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Mathematical representation of reference torque transducers in partial-range regimes

A. Brüge

Physikalisch-Technische Bundesanstalt (PTB, Germany)

Abstract

Investigations with regard to the mathematical description of reference torque transducers used in comparison calibration facilities are described. A method is presented which uses only the direct loading calibration data of a torque transducer in the full range in order to calculate its behaviour in any reasonable partial range. Examinations of typical reference torque transducers demonstrate how to find transducers which are qualified for this calculation method and provide estimations of the possible effects on the measurement uncertainty caused by the calculation of the partial-range behaviour.

Keywords: torque, calibration, reference transducer, partial range, hysteresis, comparison method, scaling transformation.

1. Introduction

Torque calibration machines working with comparison methods [1] need reference transducers to determine the torque acting on the transducer under test. Thereby, the control unit of the machine translates the output signal of the reference transducer into torque values with the help of a mathematical representation of the reference transducer. The reference transducer's sensitivity curve is to be measured in a higher-level calibration machine, usually with direct loading. The representation makes it possible to calculate values between the calibrated steps and enables the control unit to drive the machine in stepless manner.

Furthermore, the representation does not only yield values for the full range of the reference transducer, but also for partial ranges.

Due to the hysteresis effect, in partial ranges only the values of increasing load are invariant. When the maximum value of an up-down-sequence is reduced, the hysteresis diminishes more and more and the primal description of the decreasing sensitivity becomes invalid (fig. 1).

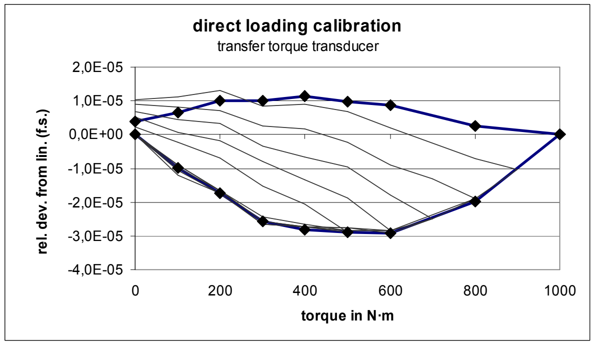

Fig. 1: Direct loading calibration of a transfer torque transducer in full range and in partial ranges from 90% to 50%. Shown is the relative deviation of the signal values from linearity

One solution is to carry out direct load calibrations of the reference transducer in each range needed at the comparison calibration machine. But the operation of direct loading machines is time- and cost-intensive. Also, some partial ranges may not be available in the mass stacks of the direct loading machine. Therefore, most reference transducers were calibrated in the full range only.

2. Representation Methods

In order to exploit the flexibility of comparison calibration machines, a mathematical representation is needed which is able to reproduce the behaviour of the reference torque transducer for both loading and unloading sequences and for all possible partial ranges. The deviation of the calculated values should be much smaller than the measurement uncertainty which is intended for the comparison calibration.

A selection of the representation methods currently in use will be discussed in the following, whereat tared signals S are presumed, i.e. $S(0\,\mathrm{N\cdot m}) = 0$.

2.1 Linearisation

The simplest representation approximates the sensitivity curve of the reference transducer to be linear (fig. 2). The only parameter of the correlation between the signal S and the torque M is the slope m (1):

$$S_{\mathrm{part}}(M) = m\,M \qquad (1)$$

$$m = \frac{S(M_{\mathrm{nom}})}{M_{\mathrm{nom}}} \qquad (2)$$

A usual application of the linearisation is the adaptation of amplifier displays from mV/V to N·m.

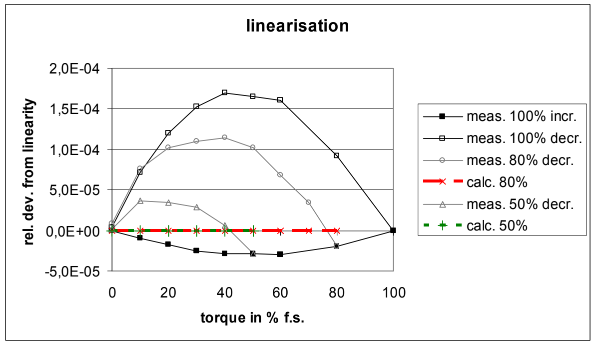

Fig 2. : Partial-range calibration and its representation as linearisation. The signals are shown as relative deviation from linearity relative to the full-scale value.

2.2 Range Linearization

An enhancement of the linearisation is the range linearisation, which assigns a specific slope m_{part} to each partial range (3), related to the maximum torque value at this partial range $M_{\text{part,max}}$:

$$m_{\text{part}} = \frac{S\left(M_{\text{part,max}}\right)}{M_{\text{part,max}}} \quad . \qquad (3)$$

The best results are to be expected at transducers having a low hysteresis and at partial ranges of small percentages, which give rise to a considerably reduced amount of the primal nominal-range-hysteresis.

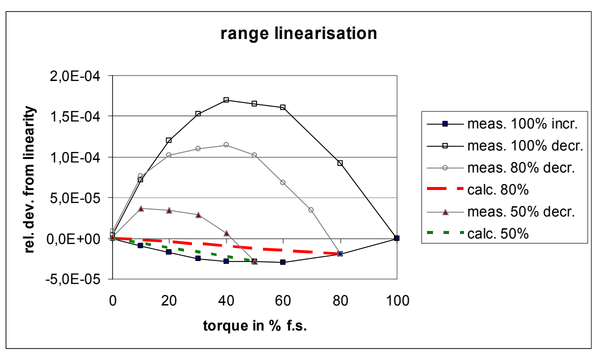

Fig 3. : Partial-range calibration and its representation as range linearisation. The signals are shown as relative deviation from linearity relative to the full-scale value.

2.*3 Nodes, Polynomial of Increasing Signal Only*

The two representations shown above fit well only at the ends of the examined ranges. To obtain a closer fit to the sensitivity curve, some amplifiers are provided with a display adaptation with a higher number of nodes (typically

10). This results in low deviations for the representation of the increasing sequence, but the method is not able to distinguish this sequence from the decreasing one. Hence, the maximum deviation for a complete calibration series equals the amount of the hysteresis. The same applies for a polynomial representation based on the increasing signal only, which is often chosen because most calibration certificates provides polynomial parameters for the increasing part of the sensitivity curve only.

2.4 Average Polynomial

To avoid the disadvantages of representations based exclusively on the increasing branch, some laboratories work with a polynomial representation based on the average of increasing signals $S^{\uparrow}(M)$ and decreasing signals $S^{\downarrow}(M)$ (fig. 4). It consists of the averaged polynomial parameters $\overline{a}_{S}$.

$$S_{avg}(M) = \frac{S^{\uparrow}(M) + S^{\downarrow}(M)}{2} \quad (4)$$

$$S_{part}(M) = \overline{a}_{S,0} + \overline{a}_{S,1} \cdot M + \overline{a}_{S,2} \cdot M^2 + \overline{a}_{S,3} \cdot M^3 . \quad (5)$$

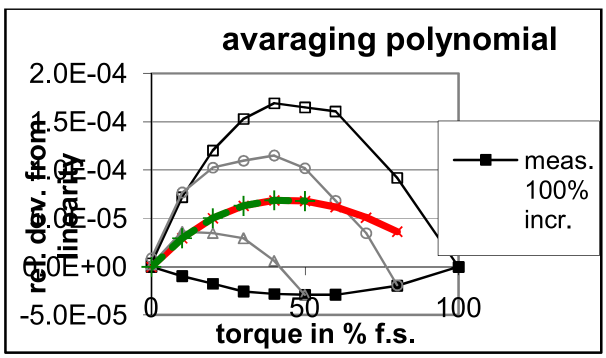

Fig 4. : Partial-range calibration and its representation as averaging polynomial. The signals are shown as relative deviation from linearity relative to the full-scale value.

2.5 Scaling Transformation

For scaling transformation, the representation of the sensitivity curve of the reference torque transducer is a polynomial of the third degree. Then the dependence of the output signal *S* in mV/V on the torque *M* in N·m is expressed by four parameters $a_{S,n}$, n=0, 1, 2, 3, whereas the effect of the hysteresis requires separate equations for the increasing and decreasing values of *M*, respectively *S* :

$$S^{\uparrow}(M) = a^{\uparrow}_{S,0} + a^{\uparrow}_{S,1} \cdot M + a^{\uparrow}_{S,2} \cdot M^2 + a^{\uparrow}_{S,3} \cdot M^3 \text{ , increasing loading} \quad (6)$$

$$S^{\downarrow}(M) = a^{\downarrow}_{S,0} + a^{\downarrow}_{S,1} \cdot M + a^{\downarrow}_{S,2} \cdot M^2 + a^{\downarrow}_{S,3} \cdot M^3 \text{ , decreasing loading.} \qquad (7)$$

Accordingly, the torque is given by the inverse description:

$$M^{\uparrow}(S) = a^{\uparrow}_{T,0} + a^{\uparrow}_{T,1} \cdot S + a^{\uparrow}_{T,2} \cdot S^2 + a^{\uparrow}_{T,3} \cdot S^3 \text{ , increasing signal} \qquad (8)$$

$$M^{\downarrow}(S) = a^{\downarrow}_{T,0} + a^{\downarrow}_{T,1} \cdot S + a^{\downarrow}_{T,2} \cdot S^2 + a^{\downarrow}_{T,3} \cdot S^3 \text{, decreasing signal.} \qquad (9)$$

Using the reference transducer in the partial-range mode, the hysteresis h will be smaller than in the nominal range. Assuming that the shape of the sensitivity curve in the partial range is similar to that in the nominal range, the transformation of the measured nominal range sensitivity curve into the partial range sensitivity curve can be achieved by a scaling operation. This means mapping of the sensitivity curve by multiplying the abscissa by a factor k_a and the ordinate by a factor k_o (fig. 5).

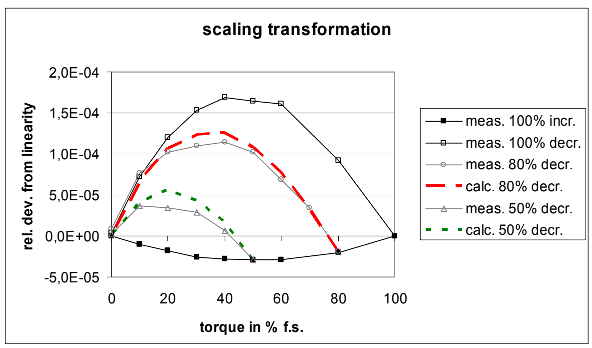

Fig 5. : Partial-range calibration and its representation as scaling transformation. The signals are shown as relative deviation from linearity relative to the full-scale value.

The abscissa factor is given by the ratio between the nominal torque value M_{nom} and the maximum torque value of the partial range $M_{max,part}$:

$$k_a = \frac{M_{max,part}}{M_{nom}} \qquad (10)$$

To find k_o, the maximum of the hysteresis h in the nominal range mode is needed. Using (8) and (9), the relative hysteresis h_{rel} (in N·m) is given by

$$h_{\mathrm{rel}}(S) = \frac{(a^{\uparrow}_{\mathrm{T},0} - a^{\downarrow}_{\mathrm{T},0}) + (a^{\uparrow}_{\mathrm{T},1} - a^{\downarrow}_{\mathrm{T},1})S + (a^{\uparrow}_{\mathrm{T},2} - a^{\downarrow}_{\mathrm{T},2})S^2 + (a^{\uparrow}_{\mathrm{T},3} - a^{\downarrow}_{\mathrm{T},3})S^3}{M_{\mathrm{nom}}} \quad , \qquad (11)$$

with the nominal torque M_{nom}.

The maximum of the hysteresis in the nominal range mode can be found at the signal value

$$S_{\mathrm{maxh,nom}} = -\frac{a^{\uparrow}_{\mathrm{T},2} - a^{\downarrow}_{\mathrm{T},2}}{3\left(a^{\uparrow}_{\mathrm{T},3} - a^{\downarrow}_{\mathrm{T},3}\right)} \pm \sqrt{\left(\frac{a^{\uparrow}_{\mathrm{T},2} - a^{\downarrow}_{\mathrm{T},2}}{3\left(a^{\uparrow}_{\mathrm{T},3} - a^{\downarrow}_{\mathrm{T},3}\right)}\right)^2 - \frac{a^{\uparrow}_{\mathrm{T},1} - a^{\downarrow}_{\mathrm{T},1}}{3\left(a^{\uparrow}_{\mathrm{T},3} - a^{\downarrow}_{\mathrm{T},3}\right)}} \quad , \qquad (12)$$

which yields two solutions, but only one of them is reasonable.

The maximum hysteresis in the partial-range mode can be found at the signal value

$$S_{\mathrm{maxh,part}} = S_{\mathrm{maxh,nom}} k_{\mathrm{a}} \quad . \qquad (13)$$

Using (11), the relative hysteresis at the signal values $S_{\mathrm{maxh,part}}$, resp. $S_{\mathrm{maxh,nom}}$, can be calculated. The relation between these relative hysteresis values yields the ordinate scaling factor

$$k_{\mathrm{o}} = \frac{h_{\mathrm{rel}}\left(S_{\mathrm{maxh,part}}\right)}{h_{\mathrm{rel}}\left(S_{\mathrm{maxh,nom}}\right)} \; k_{\mathrm{a}} \quad . \qquad (14)$$

Thus, the hysteresis in the partial-range mode can be calculated by:

$$h_{\mathrm{part}}(M) = h_{\mathrm{nom}}\left(\frac{M}{k_{\mathrm{a}}}\right) \; k_{\mathrm{o}} \quad . \qquad (15)$$

And the decreasing sensitivity curve of a partial range is

$$S^{\downarrow}_{\mathrm{part}}(M) = a^{\uparrow}_{\mathrm{S},0} + a^{\uparrow}_{\mathrm{S},1} \cdot M + a^{\uparrow}_{\mathrm{S},2} \cdot M^2 + a^{\uparrow}_{\mathrm{S},3} \cdot M^3 - h_{\mathrm{part}}(M) \; . \qquad (16)$$

The amount of deviation between the calculated and the measured values depends on how far the assumption about the constant shape of sensitivity curves is applies, i.e. if the transducer is qualified for scaling. This is to be taken into account for the selection of the transfer transducers.

That scaling ability is possible was demonstrated by Wegener et al. [2] in 2006 for a number of rotating torque transducers.

3 Measurements

3.1 Partial-Range Data Survey

In the PTB, investigations in addition to the normal calibration schedule were carried out for many transducers, by way of creep tests, alternating torque tests and partial-range tests. The partial-range test consists of a series of load steps with the vertices 0%-20%-0%-40%-0%-100%-0%. Thanks to these tests, we now possess partial-range data for a great number of different torque transducers and torque wrenches. These are now re used to test the suitability of the representation methods described in chapter 2.

For each method, the deviation f_a between the measurement and the calculation was determined for all steps of a partial-range sequence (including full-scale sequences). The maximum deviation found in a sequence, related to the measured signal at the load of this maximum, was taken as a parameter of suitability assigned to the examined representation method. Because this is a parameter relative to the measurement value (m.v.), the maximum is usually located at the lower end of the loading sequence, which was 10% in this investigation (figs. 9 and 10). This partial-range parameter is shown as a function of the maximum relative hysteresis of the full-scale sequence in the figures 6, 7 and 8.

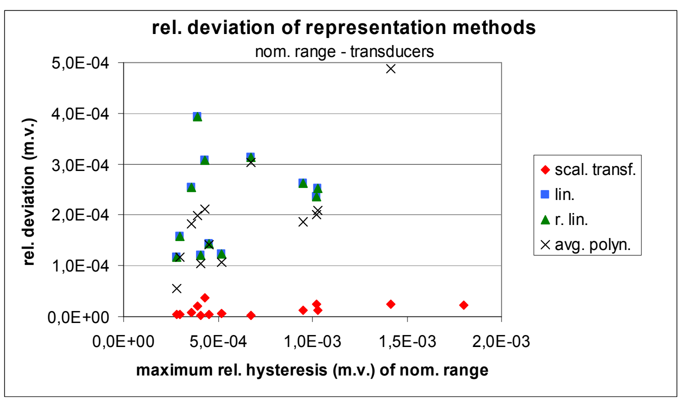

Fig 6. : Relative deviation of some representation methods in respect of measurement value (m.v.) vs. the maximum rel. hysteresis in the nominal range of high quality torque transducers used in nominal range.

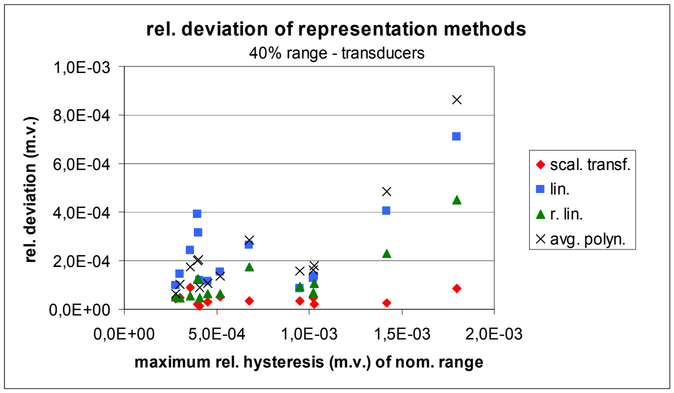

Fig 7. : Relative deviation of some representation methods in respect of measurement value (m.v.) vs. the maximum rel. hysteresis in the nominal range of high quality torque transducers used in 40% range.

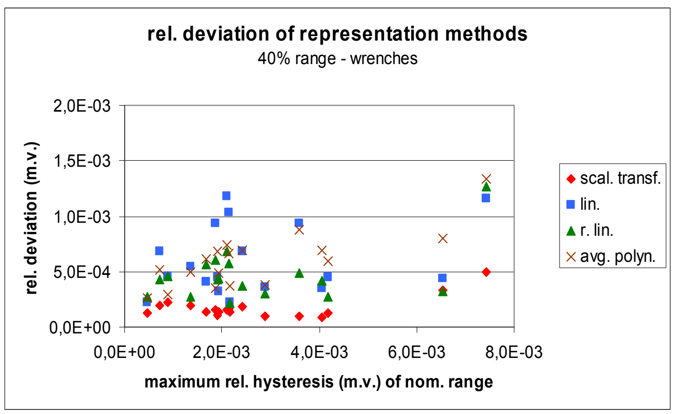

Fig 8. : Relative deviation of some representation methods in respect of measurement value (m.v.) vs. the maximum rel. hysteresis in the nominal range high quality torque wrenches used in 40% range

A representation of reference transducers in the partial-range regime must be able to describe the full-scale behaviour too. Therefore, in figure 6, the results for transfer torque transducers in full scale are presented. Here, linearisation and range linearisation are identical, and their partial-range parameters are the same. The parameter values of all methods, except for the scaling transformation, are close to a line of slope=1/2, which stands for the average of increasing and decreasing sequences. Only the scaling transformation's relative deviations are smaller than $5{\cdot}10^{-5}$, which is close to the amount of f_a, due to the bmc (best measurement capability) of the direct loading machine used for this measurement. A bmc of $2{\cdot}10^{-6}$ correlates with possible relative deviations of $2{\cdot}10^{-5}$ at 10% of the full scale. The result for the scaling transformation is independent of the maximum relative hysteresis up to values of $2 \cdot 10^{-3}$, i.e. the corresponding dots in figure 6 follow a horizontal line.

In the partial range of 40%, relative deviations of less than $5 \cdot 10^{-5}$ are achievable for the scaling transformation if the transducers are selected accordingly (fig. 7).

Using torque wrenches for reference purposes brings about greater difficulties with square drives and creep effects. The latter occur because partial sequences need less time than full-scale sequences in most of the calibration machines. Nevertheless, the partial-range parameter for the scaling transformation can be better than $2.5 \cdot 10^{-4}$ for selected wrenches (fig. 8). This result is valid for wrenches with maximum relative hysteresis values of at least up to $4 \cdot 10^{-3}$.

In all cases, the deviations of the other methods are significantly higher than for the scaling transformation. Besides, the fitting quality of these methods depends much more on the linearity of the sensitivity curve than in the case of the scaling transformation.

The contribution to the relative measurement uncertainty due to the reference transducer representation can be calculated according to the interpolation error in the German standard DIN 51309 [3] :

$$w_{f_\mathrm{a}}(M) = \frac{1}{\sqrt{6}} \frac{f_\mathrm{a}(M)}{2 \;\; S(M)} \quad . \qquad (17)$$

For the scaling transformation, the values for w_{f_a} . are $1 \cdot 10^{-5}$ for transducers in the full-range mode, $2 \cdot 10^{-5}$ in the 40% range, and $5 \cdot 10^{-5}$ for wrenches in the 40% range.

3.2 Multi-Range-Measurement

On a typical type of a transfer torque transducer, partial-range measurements were performed at ranges from 100% to 50% (fig. 1). The deviation between the representation and the measurement in respect of the maximum percentage of the range is, in the case of the scaling transformation, stochastical (fig. 9). The deviation increases with decreasing load, but the maximum deviation is independent of the range percentage. This confirms the results of the partial-range tests described in chapter 3.1 and shows that a single test in a partial range of 50% would deliver a reliable estimation for the suitability of a transducer as reference. The relative deviations are smaller than $5 \cdot 10^{-5}$.

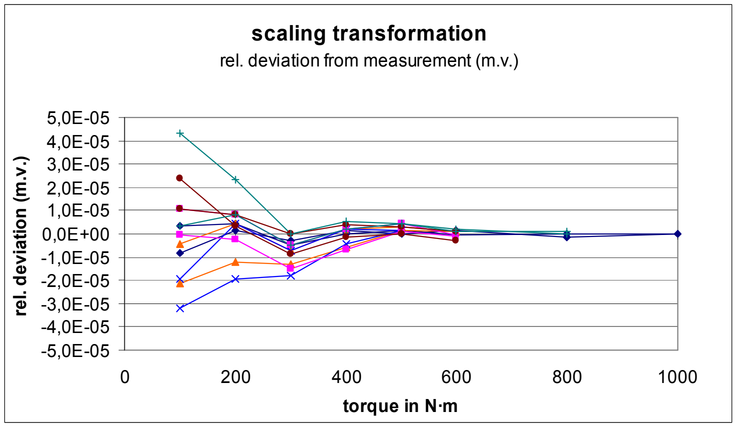

Fig 9. : Relative deviation of scaling transformation in respect of measurement value (m.v.) vs. torque with range percentages from 100% to 50%. Result of measurement shown in fig. 1.

In the case of the range linearisation, which yields the second best results in the data survey of chapter 3.1, the relative deviations are systematical and increase with decreasing load to (fig. 10). But in contrast to the scaling transformation, the maximum deviation increases with the range percentage. This confirms the expectations discussed in chapter 2.2. The results found in fig. 6 for the 100% range and in fig. 7 for the 40% range can be interpolated linearly to obtain estimations for the ranges in between. For the 90% range, the maximum deviation is about twice the deviation in the 50% range where the deviation is comparable to the deviation for the scaling transformation.

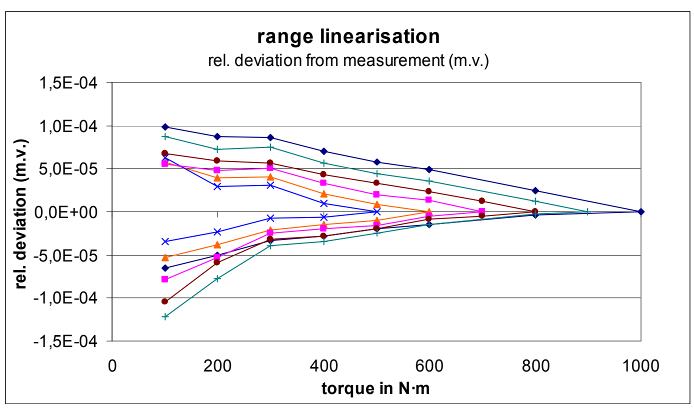

Fig 10. : Relative deviation of range linearisation in respect of measurement value (m.v.) vs. torque with range percentages from 100% to 50%. Result of measurement shown in fig. 1.

4 Conclusions

For high-quality comparison calibrations, reference transducer representations by polynomials of the 3rd degree, separately for the increasing and the decreasing load, are recommended. In partial-range regimes, a scaling transformation of these polynomials can describe the change in the sensitivity curves especially for the unloading sequence. Provided that adequate reference transducers are used, these mathematical representations cause contributions $\left(w_{f_a}\right)$ to the relative measurement uncertainty of less than $1\cdot10^{-5}$ for transducers in the full range, of $2\cdot10^{-5}$ down to the 40% range and of $5\cdot10^{-5}$ for wrenches down to the 40% range. The requirements for the hysteresis and linearity of the reference transducers are not strict. Suitable reference transducers can be selected by a single partial-range measurement of 40%. To enable the implementation of the scaling transformation, the calibration certificates of reference transducers should include polynomial parameters of the 3rd degree both for increasing and decreasing loading.

References

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