

## Voltage measurements for distorted waveforms.

H. Ndilimabaka, P. Espel, A. Poletaeff

LNE, 29 avenue Roger Hennequin, 78197 Trappes.  
patrick.espel@lne.fr ; andre.poletaeff@lne.fr

*Abstract-* This paper describes a standard system at LNE to measure RMS value of distorted voltage waveforms by means of a sampling voltmeter. Thermal converters are used to trace this quantity to the SI units. Deviations within 15  $\mu\text{V/V}$  are obtained whatever the total harmonic distortion of the waveforms.

### I. Introduction

Over the past few years the number of loads producing harmonic distortion on the power grid has considerably grown. As a consequence power analysers are increasingly being used to insure the quality of the grids. This has created a need for national laboratories to provide calibration facilities and traceability for the harmonic related quantities measured by these new instruments.

### II. Definition

The distorted voltage waveforms have a dominant fundamental component at 50 Hz and the harmonic content is limited to 50 stationary harmonics (periodic signals). These waveforms can be decomposed into Fourier series as:

$$u(t) = \sum_{k=1}^{50} U_k \sin(2\pi f_k t + \varphi_k) \quad (1)$$

$U_1$ ,  $U_k$  are the amplitudes of the fundamental and the  $k^{\text{th}}$  harmonic component which frequencies are respectively  $f_1$  and  $f_k = k f_1$ .

The two quantities of interest for our work are :

- the RMS value  $U_{\text{eff}}$  of the voltage signal which is defined as:

$$U_{\text{eff}} = \left( U_1^2 + U_2^2 + \dots + U_k^2 + \dots \right)^{\frac{1}{2}} \quad (2)$$

- the total harmonic distortion of the signal  $THD$  which is defined as:

$$THD = \left( \frac{\sum_{k>1}^{50} U_k^2}{U_1^2} \right)^{\frac{1}{2}} \quad (3)$$

The objective of the study is to measure the RMS value  $U_{\text{eff}}$  of distorted waveforms by means of a sampling voltmeter. Thermal converters are used to trace this quantity to the SI units. They compare the amount of heat produced in a heater resistor when an AC and a DC signal are successively applied by means of thermocouples fixed on the heater. The uncertainties attainable by AC-DC transfer technique are of the order of 1  $\mu\text{V/V}$  for measurements at voltages of about 1 V at low frequency.

### III. Measurement method

The block diagram of the set-up is given in figure 1. The sampling voltmeter (DVM) used for the measurement of the RMS value  $U_{eff}$  and the reference thermal converter are connected in parallel and can be fed by an AC or a DC source by means of an AC-DC switch. The output voltage of the thermal converter is measured by a precision nano-voltmeter. The sampling voltmeter is periodically calibrated against a Zener source as it also serves to measure the voltage delivered by the DC source (reference voltage). The sequence AC, DC+, DC- and AC (DC signal is applied in both polarities to eliminate the reversal error of the thermal converter) is then applied. Voltages measured by the DVM and the precision nano-voltmeter (output voltage of the thermal converter) are each time recorded. The error of the DVM is computed from this set of data.

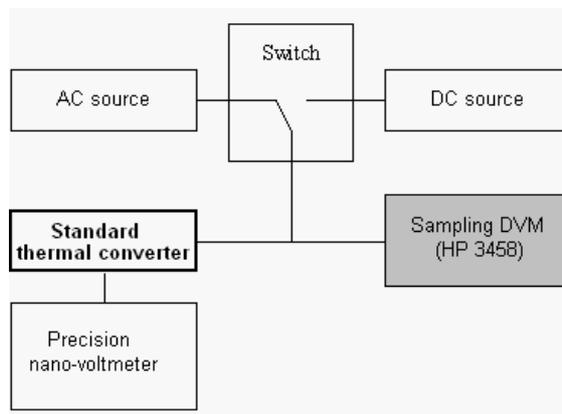


Figure 1. Experimental set-up.

For AC voltage measurements, the DVM is used in the DCV sampling mode. It is possible to control the sampling aperture time  $T_a$ , which corresponds to the integration of the signal over a finite time.  $N$  samples are taken at equally spaced discrete instants, covering an integer odd number  $M$  of periods of the input waveform. The samples are then analysed by applying discrete Fourier transform (DFT) and all the components of the signal are deduced from the amplitude spectrum. The correction terms due to the limited bandwidth of the DVM and its non zero aperture time  $T_a$  are taken into account.

### IV. Experimental results

#### A. Characterization of the analog-to-digital converters (ADC) of the DVM

The ADCs of the DVM have already been characterized for static [1] and dynamic [2] conditions, for sine waves at frequencies up to 400 Hz. In this frequency range, selecting optimized sampling parameters, the agreement between the sampling technique and AC-DC transfer using thermal converters has been found better than  $3 \mu\text{V/V}$ . The highest harmonic frequency of the distorted waveform studied is 2500 Hz. Then, first experiments have been performed to characterize the ADCs for frequencies from 400 Hz to 2500 Hz. Figure 2 shows the error  $e_{AC}$  of the DVM (relative to the voltage reference value measured by AC-DC transfer technique) according to the frequency  $f_l$  of the signal. The aperture time selected is  $60 \mu\text{s}$ . The error which vary from  $-3 \mu\text{V/V}$  at 400 Hz to  $+48 \mu\text{V/V}$  at 2500 Hz has to be taken into account for the measurements of the RMS value of the distorted waveforms.

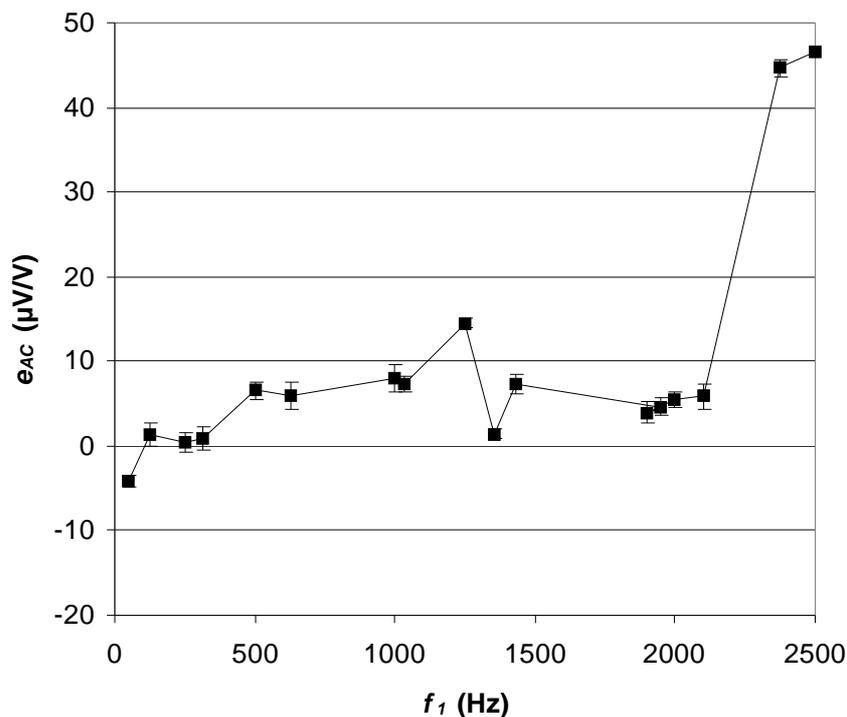


Figure 2. Relative error  $e_{AC}$  of the DVM according to the frequency  $f_1$  of the sine wave ( $f_e = 10.24$  kHz and  $T_a = 60$   $\mu\text{s}$ ).

### B. Measurements of the RMS value of distorted waveforms

For digitizing the signals, 1024 samples are taken over 5 periods and the sampling frequency is 10.24 kHz to satisfy the Shannon condition. The aperture time is 60  $\mu\text{s}$ . Twenty voltage waveforms are available, one sinusoidal waveform and nineteen distorted waveforms. The distorted waveforms have a dominant fundamental component at 50 Hz and one or several stationary harmonics. The  $THD$  varies from 0 % (sine wave) to 84 %. Figure 3 shows the error  $e_{AC}$  of the DVM as a function of the  $THD$  of the distorted waveforms. It appears that the differences between the two techniques is always below 15 parts in  $10^6$ . Most of the measurements were repeatable to within 2 parts in  $10^6$ .

These results are not valuable for all harmonic amplitudes. Indeed, in the phase of validation of this method, one of the possible limitations may come from the sensitivity of thermal converters to harmonics of very low amplitude. For example adding a harmonic whose amplitude relatively to the fundamental component is  $10^{-5}$  corresponds to a contribution of about  $10^{-10}$  times that of the fundamental to add to the total RMS value of pure signal.

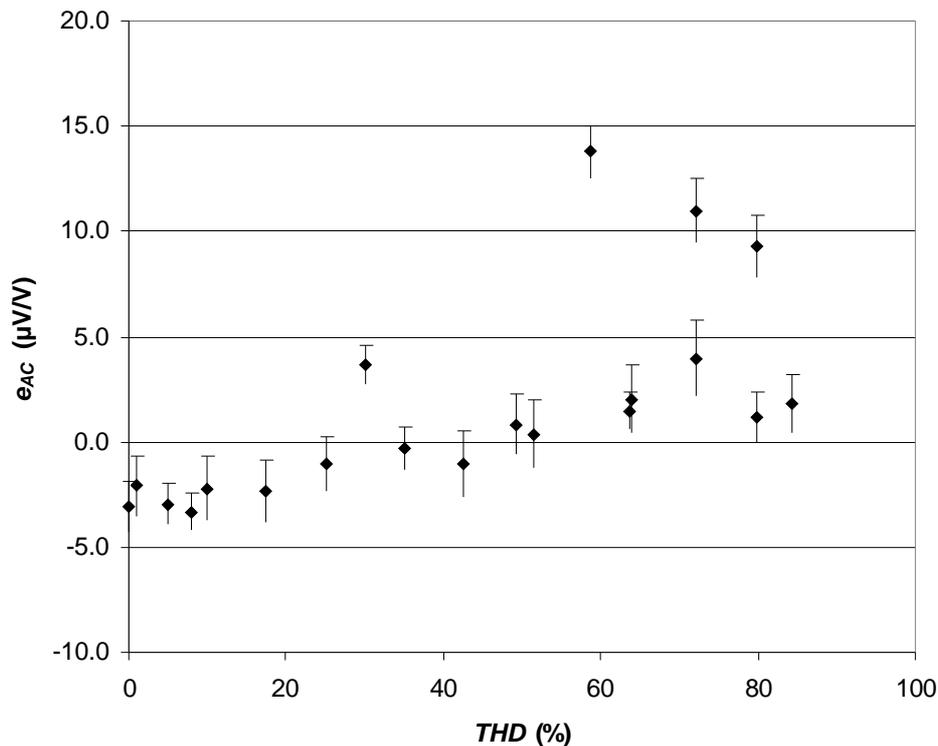


Figure 3. Relative error  $e_{AC}$  of the DVM according to the  $THD$  of the distorted waveforms studied.

## V. Conclusion

A standard method for measuring RMS value of distorted voltage waveforms has been developed at LNE. This method relies on two comparative approaches: a sampling approach by means of a digital voltmeter and a thermal approach based upon thermal converters. The sampling approach is faster and easier to implement compared to the thermal one which necessitates some constraints concerning for instance amplitude of the input voltage and a measurement sequence consisting of a succession of AC, DC+, DC-, AC power supply. Measurements have been performed on waveforms with THD ranging between 0 and 84 %. Good agreement with thermal converters, always better than 15 parts in  $10^6$ , has been obtained in all cases. Both RMS value and THD are parameters largely required by industrials (RMS value being directly related to energetic quantities). Another important parameter is the amplitude of each individual harmonic. Nevertheless the present validation method does not allow tracing this quantity. That is the reason why we are currently improving this method.

Acknowledgement - This research, conducted within the EURAMET joint research project 'Power and Energy', has received partial support from the European Community's Seventh Framework Programme, ERANET Plus, under Grant Agreement No. 217257.

## References

- [1] P. Espel, A. Poletaëff, A. Bounouh, "Static characterization of analog-to-digital converter." XIX IMEKO, September 2009, Lisbon (Portugal).
- [2] P. Espel, A. Poletaëff, A. Bounouh, "Characterization of analog-to-digital converters of a commercial digital voltmeter in the 20 Hz to 400 Hz frequency range." 2009 *Metrologia* **46** 578-584