

## MINIMISING THE EFFECTS OF SPURIOUS RESONANCES IN SECONDARY VIBRATION CALIBRATION

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### Abstract:

This paper explores the use of fitted sensitivity values in order to minimize the effects of spurious resonances in the frequency response of reference accelerometers used in secondary vibration calibration.

**Keywords:** Back to back vibration calibration; non-linear regression; reference accelerometer; curve fitting

## 1. INTRODUCTION

KEBS performs calibration of vibration accelerometers and vibration measuring instruments using back to back method according to ISO 16063-21 [1]. To calibrate these instruments, a back-to-back reference standard accelerometer (B&K 8305, Serial No. 2519460, from Brüel & Kjær Company, Denmark) is used within a frequency range of 10 Hz to 10 kHz. KEBS took part in inter-laboratory comparisons (ILCs) [2]-[4], where results in the 8 kHz to 10 kHz frequency range showed deviations that were greater than the associated uncertainty of measurement. Similar deviations were evident during calibration of KEBS' internal single-ended accelerometer (PCB J353B01, Serial No. 56216, from PCB®, USA) using the KEBS reference accelerometer. Consequently, investigations were launched into the possible cause(s) for the discrepancy as presented in the next sections.

## 2. POSSIBLE CAUSES OF DISCREPANT VIBRATION CALIBRATION RESULTS

### 2.1. Mass loading

The B&K 8305 accelerometer was calibrated at NMISA according to ISO 16063-11 [5], with its top surface as the reference surface. To determine if there was any change in sensitivity due to mass loading, the accelerometer was calibrated again (repeated) by loading drilled 20 gram and 30 gram dummy masses (made from tungsten) on the reference sensor as shown in Figure 1.

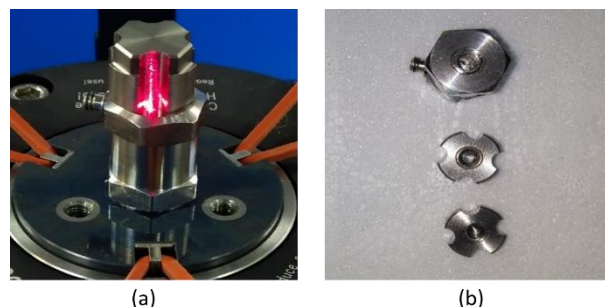


Figure 1: (a) Reference accelerometer (B&K 8305) loaded with a drilled 30 gram tungsten dummy mass. (b) Dummy masses of different nominal values.

From the on-reference sensor mass loading measurements (Figure 2), no significant change in the sensitivity was observed. However, there was a shift in the position of spurious resonances in the frequency response (points A and B).

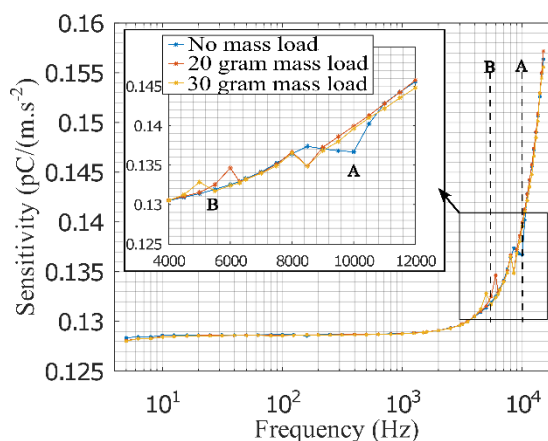


Figure 2: Dependence of spurious resonances on mass loading observed as a shift in the position of non-linear behaviour.

The results of fitting the frequency dependent accelerometer sensitivity model is shown in Figure 3, and described in details in section 3.1.

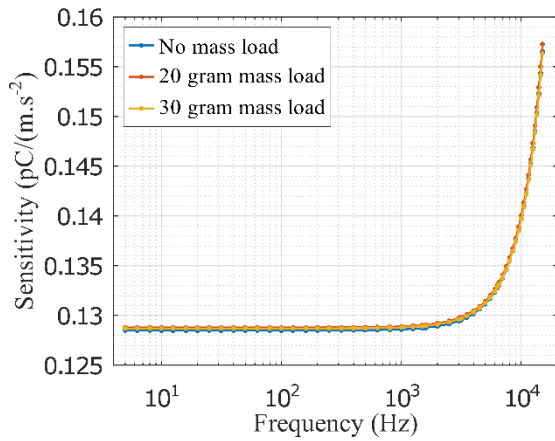


Figure 3: Fitted sensitivity (magnitude) of the B&K 8305 reference sensor under loaded and unloaded conditions.

## 2.2. Mounting stud material

Accelerometer PCB J353B01 was calibrated at NMISA according to ISO 16063-21 [1] to determine if the choice of stud material had an influence on its frequency response. From the results illustrated in Figure 4, no substantial change in sensitivity at the frequencies of interest was observed when uncertainty of measurement was taken into consideration.

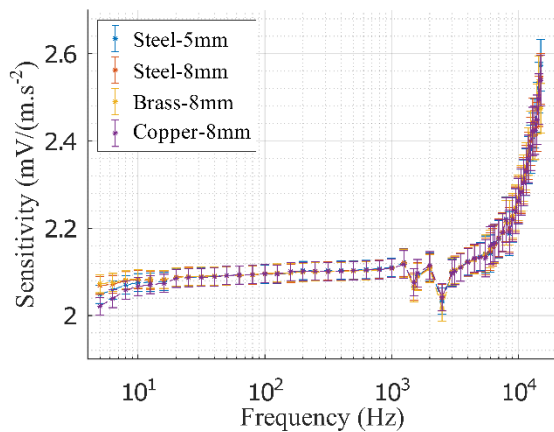


Figure 4: Effect of stud material on sensitivity of PCB J353B01.

## 2.3. Intrinsic resonances in the reference sensor

KEBS back to back accelerometer B&K 8305 and single ended accelerometer PCB J353B01 were calibrated according to ISO 16063-11 [5] and their responses evaluated for non-linearities. It was observed (Figure 5) that the back-to-back accelerometer had some non-linear behaviour around the 8 kHz region.

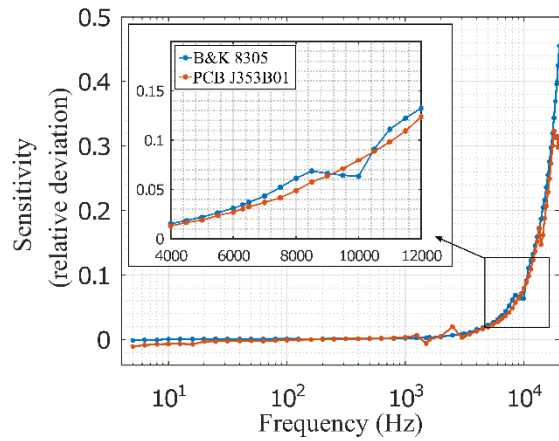


Figure 5: Frequency response (magnitude) of KEBS BK8305 (blue line) and PCB J353B01 (red line) sensors.

## 2.4. Transverse sensitivity and motion

KEBS uses a B&K 4809 flexure-based vibration exciter. Measurements according to ISO 16063-21 [1] were carried out on the SPEKTRA SE-09 air-bearing vibration exciter at NMISA. The measurements were repeated at KEBS using its B&K 4809 exciter to investigate the possible contribution of exciter transverse motion on measurement results. As depicted in Figure 6, no major deviations between the results were observed when uncertainty of measurement was taken into account.

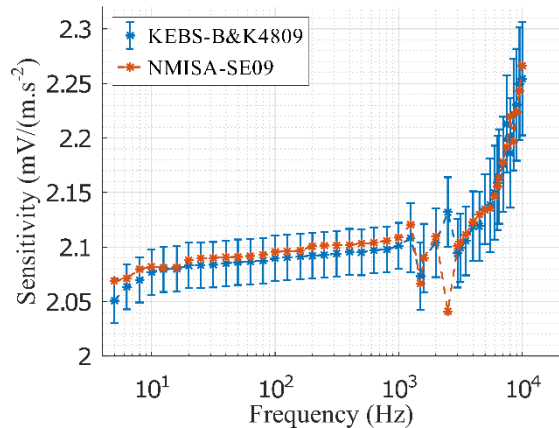


Figure 6: Comparison between results obtained using NMISA SE-09 exciter and KEBS B&K 4809 exciter

## 3. MINIMISING EFFECTS OF SPURIOUS RESONANCES USING FITTED REFERENCE SENSITIVITY

The use of fitted reference sensitivities to minimise the influence of spurious resonances in secondary calibration results was explored. The methods used and findings thereto are described in following sections.

### 3.1. Fitting of reference sensitivity

The magnitude response of accelerometers can be modelled according to equation (1) [6], which

represents piezoelectric accelerometers as single degree-of freedom spring-mass systems.

$$S(f) = \frac{S_0}{\sqrt{\left(1 - \left(\frac{f}{f_n}\right)^2\right)^2 + 4\zeta^2 \left(\frac{f}{f_n}\right)^2}}, \quad (1)$$

where,  $f_n$  is the undamped natural frequency (in Hz),  $f$  is the frequency (in Hz),  $S(f)$  is the sensitivity magnitude at frequency  $f$ ,  $S_0$  is the low frequency reference sensitivity magnitude, and  $\zeta$  is damping coefficient.

The results of calibration obtained in section 2.1 were fitted to the model by use of the Octave optimisation function, ‘leasqr’ which utilises the Levenberg-Marquardt minimisation algorithm [7]. Table 1 shows the fitting parameters and coefficient of determination ( $R^2$ ) obtained for the three cases.

The results of fitting suggested a minimal effect on sensitivity due to mass loading.

Table 1: Fitting parameters for KEBS B&K 8305 accelerometer.

Parameter	No mass load	20 gram mass load	30 gram mass load
$S_0$ (pC/(m.s <sup>-2</sup> ))	0.1285	0.1288	0.1287
$f_n$ (Hz)	34.0	35.3	35.7
$\zeta$	0.18	0.01	0.01
$R^2$	0.999	0.997	0.997

### 3.2. Use of fitted reference sensitivities

KEBS accelerometer (PCB J353B01) was calibrated using the BK8305 accelerometer using fitted and non-fitted reference sensitivities, and results were compared with those obtained from primary calibration. As depicted in Figure 7, the results of calibration using fitted reference sensitivities showed better agreement with those from primary calibration. Table 2 shows the coefficient of determination ( $R^2$ ) values between results of primary calibration and secondary calibration.

Table 2: Coefficients of determination ( $R^2$ ) between results of primary and secondary calibrations with fitted and non-fitted reference values.

	Non-fitted reference sensitivity	Fitted reference sensitivity
$R^2$	0.92	0.97

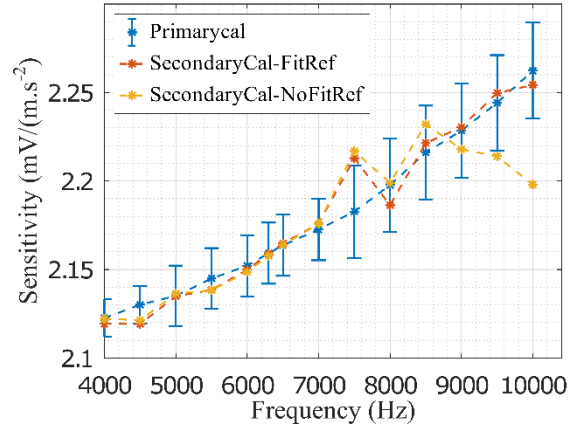


Figure 7: Results of calibration of PCB J353B01 sensor using primary method [4] and secondary method [1] with fitted and un-fitted reference sensitivities.

### 3.3. Application of fitted reference sensitivities

Corrections to KEBS results of EURAMET.AUV.V-S1 [2] were applied according to:

$$S_{\text{Corr}} = S_{\text{KE}} \cdot \frac{S_{\text{ref2}}}{S_{\text{ref1}}}, \quad (2)$$

where,  $S_{\text{Corr}}$ ,  $S_{\text{KE}}$ ,  $S_{\text{ref1}}$ , and  $S_{\text{ref2}}$  denote the corrected comparison result, the actual comparison result, un-fitted reference sensitivities, and the fitted reference sensitivities, respectively. The results of this correction for sensors BK8305-001 and EN2270 are illustrated in Figure 8 and Figure 9. Generally, the applied corrections tend to show improved results for both accelerometers.

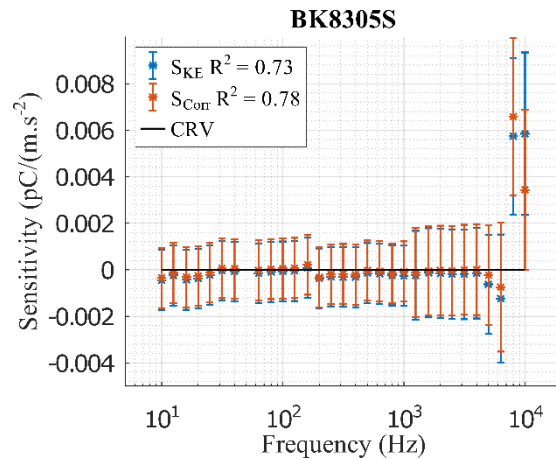


Figure 8: Correction of KEBS comparison results [2] for the calibration of the accelerometer BK8305-001.

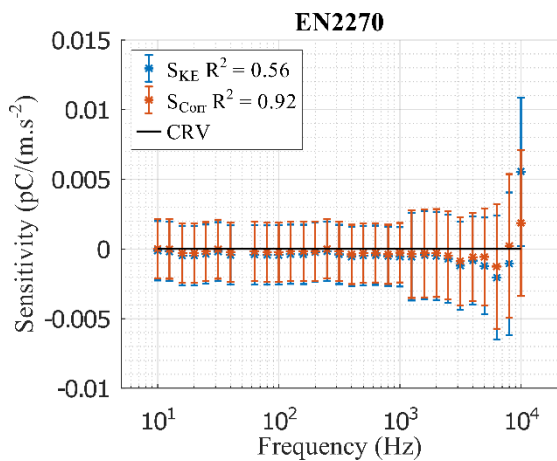


Figure 9: Correction of KEBS comparison results [2] for the calibration of the accelerometer EN2270.

Fitting of the calibration results of BK8305-001 sensor was necessary in order to improve results in the 8 kHz band. Figure 10 compares the results of fitted and non-fitted calibration values for the sensor.

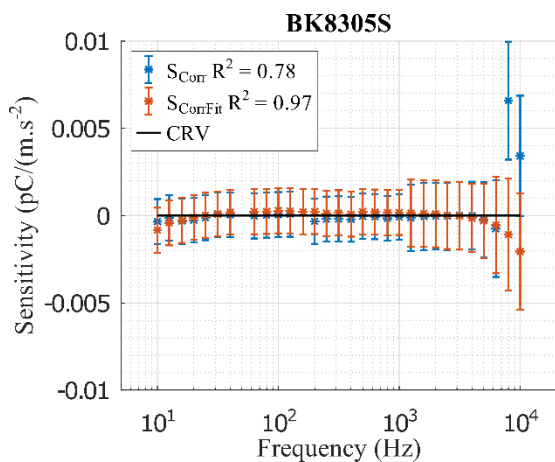


Figure 10: Results of fitting calibration results for the accelerometer BK8305.

#### 4. SUMMARY

Secondary (back-to-back) calibration systems are sensitive to non-linearity in the frequency response of the reference accelerometer. It is therefore important to establish the frequency response of the reference in order to correct non-

linear influences on the sensitivity of the device under test (DUT).

The use of mounting studs of different materials does not have a noticeable influence on calibration results at uncertainties associated with secondary calibration ( $\geq 0.5\%$ ) as long as torque requirements are fulfilled.

Calibration of the B&K 8305 accelerometer with drilled dummy mass resulted in minimal change in sensitivity. However, a dependence on mass of spurious resonances within the frequency response was observed.

A good comparability of results was obtained for primary and secondary calibration methods when fitted reference values were used.

#### 5. REFERENCES

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