Investigation in the local gravity field of a force laboratory of PTB

Andreas Lindau[1], Rolf Kumme[1], Andrea Heiker[2]

Physikalisch-Technische Bundesanstalt
Braunschweig (PTB) [1]

Geodätisches Institut, University of Hannover[2]

Abstract:

It belongs to the functions of the National Institutes of Metrology to realise a force scale of highest accuracy. The uncertainty of the resulting force basically depends on the precise knowledge of local gravity acceleration. This paper describes the spatial determination of the local gravity field in the force laboratory of the PTB on the basis of a new highly precise gravity network. A special emphasis is placed on the influence due to gravity of the surrounding masses placed in the laboratory on the single gravity value. It can be shown that the installation of the new 2 MN deadweight force standard machine has led to a significant change in the local gravity field. Temporal variations caused by earth tides are proved by long-term registrations and are considered in a correction term for the gravity value. The periodical change of gravity reaches a level of 10^{-7} *g* in the equator region and at present does not affect the uncertainty budget of the force measurement.

1 Introduction

With the installation of the 2 MN force standard machine this year the force laboratory of the PTB was extended by its largest deadweight machine. The standard was transferred in the last two years from PTB's outpost Berlin to the main location Braunschweig. For the reinstallation of the machine a considerable enlargement to the existing laboratory hall was necessary. Such enormous structional modifications always result in a change of the local gravity field.
According to the well-known principle of force standard machines for the realisation of the force [4]

$$F = m \cdot g_{\text{loc}} \cdot \left(1 - \frac{\rho_{\text{a}}}{\rho_{\text{m}}}\right) \quad , \qquad (1)$$

with m = mass, g_{loc} = acceleration due to gravity , ρ_{a} = density of the air,
ρ_{a} = density of the deadweights

especially the local acceleration due to gravity g_{loc} is considered to be known.

At the new laboratory tract information was not available about the gravity field. So gravity had to be obtained in dependence on the different altitudes of the mass bodies. For the rest of laboratory building some gravity values from a former measurement were available butit had to be assumed that the gravity was affected by the extension of the buiolding as well. A complete re-measurement became inevitable.
In co-operation with the Institute of Geodesy (IfE), University of Hannover, a local geodetic network was established in the laboratory and practically monitored with gravimetric methods. During the installation of the 2 MN machine some massive components had been installed in the hall, which also led to a change of the gravity field. The main objective of this

investigation was to develop a model to describe the results of these different gravity influences and to support it by a control network.

2 The gravimetric methods

The so-called gravity g_{loc} constitutes an acceleration that is the resultant of gravitation *b*, caused by mass attraction of the earth, and the centrifugal acceleration *z*, caused by the Earth's rotation [2]. The absolute value of the gravity acceleration g_{loc} can be expressed as a function of the location determined by the latitude φ and its height h above the sea level and of time t.

$$g_{loc} = g\,(\varphi\,,\, h,\, t) \qquad (2)$$

The latitude causes a maximum of gravity change of $5\cdot10^{-3}$ *g* between equator and the poles, Changes in gravity due to horizontal location (φ) reach a maximum of, the gravity field can be approximated through a theoretical model. When latitude and altitude are exactly known, the gravity can be computed better than $4\cdot10^{-4}$ *g*. For a higher precision the practical usage of a gravimetric sensor becomes necessary.

2.1 The gravimetric instruments

Direct determination of gravity acceleration succeeds with an absolute gravimeter, which nowadays is based on the principle of free fall. The instrument operates with a corner cube prism as a falling body inside an evacuated cylinder to decrease air friction. The acceleration can then be derived from a record of at least three measuring pairs of length and time. For the determination of length, a frequency-stabilised laser is used, which emits a signal with a well-known wavelength to the falling cube. The reflection can be analysed according to the principle of the Michelson interferometer, as is shown in figure 1. The corresponding time information is provided by a highly precise rubidium quartz standard and a universal time interval counter. Including a high-grade shield against disturbing microseismic influences, a repeatable accuracy of 10^{-9} *g* can be achieved depending on the site conditions. The new generation of absolute gravimeters offers the possibility of mobile use, but considering the complexity of the instrument and the disturbing influences an observation period of at least two days per site has to be taken into account.

A less expensive and more compact alternative to the absolute method is provided by the spring gravimeter. The major difference of this procedure is to keep the time parameter con-

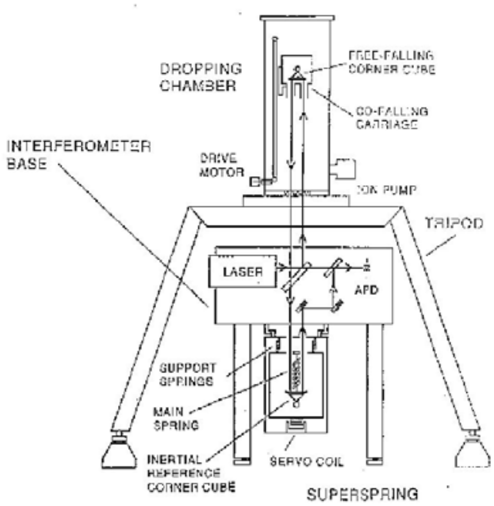

***Figure 1**: Principle of a free-fall absolute gravity meter*

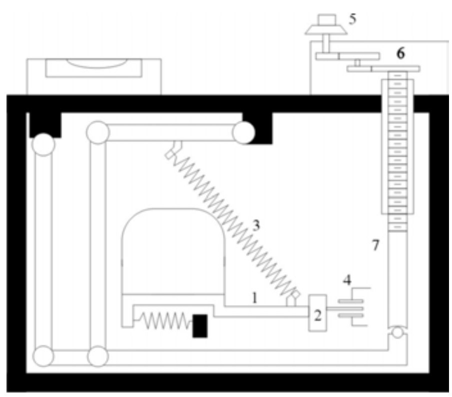

***Figure 2** : Principle of a relative spring gravity meter*

stant and to observe only the change of a sensor position. The missing information is compensated by the difference between measurements carried out at two sites. The measurand carried out by a spring gravity meter is therefor the relative change gravity between two locations.
The spring gravimeters use a counterforce to keep a test mass in equilibrium with gravity. In a model, which is widely used (figure 2), the small mass (2) is supported by a horizontal lever (1), suspended from an obliquely acting spring (3). A mechanical gearing (5,6,7) or an electrical feedback system (4) can compensate the elongation of the spring due to gravity. To convert the values directly observed into acceleration values, a calibration on a calibration line must have been carried out. The accuracy of this procedure depends first, on the redundancy of the individual site connections and second, on their spacing. In micro networks, as the present one, the achievable uncertainty compares to that of absolute gravimeters [2].
A substantial advantage of the relative method lies in the size of the sensor which is placed in a housing with the dimensions 20x18x15 cm and a 3 kg weight. An experienced observer should be able to record about 30 settings per day.

The two procedures described are affected by temporal variations in the gravity field due to the gravitation of the nearest celestial bodies, i.e. the sun and the moon. The so-called earth tides can be sufficiently approximated by a complex model so that the gravity acceleration after correction is independent of time.

2.2 Installing a gravimetric network

The character of a gravimetric network is mainly defined by its geometry and by the density of the nodal points. In the case of the force laboratory, this depends on the location of the force standard machines and the requirement of a small uncertainty, which presupposes at

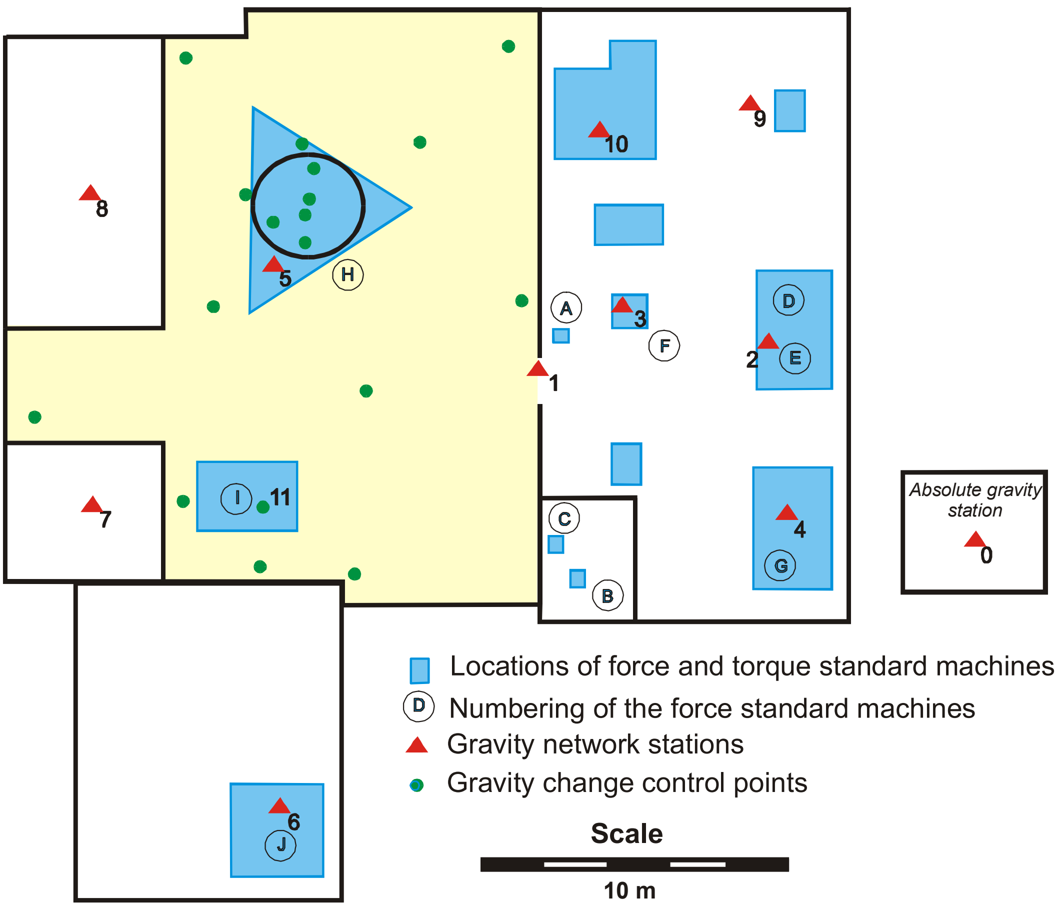

Figure 3 : *Ground plan of the PTB force lab with gravity network stations*

least one station per machine. This includes also the torque standard machines which are also arranged at the laboratory. Concerning the machines of a large vertical extension some stations were added on different floors, so that the vertical gradient can be derived. Table 1 and Figure 3 give an overview of the locations of the machines and measuring stations at the force laboratory.

As to the aspect of economicalness and efficiency due to the density of the network, it is suitable to apply the relative gravimetric method. The use of an absolute gravimeter would have failed due to the space limitations at most points and the size of the instrument. The computation of absolute gravity values from a network obtained in relative terms requires at least one connected site providing the gravity level. For this purpose, it was decided to observe an additional station in a separate basement room using the absolute gravimeter. The rest of the network was connected with relative gravity meters.

Another task in establishing the network was to identify changes in the gravity field during the installation of the 2 MN force standard machine. So some further stations in the big hall were chosen as gravity control points, as can be seen in figure 3. The number and distribution of the stations both correspond with the requirement of a high homogenous resolution of the field and with the expected changes due to mass transfer in the big hall.

Nr	Gravity stations	Force Standard Machines	Force Range			Relative Uncertainty (k=2)
		Type	Min. value	Max. value	Units	%
A	1	deadweight (manual)	0.5	200	N	0.002
B		deadweight	2	20	N	0.002
C		deadweight	5	200	N	0.002
D	2	deadweight	50	2000	N	0.002
E		deadweight	0.25	20	kN	0.002
F	3	deadweight	2	100	kN	0.002
G	4	deadweight	20	1000	kN	0.002
H	5	deadweight	50	2000	kN	0.002
I	11	hydraulic amplification	0.1	5	MN	0.01
J	6	hydraulic amplification	0.1	16.5	MN	0.01

Table 1: *Overview of the force standard machines available at the PTB*

Nr	Inst.	Height	Gravity	Standard deviation	Gradients $\Delta g = p \cdot \Delta h + q \cdot \Delta h^2$
		[m]	[m·s^{-2}]	[10^{-9} m·s^{-2}]	[m·s^{-2}]
0	-	72,545	9,812527153	37	p=-2,72·10^{-6}
1	-	75,285	9,812519218	44	p=-2,76·10^{-6}
2	a	75,285	9,812519478	55	p=-2,76·10^{-6}
	b	77,455	9,812513473	55	
3	a	72,880	9,812524994	55	p=-2,76·10^{-6}
	b	75,690	9,812518363	52	
4	a	67,815	9,812534133	56	p=-1.28·10^{-6} q=-0.08·10^{-6}
	b	74,190	9,812522456	59	
	c	78,350	9,812511064	53	
5	a	67,825	9,812535487	77	p=-2.04·10^{-6} q=-0.03·10^{-6}
	b	71,530	9,812527396	65	
	c	75,285	9,812518570	50	
	d	79,345	9,812507402	60	
	e	82,635	9,812498025	73	
6	-	84,165	9,812494368	53	-
7	-	75,285	9,812519153	56	p=-2,78·10^{-6}
8	-	75,285	9,812518860	56	p=-2,76·10^{-6}
9	-	75,885	9,812517653	56	p=-2,78·10^{-6}
10	a	73,040	9,812524158	56	p=-2,76·10^{-6}
	b	75,350	9,812518830	58	p=-2,76·10^{-6}
11*	-	75,285	9,812519104.	37	p=-2,76·10^{-6}

Table 2: *Results from the first gravity network campaign in the force lab of the PTB, April 2000*
** Gravity value based upon the results of epoch 2*

3 Results of the network campaign

The first network campaign was performed in April 2000 before the installation of the 2 MN force standard machine has started started. 19 network stations on different floors (trianglular symbols in Figure 3) were tied to the absolute reference station with the simultaneous use of four relative gravity meters. This strategy provides four decisive advantages: improved efficiency, high redundancy, simultaneous gravimeter calibration and a mutual control of the instruments used.

The final results of the campaign after a least-squares adjustment of 150 observations or 134 gravity differences respectively were listed in Table 2. The numbering of the stations in that table correspond to the that in Figure 3. Depending on the vertical extension of some larger machines up to five instances of the main point were established at different heights. This led to a maximum gravity variation of $4.2 \cdot 10^{-6}$ *g* over a height difference of 16 m. The standard deviation of the mean per adjusted gravity could be obtained with $5.8 \cdot 10^{-9}$ *g*.

For a spatial analysis of the local gravity field, the discrete values of the acceleration are not sufficient. To get an impression of the field behaviour in the third dimension one has to know the local gravity gradient. At the large force standard machines the gradient could be derived from the vertical by arranged instances. On the other stations additional measurements 1 m above ground level were necessary. In the immidiate vicinity of a point, the gradient can be considered as linear. Depending on the height and the surrounding mass influences, the value can decrease by up to 30% of the mean free air gradient. On the ground floor of the force lab, at 72.285 m height (see table 2), it reaches a mean of $2.76 \cdot 10^{-7}$ *g*/m, which is quite usual for buildings. Over a larger distance, such as one or more floors, the trend of the gradient can be described as a polynomial of second or third order. For the large 1 MN and 2 MN machines a second order polynomial represents the behaviour of the local gradient best. In table 2 the polynomial coefficients are given in the following way :

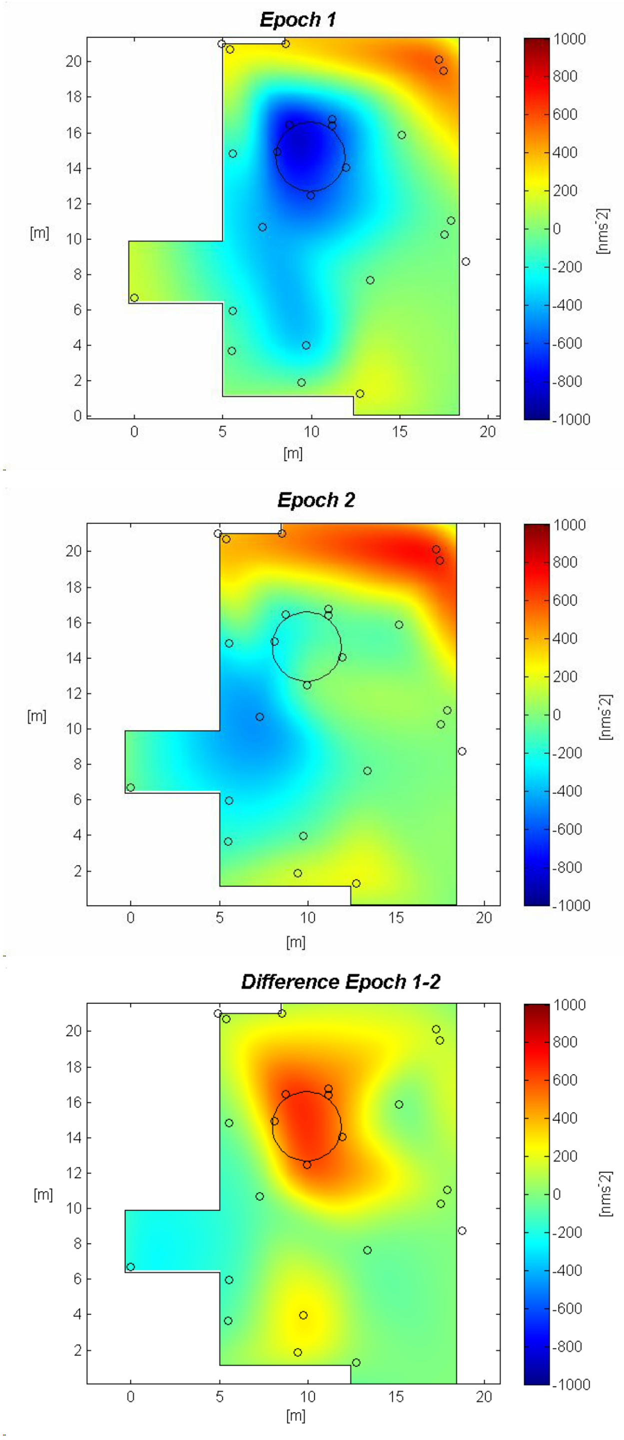

***Figure 4** : top and centre: Gravity differences related to station 1 after epoch 1 and 2 in the new hall of the force lab, bottom: differences between epoch 1 and 2*

$$\Delta g = p \cdot \Delta h + q \cdot \Delta h^2 \quad (3)$$

The local gravity for a different height can be expressed in the closed form :

$$g(h_2) = g_1 + p\cdot(h_2-h_1) + q\cdot(h_2-h_1)^2 \quad (4)$$

***Changes in the gravity field during the installation of the 2 MN machine*.**

In April 2001 the first step of the installation was finished after the 35 t steel framework for suspending the deadweights had been considered. This style was covered by a second gravimetric campaign which also included the big hall of the force lab with over 280 observations. At the top of Figure 4, the results of these observations, in the following called epoch 1, are shown as gravity differences related to station 1 (see figure 3), which is at the ground level. The maximum deviation appears at the centre of the circle, where a cylindrical hole pierces two floors for the deadweights installed later. The deviation can easily be explained by a mass deficit in the hole and a close cellar compared to the rest of the hall.

In the second step the mass bodies for the direct loading with a total weight of 200 t were suspended from the steel frame and inserted in the hole. After another gravimetric epoch, a change of the maximum deviation in the gravity field could be observed outside the hole (Figure 4, center). That means that the deadweights have nearly completely compensated the mass deficit next to the circle.

A similar, though smaller effect could be observed with the simultaneous installation of the 5 MN force standard machine in the same hall. In Figure 4 (bottom) the difference of both epochs shows two maximums due the change of gravity. The smaller one below appears at the location of the 5 MN machine.

Nr	Inst.	Height	Gravity	Standard deviation	Gradients $\Delta g = p\cdot\Delta h + q\cdot\Delta h^2$
		[m]	[$m\cdot s^{-2}$]	[10^{-9} $m\cdot s^{-2}$]	[$m\cdot s^{-2}$]
5	a	67,825	9,812534909	35	$p=-1.88\cdot10^{-6}$ $q=-0.04\cdot10^{-6}$
	b	71,530	9,812526816	35	
	c	75,285	9,812519156	26	
	d	79,345	9,812507466	32	
	e	82,635	9,812497976	33	

Table 3 : *Gravity at Station 5 after epoch 2*

After the second epoch the absolute gravity values, listed in table 2 are recalculated for station 5 close to the 2 MN machine. From this a mean deviation of $5\cdot10^{-8}$ *g* results, with its maximum directly on the mass stack where the gravity value is increased to $5\cdot10^{-8}$ *g.*

4 Approximation of the gravity influence by a model of rectangular prisms

If the nature of a body is described by its form and density, its influence due to gravity on the surrounding gravity field can be approximated by a model. According to Newton's law, the gravity effect can be strictly calculated for elementary geometries, e.g. a rectangular prism (figure 5).
With this, more complex figures can be approximated by composition through basic elements [3]. For the integration of high-resolution fields, the number of elements increases with the third power, which leads to corresponding computation effort [1].

In the case of the force lab, such a model has been established for the mass bodies of the 2 MN machine. The form of the cylindrical discs can easily be described and compared to the gravitational effect of the basic form of a cylinder. Because of the homogeneous geometry, the resulting gravity change is concentric around the cylindrical hole. This can be explained by the rest of the hall components which are not taken into consideration. Comparing the model directly to the empirically determined difference between epochs 1 and 2, the largest

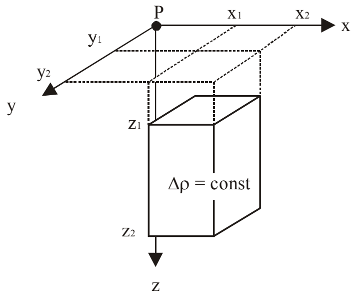

Figure 5 *: System definition of a rectangular prism [2]*

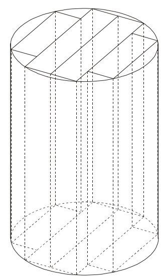

Figure 6 : *Approximation of the cylindrical mass bodies by prisms [1]*

deviations occur in the neglected areas of the model. In the intermediate vicinity of the cylindrical hole, model and measurement agree within 10^{-8} g . An extension of the model including the structure of the hall, the cellars and the machine components allow an even better fit of the existing gravity field to be expected.

5 Time dependence of the gravity measurements

The practical methods of determining the gravity acceleration are always affected by the influence of the closest celestial bodies. Especially the influence of moon and sun due to short and long periodical tidal waves and the Earth's rotation itself lead to a dependence on time and place of measurement. The sum of all waves reach their maximum at the equator with about $1.2 \cdot 10^{-7}$ g [2]. The local wave parameters for the calculation of the gravity correction can be derived from long-time recordings with a standard deviation better than 10^{-10} g . The strong periodical behaviour of the earth tides makes a synthetic computation possible, with a standard deviation of 10^{-9} g . Generally, local gravity acceleration is stated without the tidal influence to avoid a time-dependent validation of the value.

If the gravity is required for a certain instant of time, e.g. the realisation of the force in a force standard machine, the synthetically computed tidal correction has to be reduced in the inverse direction.
Figure 7 shows an example of tidal recordings at the PTB's force laboratory for a period of 24 hours.

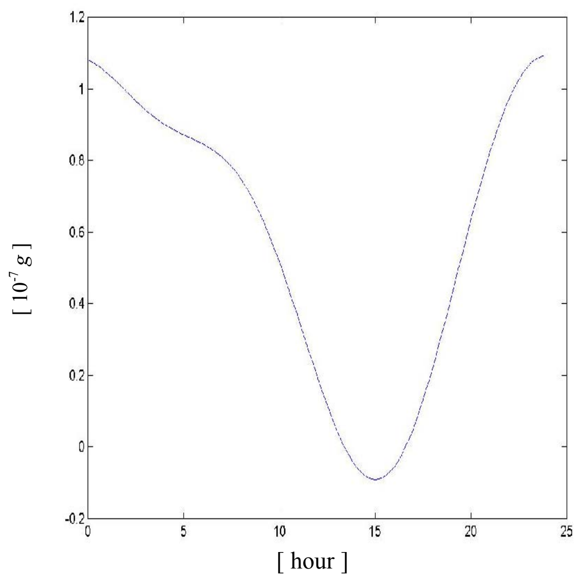

Figure 7: *Gravimetric tidal observations at PTB force lab for a period of 24 hours*

6 Uncertainty analysis

In the above analysis the achievable accuracy of gravity measurements has so far been expressed as standard deviation. A generalised form of the relative uncertainty for the gravity can be derived from equation (2) by splitting g_{loc} into its three components

$$\frac{\Delta g}{g} = \frac{\Delta \varphi}{\varphi} + \frac{\Delta h}{h} + \frac{\Delta t}{t} . \qquad (5)$$

Equation (5) can even be simplified by combination of the first two addends to a shared function of the location. From several observations (> 600) the standard deviation of the mean of this first part $s(\Delta\varphi/\varphi, \Delta h/h)$ can be set to a level of $s(\Delta\varphi/\varphi, \Delta h/h) < 10^{-8}\, g$. The time-dependent influence by the earth tides can be determined with high precision, but for the realisation of a force scale with force standard machines it is normally neglected. For the uncertainty budget the full tidal variation has to be considered, which reaches a maximum of $s(\Delta t/t) < 1.2 \cdot 10^{-7}\, g$ in the equator region. Because of the systematic behaviour of the tides, normal distribution can not be assumed. Following the mathematical rules for relative uncertainties and considering the strong domination of the time dependent component, the relative uncertainty of the gravity can be set to

$$u(g) = 1 \cdot 10^{-7}\, g.$$

7 Summary and conclusions

In the course of the investigations in the local gravity field, a close-meshed gravimetric network directly related to the force standard machines has been established. With the combination of relative and absolute gravimetric methods gravity accelerations and gradients could be derived with a standard deviation of $5 \cdot 10^{-9}\, g$.
During the installation of the new 2 MN force standard machine, the change of the gravity field due to the large mass transfer has been controlled in two independent measurement epochs. With the aid of high-resolution gravimetric sensors a gravity change of $5 \cdot 10^{-8}\, g$ could be demonstrated.
A theoretical model for the gravitational effect of the mass stack has confirmed the empirical results. This model can easily be generalised and extended with a more detailed structure, to deliver a highly resoluted gravity field without measurements.
The empirically obtained gravity accelerations are usually reduced by the periodical earth tides for being independent of the time. Nowadays, these tidal effects can be modelled and corrected with a standard deviation of $10^{-9}\, g$. If the time dependent component is neglected in force measurements, the relative uncertainty of the gravity can be set to $1 \cdot 10^{-7}\, g$. If the correction term for the tides is taken into account, the uncertainty can be reduced to about $10^{-8}\, g$.

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Contact : Dipl.Ing. Andreas Lindau
Physikalisch-Technische Bundesanstalt (PTB)
Bundesallee 100, 38116 Braunschweig
email : andreas.lindau@ptb.de