mathrm{i}\omega)}{\frac{\omega^2 H_{\mathrm{top}}(\mathrm{i}\omega)}{\rho} \cdot \left(\frac{-\omega^2 J_{\mathrm{B}}}{\mathrm{i}\omega d_{\mathrm{E}} + c_{\mathrm{E}}} + \frac{\mathrm{i}\omega d_{\mathrm{T}} + c_{\mathrm{T}}}{\mathrm{i}\omega d_{\mathrm{E}} + c_{\mathrm{E}}} + b(\mathrm{i}\omega) \right) + t(\mathrm{i}\omega) \cdot \left(\frac{-\omega^2 J_{\mathrm{B}}}{\mathrm{i}\omega d_{\mathrm{E}} + c_{\mathrm{E}}} + b(\mathrm{i}\omega) \right)}\ . \tag{11}$$

Examining Eq. (11), the simple enumerator and complex denominator suggest considering the inverse $H_{\mathrm{bott}}(i\omega)^{-1}$ instead:

$$\frac{1}{H_{\mathrm{bott}}(i\omega)} = \frac{\omega^2}{\rho} \cdot \left(\frac{-\omega^2 J_{\mathrm{B}}}{\mathrm{i}\omega d_{\mathrm{E}} + c_{\mathrm{E}}} + \frac{\mathrm{i}\omega d_{\mathrm{T}} + c_{\mathrm{T}}}{\mathrm{i}\omega d_{\mathrm{E}} + c_{\mathrm{E}}} + b(\mathrm{i}\omega) \right) + \frac{t(\mathrm{i}\omega)}{H_{\mathrm{top}}(\mathrm{i}\omega)} \cdot \left(\frac{-\omega^2 J_{\mathrm{B}}}{\mathrm{i}\omega d_{\mathrm{E}} + c_{\mathrm{E}}} + b(\mathrm{i}\omega) \right)\ , \tag{12}$$

or alternatively from Eq. (8)

$$\frac{1}{H_{\mathrm{bott}}(i\omega)} = \frac{\omega^2}{\rho} \cdot \left(\frac{-\omega^2 J_{\mathrm{B}}}{\mathrm{i}\omega d_{\mathrm{E}} + c_{\mathrm{E}}} + \frac{\mathrm{i}\omega d_{\mathrm{T}} + c_{\mathrm{T}}}{\mathrm{i}\omega d_{\mathrm{E}} + c_{\mathrm{E}}} + b(\mathrm{i}\omega) \right) - \frac{t(\mathrm{i}\omega)}{\rho} \cdot \left(\frac{\left(\frac{-\omega^2 J_{\mathrm{B}}}{\mathrm{i}\omega d_{\mathrm{E}} + c_{\mathrm{E}}} + b(\mathrm{i}\omega) \right) \cdot (\mathrm{i}\omega d_{\mathrm{T}} + c_{\mathrm{T}})}{J_{\mathrm{M2}} + (J_{\mathrm{M1}} + J_{\mathrm{H}}) \cdot t(\mathrm{i}\omega)} \right)\ , \tag{13}$$

which shows the dependencies on the unknown parameters $\rho,\ c_{\mathrm{T}},\ d_{\mathrm{T}},\ J_{\mathrm{H}},\ J_{\mathrm{B}}$.

To get a closer look into the structure of Eq. (13), we denote

$$g(\mathrm{i}\omega, J_\mathrm{B}, J_\mathrm{H}) = \frac{\omega^2}{\mathrm{i}\omega \mathrm{d_E} + \mathrm{c_E}} - \frac{t(\mathrm{i}\omega) \cdot \left(\frac{-\omega^2 J_\mathrm{B}}{\mathrm{i}\omega d_\mathrm{E} + c_\mathrm{E}} + b(\mathrm{i}\omega) \right)}{J_\mathrm{M2} + (J_\mathrm{M1} + J_\mathrm{H}) \cdot t(\mathrm{i}\omega)} . \quad (14)$$

Then, using Eq. (14) leads to

$$\frac{1}{H_\mathrm{bott}(i\omega)} = \omega^2 \frac{1}{\rho} \cdot \left(\frac{-\omega^2 J_\mathrm{B}}{\mathrm{i}\omega d_\mathrm{E} + c_\mathrm{E}} + b(\mathrm{i}\omega) \right) + \frac{\mathrm{i}\omega d_\mathrm{T}}{\rho} \cdot g(\mathrm{i}\omega, J_\mathrm{B}, J_\mathrm{H}) + \frac{c_\mathrm{T}}{\rho} \cdot g(\mathrm{i}\omega, J_\mathrm{B}, J_\mathrm{H}) , \quad (15)$$

expressing the partial linearity of the model for $1/\rho$, c_T/ρ, d_T/ρ. The same applies from Eq. (8) to $H_\mathrm{top}(\mathrm{i}\omega)^{-1}$ giving

$$\frac{1}{H_\mathrm{top}(\mathrm{i}\omega)} = -\frac{\mathrm{i}\omega d_\mathrm{T}}{\rho} \cdot \frac{1}{J_\mathrm{M2} + (J_\mathrm{M1} + J_\mathrm{H}) \cdot t(\mathrm{i}\omega)} - \frac{c_\mathrm{T}}{\rho} \cdot \frac{1}{J_\mathrm{M2} + (J_\mathrm{M1} + J_\mathrm{H}) \cdot t(\mathrm{i}\omega)} , \quad (16)$$

which is partially linear for c_T/ρ, d_T/ρ as well.

Assuming that J_B, J_H were known in Eq. (15) or J_H in Eq. (16) additionally to the known parameters of the measuring device, there would be closed form formulas for the estimation of the remaining parameters $1/\rho$, c_T/ρ, d_T/ρ Eq. (15) or c_T/ρ, d_T/ρ Eq. (16) by means of a linear least squares approach. These formulas would of course depend on J_B, J_H, which has implications on the estimation procedure for all unknown model parameters (see Table 1).

Instead of estimating the 5 parameters of Eq. (15) or the 4 parameters of Eq. (16), respectively, by a least squares approach in one nonlinear optimisation step, the parameters $1/\rho$, c_T/ρ, d_T/ρ may be replaced by closed form formulas, and a nonlinear minimisation over two dimensions (J_B, J_H) can be carried out. Consecutively, the estimates for $1/\rho$, c_T/ρ, d_T/ρ will be obtained from the closed form formulas, with the estimated values of J_B, J_H.

6. CONCLUSIONS

The presented identification scheme for the model parameters is a necessary component for the dynamic calibration of torque transducers. The dynamic behaviour of torque transducers is described by a physical model. The model parameters of each transducer will be identified from measurement data acquired with the calibration. The model parameters of the measuring device have been determined prior to the measurement to be able to identify the parameters of the transducer's model.

For their identification, the angular acceleration at the top and at the bottom of the transducer under test, as well as the transducer's output, will be analysed. Based on this input, a parameter identification based on the method of least squares is presented, and a two stage procedure utilizing the partial linearity of the model for a consecutive linear and nonlinear optimisation is described.

REFERENCES

[1] C. Bartoli et al., "Traceable Dynamic Measurement of Mechanical Quantities: Objectives and First Results of this European Project" in International Journal of Metrology and Quality Engineering; 3, 127–135 (2012) DOI: 10.1051/ijmqe/2012020

[2] T. Bruns, "Sinusoidal Torque Calibration: A Design for Traceability in Dynamic Torque Calibration" in Proc. of XVII IMEKO World Congress; 2003, Dubrovnik, Croatia, online at www.imeko.org: http://www.imeko.org/publications/wc-2003/PWC-2003-TC3-008.pdf

[3] L. Klaus, T. Bruns, M. Kobusch, "Determination of Model Parameters of a Dynamic Torque Calibration Device" in Proc. of XX IMEKO World Congress; 2012, Busan, Republic of Korea, online at imeko.org: http://www.imeko.org/publications/wc-2012/IMEKO-WC-2012-TC3-O33.pdf

[4] L. Klaus, M. Kobusch, "Experimental Method for the Non-Contact Measurement of Rotational Damping" in Proc. of Joint IMEKO International TC3, TC5 and TC22 Conference, 2014, Cape Town, South Africa, to be published

ACKNOWLEDGEMENT

The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union.