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ELECTRICAL IMPEDANCE BASED CALIBRATION OF ACCELEROMETERS

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Abstract:
This contribution proposes a methodology to predict the transfer function of an accelerometer solely based on an electrical impedance measurement. Such impedance measurement can be conducted in-situ and has certain advantages for some types of accelerometers and under certain measuring conditions. A proof-of-concept study has been conducted where the methodology is used to estimate the sensitivity at 160 Hz, the proposed method underestimates the sensitivity by 9% compared to a conventional vibration experiment.

Keywords: Accelerometer; in-situ characterization; electrical impedance; transfer function

1. INTRODUCTION

The transfer function of accelerometers is conventionally characterized by applying a known acceleration using a shaker. This method is superior for most accelerometer characterization. However, a precise and controlled mechanical excitation can become difficult to achieve in some cases. One such case is characterization of accelerometers with large housing where structural resonances will occur at low frequencies, another case is calibration of triaxial accelerometers, where two of the three directions are characterized by mounting the accelerometer with tape or vacuum glue in the unintended directions. Finally, a shaker calibration characterizes the response under the mounting conditions at the shaker, however these conditions will not be the same as the mounting conditions at the in-situ application. This work proposes a methodology to characterize the accelerometer transfer function using electrical impedance analysis. This methodology is advantageous in the afore mentions cases where mechanical excitation can be difficult.

2. DESCRIPTION OF THE WORK

The presented analysis can be divided into the mechanical and the electrical domain. The harmonic oscillator model is used to describe the mechanical properties, whereas electrical domain is modelled by a capacitor. The two domains are coupled by a transformer ratio Γ. The model can be visualized by the lumped element diagram shown in Figure 1, and is inspired by the analysis presented in [1] and [2].

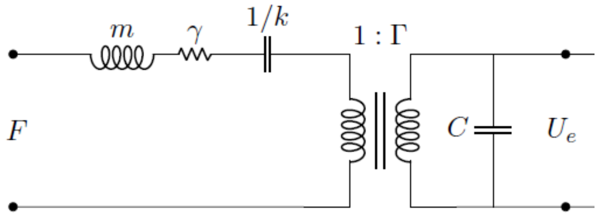

Figure 1. Sketch of the lumped element model. Mechanical domain on the left side of the transformer and the electrical domain on the right side of the transformer.

The mechanical impedance is given by

$$Z_m = sm + \gamma + \frac{k}{s}, \tag{1}$$

where m is the mass, γ is the damping coefficient, k is the spring constant, and $s = i\omega$, with ω being the angular frequency. The mechanical forces are in the frequency domain governed by

$$Z_m V = F + \Gamma\, U_e\,, \tag{2}$$

where V is the velocity, F is an excitation force and U_e is the potential across the piezoelectric material. The electrical domain can be described by Kirchhoff's law and the current, I, in this case yields

$$I = \Gamma\, V + s\, C\, U_e\,, \tag{3}$$

where C is the capacitance. The electrical impedance, Z_e , can be found by substituting Eqn. (2) into (3), which yields

$$Z_e = \frac{Z_m}{\Gamma^2 + s\, C\, Z_m}, \tag{4}$$

the well-known Butter-Worth Van Dyke lumped element impedance. The input parameters to Eqn. (4) can be found by fitting to experimental impedance measurements, and when these parameters are estimated the transfer function can be predicted by

$$\frac{Q}{A} = \frac{\Gamma m}{s Z_m}, \tag{5}$$

where Q is charge and A is acceleration both quantities defined in the frequency domain.

Hence, one can measure the electrical impedance in-situ, fit to the experimental data, and then estimate the transfer function afterwards.

The only requirement for the measuring equipment is the capability of applying an AC voltage while measuring the corresponding AC current. Such task can be performed by a LAN-XI module, which has been used to verify the proposed methodology.

Figure 2 A) shows the measured impedance of a HBK accelerometer with Eqn. (4) fitted to the frequency regions around the main resonance at approximately 40 kHz. The model and the experimental data show excellent agreement, and a clear resonance at 40.21 kHz and an anti-resonance at 40.36 kHz can be identified. The electromechanical coupling coefficient can be determined from the resonance, f_0, and anti-resonance, f_1, and is governed by

$$\kappa^2 = 1 - {f_0^2}/{f_1^2}, \tag{6}$$

and is estimated to be 0.007 for this particular accelerometer. Thus, the impedance analysis allows for additional information, which is not easily determined from conventional vibration measurements. Figure 2 B) shows the estimated transfer function based on the measured impedance together with the sensitivity measured at 160 Hz using a HBK type 4808 shaker.

Table 1: Fitted parameters

Parameter	**Value**	**Unit**
m	2.85	g
γ	7.10	N s/m
k	1.82×10^8	N/m
C	228	pF
Γ	0.012	N/V

The fitted parameters from the impedance measurement are provided in Table 1. The capacitance and the mass are two parameters which are easy to measure prior to the impedance analysis. Hence, an initial guess for the capacitance can be obtained by a simple capacitance meter and the weight of the sensing unit can be used as an initial guess for the mass. Since the mass is known with high precision an initial guess for the spring constant can also be obtained by identifying the resonance in the impedance spectra and subsequently use the conventional formula for the resonance frequency given by

$$2\pi f_0 = \omega = \sqrt{\frac{k}{m}}. \tag{7}$$

An educated initial guess for the transformer ratio, Γ, can also be obtained by evaluating the ratio between the resonance and anti-resonance frequency, which can be expressed as

$$\frac{f_0^2}{f_1^2} = \left(1 + \frac{\Gamma^2}{kC}\right)^{-1}. \tag{8}$$

Hence, by combining the initial guesses for the spring constant, the capacitance and the resonance and anti-resonance from the in the impedance spectrum it is possible to obtain an educated initial guess for the transformer ratio. The damping coefficient is the final input parameter for which no obvious initial guess can be derived. Thus, the damping coefficient must be found by manually tuning the value. The final parameter tweaking is done by a least square minimization routine yielding the fitted model shown in Figure 2 A).

Notice, the impedance measurements are conducted at the frequency range from 20 kHz to 70 kHz and the information around resonance at 40 kHz is sufficient to estimate the sensitivity at 160 Hz with a 9% accuracy.

The estimated sensitivity is 1.87 *pC/g* by the proposed impedance methodology, whereas the sensitivity is measured to be 2.06 *pC/g* using the conventional shaker set-up.

Hence, the impedance method underestimates the sensitivity by 9%.

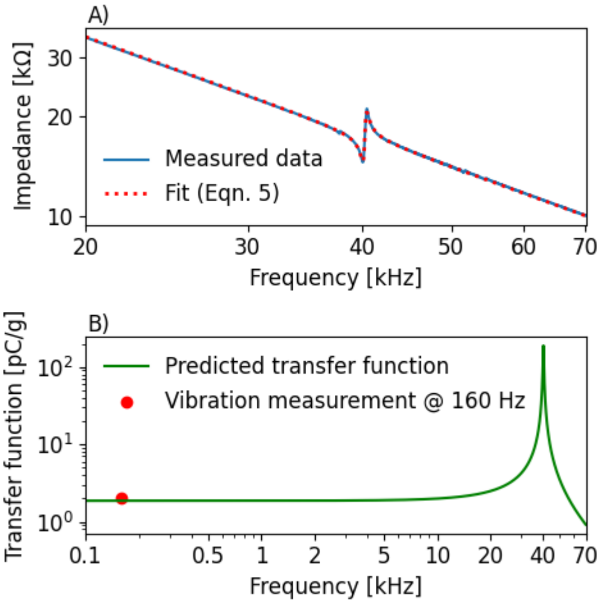

Figure 2: Proposed methodology. The measured impedance together with a fitted Eqn. (5) are shown in A). The fitted parameters are used to predict the transfer function in B), which is compared with a sensitivity measurement at 160 Hz. Notice, the x-axis differs between the two plots.

The accuracy of the impedance-based estimate is not yet satisfying, however, it is believed to be further improved by including additional modes, such as the transverse resonance frequency, as well as optimizing the electrical conditioning during the impedance measurements, both of these tasks will be investigated in future work.

3. SUMMARY

A novel impedance-based methodology for estimating the sensitivity of an accelerometer is presented. This method can be conducted in-situ and a proof-of-concept measurement has been presented where the difference between the proposed impedance methodology and a conventional vibration measurement at 160 Hz show a discrepancy of 9 %.

4. REFERENCES

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