

## REALIZATION OF SINUSOIDAL FORCES AT CEM

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**Abstract:** This paper describes the procedure implemented at CEM for dynamic force calibration using sinusoidal excitations of force transducers. The method is based on a sinusoidal excitation of force transducers equipped with an additional top mass excited with an electrodynamic shaker system. The acceleration is measured by means of a laser vibrometer.

**Keywords:** Dynamic force, sinusoidal excitation, laser vibrometer, harmonic distortion, rocking motion.

### 1. INTRODUCTION

Static calibration of force transducers has a well established procedure, but most applications of force transducers are in dynamic conditions. That is the reason why dynamic calibration is essential for a proper characterization of a force transducer. This also happens with other quantities so that more and more research is being performed in metrology for dynamic conditions.

This work is part of a project called “Traceable dynamic measurement of mechanical quantities” financed by the European Union under the European Research Metrology Program. The aim of the project is to provide traceability for pressure, torque and force in dynamic conditions [1].

The realization at CEM is focused on the development of a primary standard for sinusoidal forces. This standard is based on the direct definition of force as mass times acceleration. The transducer is loaded with different calibrated masses and different accelerations are generated by a vibration shaker system. The acceleration is measured by a laser vibrometer traceable to the unit of length (laser wavelength). The transducer is characterised by its dynamic sensitivity, which is the ratio of its electrical output signal of the force transducer and the acting dynamic force.

Being a fully dynamic measurement it requires a multichannel data acquisition system in real time. This system will acquire electrical signals from the laser vibrometer, sensor under calibration and other auxiliary accelerometers in real time. This system is very important,

and its correct implementation is crucial for the optimal functioning of the whole system.

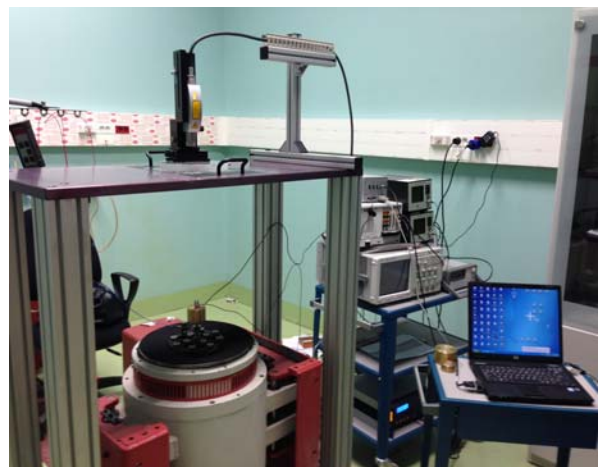


Fig 1: Overview of the system

### 2. DESCRIPTION

A special table for placing the laser vibrometer (Polytec CLV 2534) over the shaker (LDS 726 with power amplifier PA 2000, which can work up to 2400 Hz) has been designed (Fig. 1). The laser vibrometer is a modified Mach-Zehnder interferometer and it incorporates a Bragg cell (opto-acoustic modulator). The signal from the photodetector is mixed with two phase-shifted  $\pi/2$  signals with the Bragg cell frequency in separate branches. When they pass through their corresponding low pass filters, two signals in quadrature are obtained that contain the information of displacement against time. Such information is obtained by the arc tangent method with a special algorithm based on the establishment of the function continuity for  $\pi$  multiples. As a result a voltage output proportional to the velocity of the sinusoidal movement under measurement is obtained from the laser vibrometer.

As an auxiliary sensor a B&K 8305 accelerometer with its signal conditioner B&K 2525 is used.

The required masses for generating the forces on the transducers have been manufactured and calibrated to

determine their mass and their corresponding uncertainty. The masses have nominal values 347 g, 1 kg, 2 kg, 7.3 kg and 12.3 kg. The three smaller masses are connected directly to the sensor under calibration, the bigger ones are connected by means of a special adaptor. These masses define the maximum force the system can generate, but further studies with larger masses are planned to be implemented in the future.

Depending on the sensor to be calibrated special adaptors may be required in order to attach the masses to the sensor or the sensor to the shaker. Special care has to be taken to ensure that the contact between pieces is made by means of flat surfaces. The torque which is applied to connect the different parts is also under careful control.



Fig 2: View of the masses and the adaptor used for that purpose also connected to a sensor.

As a first data acquisition system the Agilent Infinium oscilloscope DSO8064A (600 MHz, 4GS/s) of four channels was used. This oscilloscope acquired electrical signals directly as a function of time and it allowed to export the data for subsequent mathematical processing by means of the sine approximation method. Its problem was that signals values were not accurate enough, so a more accurate system had to be implemented.

The current data acquisition system is a NI PXI 1033 module with a 4462 card (24 bits, 204.8 kS/s), which is programmed in Labview. In the first version of the software the data acquisition system was only used to acquire the signals as functions of time and export them into a file. After different versions the current one directly calculates the transducer sensitivity (modulus and phase) in real time both with the laser vibrometer as the reference standard and with a B&K 8305 accelerometer with its signal conditioner B&K 2525 as the reference standard. As a first check of the reliability of the software the accelerometer was located in the centre on the top of one mass and acceleration measurements were performed with both, the accelerometer and the laser vibrometer, with agreement within measurement uncertainty.

The implemented software samples the signals separately with a speed of 40 kS/s. The excitation frequency is directly detected by the software, so that the conversion of

the output of the laser vibrometer as velocity into acceleration is straight forward.

Harmonic distortion and noise were very important disturbing factors. As a first step some filtering techniques were implemented in Labview to minimize its effects such as choosing just the fundamental frequency components or eliminate the noise. These techniques were implemented through Labview functions. In the end the sine approximation method, also implemented with Labview functions in real time, was the chosen option. According to this method every signal  $a(t_i)$  is fitted according to:

$$a(t_i) = A \cos(2\pi f t_i) - B \sin(2\pi f t_i) + C \quad (1)$$

$i = 1 \dots N$  and  $N$  the total number of measurements considered for the fitting. This generates an overdetermined system of equations in which  $A$ ,  $B$  and  $C$  can be found. Amplitude  $\hat{a}$  and phase,  $a_\varphi$  can then be determined as

$$\begin{aligned} \hat{a} &= \sqrt{A^2 + B^2} \\ a_\varphi &= a \tan\left(\frac{A}{B}\right) \end{aligned} \quad (2)$$

In our case this method has been implemented considering also harmonics as follows

$$\begin{aligned} a(t_i) &= A_0 \cos(2\pi f t_i) - B_0 \sin(2\pi f t_i) + C + \\ &+ A_1 \cos(4\pi f t_i) - B_1 \sin(4\pi f t_i) + \\ &+ A_2 \cos(6\pi f t_i) - B_2 \sin(6\pi f t_i) + \\ &+ A_3 \cos(8\pi f t_i) - B_3 \sin(8\pi f t_i) \end{aligned} \quad (3)$$

For sensitivity determination only the fundamental frequency components are considered

$$\begin{aligned} \hat{a} &= \sqrt{(A_0)^2 + (B_0)^2} \\ a_\varphi &= \arctan\left(\frac{A_0}{B_0}\right) \end{aligned} \quad (4)$$

This least squares approximation is performed by means of the singular value decomposition method.

### 3. MEASUREMENT MODEL AND INFLUENCE FACTORS

The parameter that characterises the force transducer is the sensitivity  $S$ , which is the ratio of the electrical output signal of the transducer to the acting dynamic force. Its modulus can be determined as

$$S = \frac{U/V}{(m + m_i) \cdot a} \quad (5)$$

where  $U$  is the output of the conditioning amplifier,  $V$  is the amplification factor of the conditioning amplifier,  $m$  is the mass for loading the transducer,  $m_i$  is the internal mass of the transducer that contributes as a load and  $a$  is the acceleration measured by the laser vibrometer.

The sensitivity phase is determined as the phase difference between the sensor output and the laser vibrometer output.

The system behaves as a mechanical resonator, this is a loading mass  $m$  connected to the sensor by a spring of stiffness  $k$  and velocity-proportional damper of damping factor  $b$ , according to equation (6)

$$m \cdot \ddot{x}_{up} = -k \cdot (x_{up} - x_{down}) - b \cdot (\dot{x}_{up} - \dot{x}_{down}) \quad (6)$$

In order to study this behaviour the accelerations of the top of the mass  $\ddot{x}_{up}$  and the top surface of the shaker  $\ddot{x}_{down}$  have to be measured (see figure 3). This is performed by the laser vibrometer at the top of the mass and an accelerometer with the signal conditioner at the top surface of the shaker. The acquisition system is also used to perform these measurements.

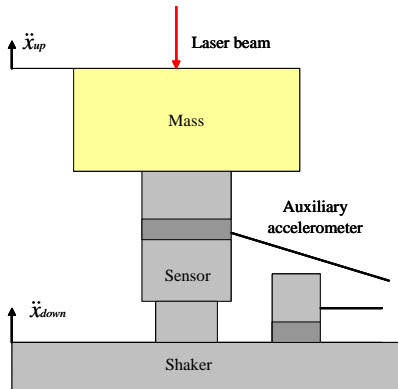


Fig 3: Simplified diagram that shows the measurement configuration..

The ratio of these two accelerations follows

$$\frac{a_{up}}{a_{down}} = \frac{k + ib\omega}{k + ib\omega - m\omega^2} \quad (7)$$

where  $m$  is the loading mass,  $\omega$  is the angular frequency,  $k$  is the stiffness and  $b$  is the damping factor. Figures 4 and 5 qualitatively show this behaviour.

There are many influences in this system which are potential sources of uncertainty. Among the potential sources of type B uncertainty the ones caused for external traceability are included in Table 1.

The laboratory conditions are stable enough so that temperature influence on the transducer can be considered negligible for most transducers.

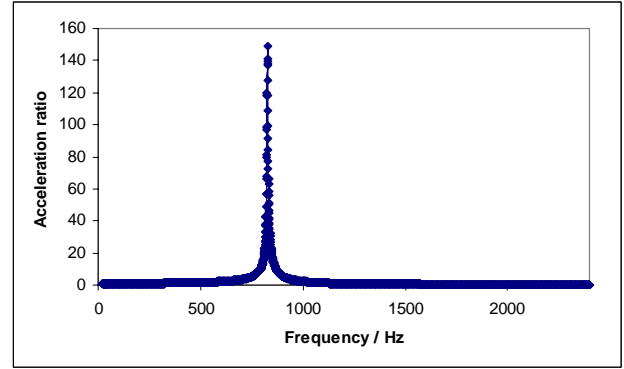


Fig 4: Acceleration ratio modulus for a Kistler 9175 B transducer and a 12.3 kg loading mass.

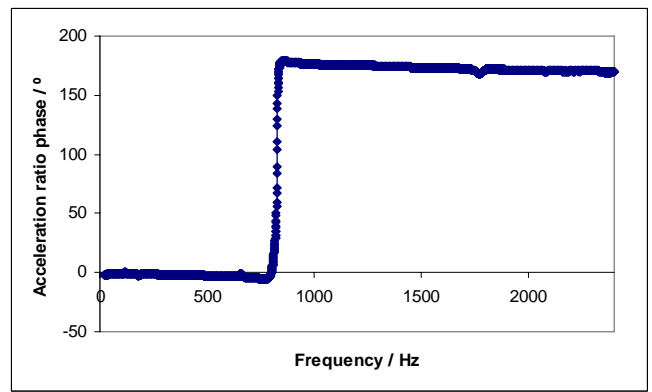


Fig 5: Acceleration ratio phase for a Kistler 9175 B transducer and a 12.3 kg loading mass.

Uncertainty contribution	Contribution to the sensitivity modulus	Contribution to the sensitivity phase
Data acquisition system	0.05 %	$2 \times 10^{-3}$ °/frequency
Laser vibrometer	0.1 %	0.25 °
Conditioning amplifier	0.05 %	0.25 °
Loading mass	0.01%	-

Table 1: Sources of type B uncertainty derived from external traceability.

The most important contributions are the sources of type A uncertainty, especially for the sensitivity modulus, but not so much for the sensitivity phase. They also depend strongly on the excitation frequency for the sensitivity modulus, but not for the sensitivity phase.

One source of type A uncertainty is the the fitting mean square error of the sine approximation. This source of uncertainty quantifies the harmonic distortion and the noise. The software performs a fitting per 40 kS, so if the harmonic distortion and the noise are small, the contribution maybe negligible.

Another important effect is rocking motion. This effect has been broadly studied for accelerometers [2]. In other to study this effect we used the laser vibrometer to measure the

velocity of several dots in a circle around the centre on the top of the mass. This effect has been found to be very important, especially for high frequencies. Figures 6, 7 and 8 show an example of this effect.

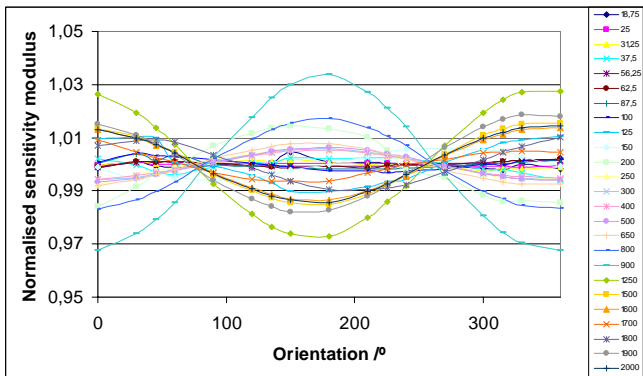


Fig 6: Each line represents the normalised sensitivity modulus to the mean value for each measurement frequency for the piezoelectric transducer Kristler 9175 B when it is loaded with the 7.3 kg mass. The laser vibrometer beam has been focused on the adaptor several dots around a circle centered on the mass top surface with radius 1 cm. In the x axis the orientation of each dot in degrees is presented.

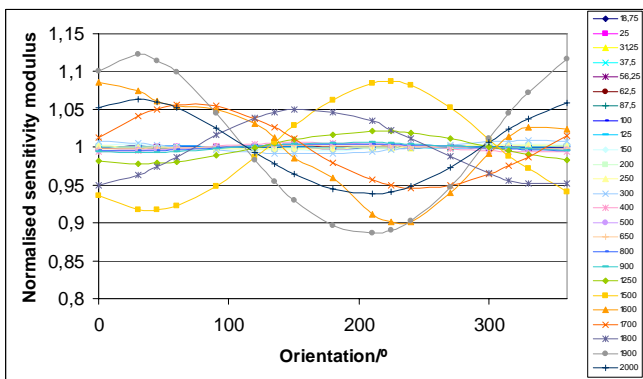


Fig 7: Each line represents the normalised sensitivity modulus to the mean value for each measurement frequency for the piezoelectric transducer Kristler 9175 B when it is loaded with the 2 kg mass. The laser vibrometer beam has been focused on the mass several dots around a circle centered on the mass top surface with radius 1,7 cm. In the x axis the orientation of each dot in degrees is presented.

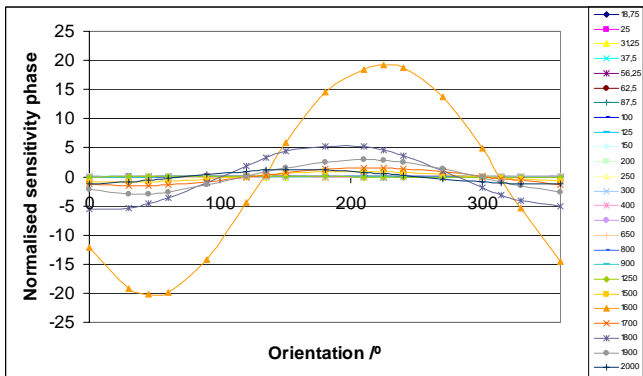


Fig 8: Each line represents the normalised sensitivity phase to the mean value for each measurement frequency for the piezoelectric transducer Kristler 9175 B when it is loaded with the 2 kg mass. The laser vibrometer beam has been focused on the mass several dots around a circle centered on

the mass top surface with radius 1,7 cm. In the x axis the orientation of each dot in degrees is presented.

Figures 9 and 10 show this effect for a fixed frequency as a function of the position of the laser beam on the top of the mass. It can be concluded then that the best dot to measure the acceleration by means of the laser vibrometer is the center of the loading mass. The uncertainty contribution is usually less than 0.01% for frequencies without transverse resonances.

Transverse acceleration can be an important influence. In practice, if the transverse acceleration is an important influence for a certain frequency, this frequency is not considered. The reason for its importance may be the existence of a transverse resonance for this frequency. As an example of the effect of transverse acceleration the line for 1600 Hz shows some distortion in 7 and the amplitude of this line is much bigger than others in figure 8.

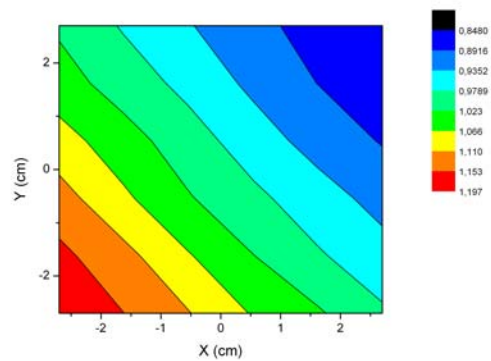


Fig 9: Contour plot of the normalised sensitivity modulus for 1500Hz as a function of the position of the laser beam on the top of the mass.

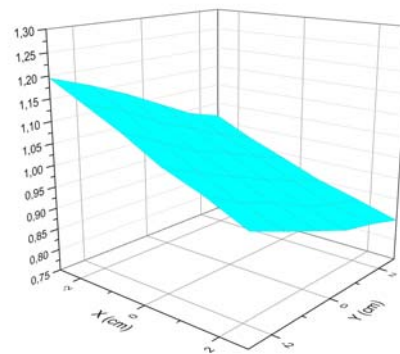


Fig 10: 3D plot of the normalised sensitivity modulus for 1500Hz as a function of the position of the laser beam on the top of the mass.

Most times the loading mass of the transducer is not known, so it has to be determined from the measurement results. The idea is to measure the sensitivity for different masses and find the loading mass of the sensor taking into account that the sensitivity for static conditions (zero frequency) has to be independent from the loading mass. It is clear that the uncertainty of the loading mass of the sensor

will affect more the sensitivity modulus the smaller the loading mass is, but it is very useful to use small masses for sensitivity determination in order to be able to obtain an accurate loading mass of the sensor.

One most important influence is the reproducibility of the mounting. Care should be taken in using an adequate and reproducible mounting torque for all the components of the system: mass, sensor, shaker and adaptors. Figure 11 shows the differences in the system resonant behaviour when mounting torque is not adequate. Further studies have to be performed in order to get more information of the mounting torque influence.

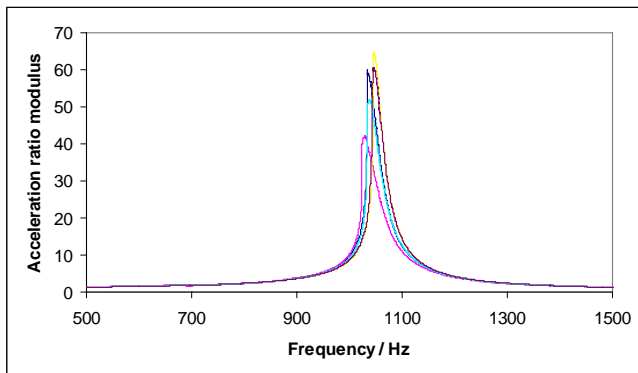


Fig 11: Acceleration ratio modulus for the same system in different test when the mounting torque is not fixed.

#### 4. CONCLUSIONS

This paper describes the procedure implemented at CEM for dynamic force calibration using sinusoidal excitations of force transducers. It is a contribution to the EMRP project “Traceable dynamic measurement of mechanical quantities”.

The minimum calibration expanded uncertainty that can be obtained is around 0.5 % for the sensitivity modulus and 1° for the sensibility phase. This uncertainty may increase depending on the excitation frequency.

#### Acknowledgement

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#### 5. REFERENCES

- [1] C. Bartoli, M. F. Beug, T.s Bruns, C. Elster, T. Esward, L. Klaus, A. Knott, M. Kobusch, S. Saxholm, C. Schlegel “Traceable dynamic measurement of mechanical quantities: objectives and first results of this European project”, Proceedings of XX IMEKO World Congress, 2012.
- [2] A. Cabral, I. Godinho, V. Oliveira, J. M. Rebordão, M. C. Nunes “Primary laser vibration metrology: evaluation of the rocking motion impact in the accuracy of acceleration measurements” Proc. of SPIE Vol. 7098 70980Z-3