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MEASUREMENT OF DYNAMIC ROCKING IN SINUSOIDAL CALIBRATION WITH AN ELECTRODYNAMIC SHAKER

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Abstract:
Rocking motion is often a significant source of uncertainty in sinusoidal calibrations of accelerometers and force transducers. We describe real-time measurements of rocking during sinusoidal excitation, performed at NIST. The measurement is contactless and can be performed simultaneously with the motion. We report rocking measurements for excitation frequencies up to 2 kHz. The results obtained have an absolute uncertainty ($k = 2$) in the range of 0.02 µrad to 3.3 µrad and allow error due to rocking motion to be largely corrected in axial acceleration measurement.

Keywords: sinusoidal calibration; dynamic force; vibration; rocking; shaker

1. INTRODUCTION

In sinusoidal calibration of accelerometers and force transducers using electrodynamic shakers, angular vibration (rocking) of the shaker table is often a leading source of uncertainty and error. Due to problems intrinsic to the shaker (load balancing, coil assembly, heating, etc.) and intrinsic to the calibration payload (unbalance, cable loads, etc.), the motion that should be purely vertical is instead a movement in six degrees of freedom, where, in addition to the desired vertical motion (Z axis), rotations around axes normal to the Z axis predominate. These rotations are one of the main sources of uncertainty in the core frequency range of approximately 100 Hz up to several kilohertz [1]-[8].

Accurate quantification of such rocking is required in order to determine its contribution to the calibration uncertainty and to correct the acceleration measurement for the effects of rocking. As the rocking is in general dependent on the payload supported by the shaker table, which varies from one calibration to another, it is desirable to have a method to measure the rocking in-situ during the calibration. We report here the demonstration of such a measurement. Such measurements can allow significant reduction of the uncertainty due to rocking in sinusoidal calibrations.

Several studies have been done on the effect of rocking and other parasitic motions on sinusoidal acceleration measurements and calibration [2]-[7]. A common approach to reduce the errors due to rocking is to measure acceleration at several points in a circle around the desired measurement location and to average the results. Sprecher et al [6] have recently demonstrated the use of a scanning mirror to measure the elliptical rocking trajectory simultaneously with axial acceleration, allowing it to be corrected. Here we present a relatively inexpensive and straightforward method of measuring the rocking simultaneously with the axial acceleration, using a quadrant photodetector to detect transverse motion of a laser spot reflected from the target surface.

2. EXPERIMENTAL METHOD

The excited system is composed of a force transducer coupling + weight, which is an assembly of the type used in sinusoidal dynamic force mounted on an electrodynamic shaker which generates a vertical sinusoidal movement. The axial acceleration was measured by means of mounted accelerometers.

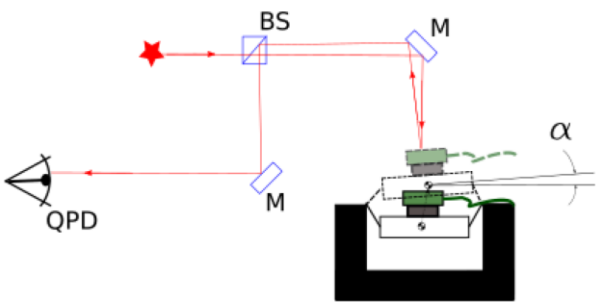

Figure 1: Dynamic rocking measurement system configuration. The optical path from the laser to the quadrant photodiode is shown, reflecting off a target on the shaker table. Tilt α of the target surface results in a transverse motion of the laser spot at the quadrant photodetector. M: mirror, BS: beamsplitter, QPD: quadrant photodetector.

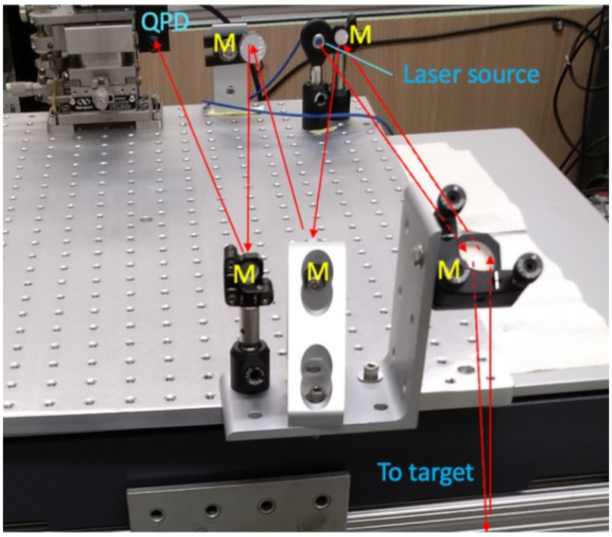

Figure 2: Photograph of experimental setup. The collimated laser beam emerges from the source and is directed downward by a mirror (M) to the target mounted on the shaker. The returning beam is directed by a series of mirrors to the surface of the QPD after traversing a path of length 3.293 m.

The rocking measurement system is mounted on an anti-vibration table and is composed of a He-Ne laser beam (λ = 633 nm) which is reflected from the upper surface of the weight onto a quadrant photodetector (QPD) [9]. A series of mirrors are used to increase the distance travelled by the beam in order to amplify the transverse displacement of the laser spot on the photodetector for a given angular rotation of the weight, and to steer the beam onto the QPD. The displacement measured at the QPD divided by twice the distance travelled by the beam provides the angle of rotation of the reflecting surface. The measurement system is shown schematically in Figure 1, and a photograph is shown in Figure 2.

Detector Calibration: A diagram of the quadrant photodetector is shown in Figure 3.

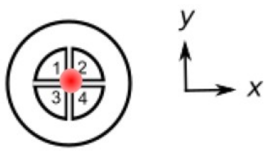

Figure 3: Diagram of quadrant photodetector. Output voltages from the four quadrants are summed, differenced, and ratioed to indicate x- and y-coordinates of the laser spot on the photodetector.

Output voltages from each of the 4 quadrants are summed and differenced internally, giving output signals V_x , V_y and V_{sum} as follows

$$V_x = V_1 + V_3 - V_2 - V_4 \,,$$
$$V_y = V_1 + V_2 - V_3 - V_4 \qquad (1)$$
$$V_{sum} = V_1 + V_2 + V_3 + V_4 \,.$$

To convert these QPD output voltages to angular deflection, it is required to know the distance travelled by the laser beam from the weight surface to the QPD, and to calibrate the QPD output voltages in terms of laser spot position on the QPD surface. The QPD calibration is performed with the same optical setup and laser spot that is used to measure the rocking, in order to accommodate the QPD response dependence on laser spot size, shape, and intensity. For the calibration, the laser beam was impinged on the QPD and kept stationary (no rocking) while the QPD was moved along the X and Y axes by means of micrometer stages. The QPD X- and Y-sensitivity data were least-squares fit with 4th order polynomials,

$$j = A\left(V_j/V_{sum}\right)^4 + B\left(V_j/V_{sum}\right)^3 + C\left(V_j/V_{sum}\right)^2 + D \cdot V_j/V_{sum} + E \,. \qquad (2)$$

where $j = (x, y)$, in millimeters. The fit coefficients for the x-calibration were

$A = (0.0859 \pm 0.2391)$ mm
$B = (0.279 \pm 0.048)$ mm
$C = (0.100 \pm 0.036)$ mm
$D = (1.026 \pm 0.005)$ mm
$E = (-0.00054 \pm 0.00093)$ mm ,

and for the y -calibration were

$A = (0.298 \pm 0.087)$ mm
$B = (0.279 \pm 0.017)$ mm
$C = (-0.112 \pm 0.013)$ mm
$D = (1.010 \pm 0.002)$ mm
$E = (-0.00058 \pm 0.00036)$ mm .

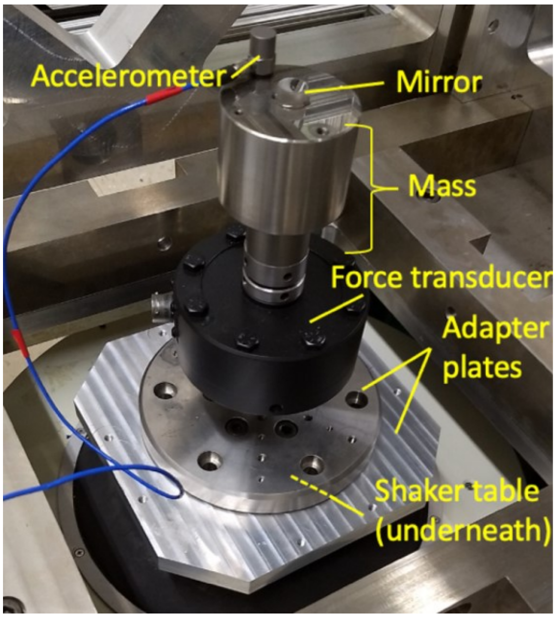

Figure 4: Mechanical stack mounted on shaker table. The mechanical stack for which rocking measurements are reported differed from the one pictured in that three additional accelerometers were mounted on top of the mass.

In both cases the voltage ratio V_j/V_{sum} is approximately equal to the laser spot displacement in millimetres. The angular deflections are then given by

$$\alpha_j = \frac{j}{2\,L}, \tag{3}$$

where L is the one-way path length between the QPD and the target surface (top of the weight) in its nominal (vertically centred) position.

The uncertainty of the displacement indicated by the QPD is estimated as

$$\begin{aligned} u_j &= R^4 u_{\text{A}} + |R^3| u_{\text{B}} + R^2 u_{\text{C}} + |R| u_{\text{D}} + u_{\text{E}} \\ &\quad + |4\,A\,R^3 + 3\,B\,R^2 + 2\,C\,R + D| u_{\text{R}}, \end{aligned} \tag{4}$$

where R is the voltage ratio V_j/V_{sum}, and the correlation of terms in the polynomial fit has been considered. As R is approximately equal to the displacement j in mm, for small j ($j < 100$ µm) we have

$$u_j \approx \left(\frac{j}{1\text{ mm}}\right)^2 u_{\text{C}} + \left|\frac{j}{1\text{ mm}}\right| u_{\text{D}} + u_{\text{E}} + |D| u_{\text{R}} \tag{5}$$

where j $(= (x, y))$ is in units of mm, and

$$u_{\text{R}} = |R| \sqrt{\left(\frac{u_{V_j}}{V_j}\right)^2 + \left(\frac{u_{V_{\text{sum}}}}{V_{\text{sum}}}\right)^2} \approx \left|R\,\frac{u_{V_j}}{V_j}\right|. \tag{6}$$

u_{V_j} was given by the resolution and noise of the oscilloscope with the shaker at rest and was 0.3 mV.

Data Processing: Rocking was measured at 100 Hz frequency intervals from 100 Hz up to 2 kHz. At a given frequency, the output voltages from the mounted accelerometers (after passing through a signal conditioner) were observed with an oscilloscope, and the drive voltage to the shaker was adjusted manually until a nominal acceleration amplitude of 15 m/s^2 was achieved. Then the output voltages V_{x}, V_{y} and V_{sum} from the QPD were simultaneously digitized at a sampling rate of 1×10^5 samples per second using a 12-bit oscilloscope. Data sets of one second duration were collected. The collected data sets were loaded into MATLAB software, where the voltages were ratioed, the ratios were converted into angles according to equations (2) and (3), the angles were fast Fourier transformed, and finally the two amplitudes and the relative phase of the two components at the excitation frequency were extracted.

3. MEASUREMENT RESULTS

Rocking measurements were made for a mechanical stack similar to the one shown in Figure 4, the only difference being the attachment

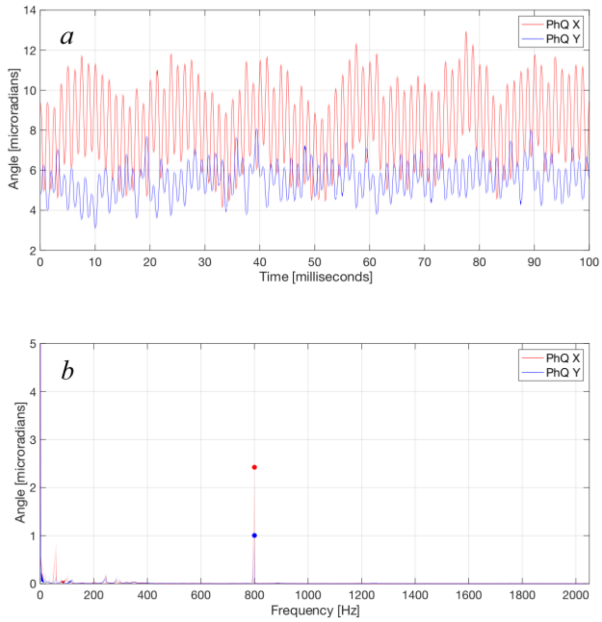

Figure 5: Example of rocking data trace acquired with a shaker excitation frequency of 800 Hz.
(a) 100 milliseconds of data is shown out of a one-second duration data collection. An approximately 180-degree phase of the y rocking component relative to the x component is evident.
(b) Fourier amplitude spectra of the x- and y-rocking angle measurements.

of three additional accelerometers on top of the weight. Rocking was measured at frequencies from 100 Hz to 2 kHz, at a constant nominal vertical acceleration amplitude of 15 m/s^2. The mechanical stack consisted of adapter plates, a 2-kilonewton capacity strain-gauge force transducer, a mechanical adapter, and a 1-kilogram weight on which were mounted a small mirror and four accelerometers. The cable of the strain gauge force transducer was not connected to the force transducer during this measurement, whereas the cables of the accelerometers were connected. Figure 5 shows an example data set plotted as x and y rocking angles versus time, and the discrete Fourier transform amplitudes. The results of the rocking motion measurement are shown in Table 1.

4. UNCERTAINTY CONTRIBUTIONS

The evaluated measurement uncertainty in the rocking angles is reported in Table 1. The sources of uncertainty in the measured rocking angles (α_x, α_y) were

1. u_{L}: Uncertainty in the distance traveled by the laser beam from the weight to the QPD, including uncertainty of the center position of the weight reflective surface during motion. The optical path length was measured as 3293 mm ± 8 mm.

2. u_j: Uncertainty in the displacement determined from the QPD outputs, as given by equation (5).
3. u_{oth}: Other contributions such as table vibration, noise during shaker motion, and variation of the signal amplitude during a measurement. These uncertainty contributions were estimated by the disagreement between measurements made using two different path lengths, and were generally larger at lower frequencies

The relative uncertainty contributions from u_{L} and u_j were added in quadrature, and u_{oth} was added to the result. The resulting combined uncertainty was then expanded by multiplying by 2. The combined and expanded uncertainties are shown in Table 1.

Table 1: Rocking measurement results

Frequency	X axis	$U_{(k=2)}$ X	Y axis	$U_{(k=2)}$ Y
Hz	µrad	µrad	µrad	µrad
100	11.50	1.29	9.52	1.27
200	12.27	3.29	3.72	0.98
300	1.25	1.45	0.77	2.35
400	1.47	3.07	0.64	0.37
500	0.46	0.24	0.61	0.23
600	0.85	0.31	0.64	0.17
700	1.24	0.15	0.68	0.09
800	2.42	0.38	1.00	0.12
900	13.37	1.44	9.91	1.04
1000	2.06	0.23	0.29	0.04
1100	1.02	0.11	0.63	0.08
1200	0.65	0.08	1.16	0.13
1300	0.28	0.04	0.59	0.07
1400	0.25	0.03	0.15	0.02
1500	0.47	0.06	0.21	0.03
1600	0.25	0.04	0.37	0.06
1700	0.28	0.03	0.54	0.08
1800	0.34	0.07	0.89	0.13
1900	0.41	0.06	0.66	0.15
2000	0.57	0.07	1.06	0.11

5. DISCUSSION

The measurements achieved expanded relative uncertainties of below 0.2 for rocking measurements outside the frequency range 200 Hz – 600 Hz, as shown in Table 1. The measurement method is straightforward and can be applied simultaneously (in-situ) with vibrometer or accelerometer axial acceleration measurement. It is rapid (measurement times one the order of one second), and therefore avoids errors due to variation of the rocking motion with time, which is often observed and may be attributed to heating up of the shaker.

The method requires a mirror surface on the target, in order to provide a well-defined and consistent laser spot shape on the QPD. In calibration systems employing laser vibrometers, the laser beam used for measuring the rocking can be the same vibrometer beam that is used for axial acceleration measurement, achieved by using a beam splitter. If focusing optical elements are employed in the optical path between the target and the QPD, the calibration of the QPD must be done in a way that includes the effect of such elements.

The rocking measurements provide not only the two components of the rocking motion but also the relative phase between them. Additionally, the relative phase of the rocking motions with respect to the axial acceleration measurement is provided. The measurements allow the acceleration uncertainty due to rocking to be analytically reduced [6], [8].

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† Certain commercial instruments are identified in this article in order to describe the experimental procedure adequately. Such identification is not intended to imply recommendation or endorsement by the National Institute of Standards and Technology, nor is it intended to imply that the instruments identified are necessarily the best available for the purpose.