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An approach to the evaluation of dimensional measurements on pressure-measuring piston-cylinder assemblies

W. Sabuga[1, 3], T. Priruenrom[2]

[1] Physikalisch-Technische Bundesanstalt (PTB), Bundesallee 100, 38116 Braunschweig, Germany, wladimir.sabuga@ptb.de, phone: +49-531-5923230, fax: +49-531-592693230
[2] National Institute of Metrology (Thailand) (NIMT), 3/4-5 Moo 3, Klong 5, Klong Luang, Pathumthani 12120, Thailand, currently researcher at PTB, tasanee.priruenrom@ptb.de, phone: +49-531-5923212, fax: +49-531-592693212
[3] responsible for correspondence

Abstract

To improve the consistency of 3-dimensional data describing the geometry of pressure-measuring piston-cylinder assemblies, a new approach, based on the least-squares method, is proposed. It allows the results of diameter-, straightness- and roundness measurements to be linked with each other, with only minimum discrepancies between them. When processing the dimensional data, it is possible thanks to this new approach to weight them differently - according to their measurement uncertainties. The new approach was applied to three gas-operated piston-cylinder assemblies with nominal effective areas of 10 cm^2 and 5 cm^2 which are used at PTB as primary gas pressure standards for the range up to 2 MPa. The dimensional measurements were carried out by means of different instruments and the results were analysed. The discrepancies in the dimensional data sets obtained within the scope of the new approach are typically smaller than 16 nm, which agrees with the uncertainties claimed for each kind of dimensional measurement. The effective areas of the three piston-cylinder assemblies were calculated using the Dadson theory and then adjusted taking into account the results of both cross-float measurements carried out between them and pressure measurements carried out against a primary mercury manometer. Finally, relative standard uncertainties smaller than $2 \cdot 10^{-6}$ could be obtained.

Keywords: pressure balance, piston-cylinder assembly, dimensional measurement, least-squares method

1. Introduction

The effective area of piston-cylinder assemblies which are used as primary pressure standards in pressure balances is usually determined from the dimensional properties of the pistons and cylinders by means of Dadson's theory [1]. At PTB, the lowest uncertainty of the dimensional data is achieved when the straightness of the pistons and cylinders (S), the roundness (R) and the diameters (D) are measured separately (sometimes using different measurement instruments) and when S and R are then linked to D in order to generate a 3-dimensional (3D) data set which describes the generatrix surfaces of the cylinder bore and the piston. One of the contributions to the uncertainty of the 3D data produced in this way originates from the contradictions between S, R and D measurements.

So far [2], only two pairs of diameters, measured in two reference levels and in two orthogonal directions, have been used for a 3D data generation which was performed successively: First, two circle traces, measured in the reference levels, were linked to the two diameters pairs; then the generatrix traces were positioned in the space to meet the defined two circle traces; finally, the remaining circle traces were adjusted to the generatrix traces. Additional diameters, measured in other than the reference levels, were used to check the results of the 3D generation. The main disadvantage of this method is, in the first place, that not all measurement data being available are included in the analysis and, in the second place, that a potential anomaly of S, R or D data at the reference levels and azimuths has a strong effect on the output data.

An improvement can be achieved using the least-squares (LS) method because an arbitrary choice of the reference heights and directions is no longer necessary and, due to a link of all data at all cross points, the effect of an anomaly is reduced. In addition, when processing the S, R and D data, the LS method allows them to be weighted according to their measurement uncertainties.

2. Least-squares methods

Normally, results of dimensional measurements are present in the form of
straightness deviations $S_j(z)$, j = 1, 2,…,m,
roundness deviations $R_i(\varphi)$, i = 1,2,…,l,
and diameters D_{ij},

where z and φ mean the axial and angular coordinates, j enumerates the angle directions of the straightness measurements, i counts levels of roundness measurements, m and l are total numbers of generatrix and circle traces, respectively, characterised by the dimensional measurements. (For D_{ij}, i and j have the same meaning as given above, however it is not necessary that diameter values are available for all cross points of the generatrix and circle traces). The radii of the generatrix and circle traces, $r_{S,j}(z)$ and $r_{R,i}(\varphi)$, can be obtained from the shape deviations using the following relations:

$$r_{S,j}(z) = S_j(z) + a_j + b_j z\,, \qquad (1)$$

$$r_{R,i}(\varphi) = R_i(\varphi) + P_i + w_i \cdot \cos(\varphi) + v_i \cdot \sin(\varphi)\,, \qquad (2)$$

where a_j, b_j, P_i, w_i and v_i are unknown parameters. They are found, by means of the LS method, from the condition of minimum discrepancies between $r_{S,j}(z)$, $r_{R,i}(\varphi)$ and diameters at the cross points of the generatrix and circle traces. Two cases will be considered in the following: (i) when the uncertainties of S, R and D measurements are equal, and (ii) a more general case, when they are different.

Equal uncertainties of dimensional measurements

The cross points of generatrix and circle traces are given by the coordinates (z_i, φ_j), the radii of the generatrix and circle traces are defined as $r_{S,ij} = r_{S,j}(z_i)$, $r_{R,ij} = r_{R,i}(\varphi_j)$. As diameters are not necessarily measured for all (z_i, φ_j) points, it is useful to introduce the parameter d_{ij}, which is 1 if a diameter is available and 0, if it is not.

Four diameters, lying in two different levels and in two different angle directions, should be chosen to identify an initial coordinate system for the 3D data to be created. The choice of these diameters is unimportant because in the end, the z axis of the coordinate system will be redefined by the axis of the LS cylinder, as described in section 3. The chosen reference diameters are indicated by parameter c_{ij} equal 1 for reference diameters and equal 0 for non-reference diameters.

With these definitions, the differences between the diameters and radii of the generatrix and circle traces are given by the following equations:

S and R: $$\Delta_{ij} = r_{S,ij} - r_{R,ij} , \tag{5}$$

S and non-ref. D: $$\mu_{ij} = 0.5 \cdot \left(D_{ij} - r_{S,ij} - r_{S,ij^*}\right) \cdot \left(1 - c_{ij}\right) \cdot d_{ij} , \tag{6}$$

R and non-ref. D: $$\nu_{ij} = 0.5 \cdot \left(D_{ij} - r_{R,ij} - r_{R,ij^*}\right) \cdot \left(1 - c_{ij}\right) \cdot d_{ij} , \tag{7}$$

S and ref. D: $$\sigma_{ij} = \left(0.5 \cdot D_{ij} - r_{S,ij}\right) \cdot c_{ij} , \tag{8}$$

R and ref. D: $$\tau_{ij} = \left(0.5 \cdot D_{ij} - r_{R,ij}\right) \cdot c_{ij} , \tag{9}$$

where $$j^* = \begin{cases} j + m/2, & j \le m/2 \\ j - m/2, & j > m/2. \end{cases} \tag{10}$$

The coefficient of 0.5 in (6, 7) takes into account the fact that these relations contain two differences between the shape and the diameter data. The coefficient of 0.5 in (8, 9) means that half reference diameters are taken as reference radii.

A minimum of the sum of the squared differences χ^2,

$$\chi^2 = \sum_{i,j} \left(\Delta_{ij}^2 + \mu_{ij}^2 + \nu_{ij}^2 + \sigma_{ij}^2 + \tau_{ij}^2\right) \rightarrow \min , \tag{11}$$

is obtained by differentiating it by each of the $2 \times m + 3 \times l$ unknown parameters a_j, b_j, P_i, w_i and v_i and by equating each result to zero. This leads to a system of $2 \times m + 3 \times l$ linear equations which can easily be solved.

Different uncertainties of dimensional measurements

In the case of different uncertainties of the S, R and D measurements, u_S, u_R, and u_D, respectively, differences between $r_{S,ij}$, $r_{R,ij}$ and D_{ij} on the one hand and unknown "true" radii r_{ij} on the other hand can be considered:

r_{ij} and S : $$\alpha_{ij} = r_{ij} - r_{S,ij}\,, \tag{12}$$

r_{ij} and R : $$\beta_{ij} = r_{ij} - r_{R,ij}\,, \tag{13}$$

r_{ij} and non-ref. D: $$\gamma_{ij} = 0.5 \cdot \left(r_{ij} + r_{ij^*} - D_{ij}\right) \cdot \left(1 - c_{ij}\right) \cdot d_{ij}\,, \tag{14}$$

r_{ij} and ref. D: $$\delta_{ij} = \left(r_{ij} - 0.5 \cdot D\right) \cdot c_{ij}\,. \tag{15}$$

In addition to the unknown parameters a_j, b_j, P_i, w_i and v_i, these relations contain $l \times m$ unknown variables r_{ij}. As in the previous subsection, all unknown parameters are found by minimising the sum of the squared differences, which are now weighted by the inversely proportional squares of the uncertainties:

$$\chi^2 = \sum_{i,j} \left[\left(\frac{\alpha_{ij}}{u_S} \right)^2 + \left(\frac{\beta_{ij}}{u_R} \right)^2 + \left(\frac{\gamma_{ij}}{0.5 \cdot u_D} \right)^2 + \left(\frac{\delta_{ij}}{0.5 \cdot u_D} \right)^2 \right] \rightarrow \min\,. \tag{16}$$

Factor 0.5 in (16) takes into account that γ_{ij} and δ_{ij} depend only on half diameters. Differentiating (16) by each of the unknown parameters leads to a system of $l \times m + 2 \times m + 3 \times l$ linear equations with the same amount of variables.

3. Data processing and effective area calculation

Since 1989, different measurement techniques and instruments, including a one-dimensional Abbe-comparator, a universal measurement machine Moore No. 3, an RTH Talyrond 73, a state-of-the-art comparator for diametric measurements (KOMF) and an MFU-8 instrument have been applied for the dimensional characterisation of piston-cylinder assemblies [2-5]. With these instruments, the shape deviations are measured dynamically so that the results contain noise and surface roughness components. To remove them, the Gaussian filters with the cut-off wavelength of 0.8 mm for S data and with the cut-off wave number of 150 upr for R data, 50 % transmission [6], are applied prior to linking the S, R and D data. After the 3D data sets have been generated, they are transformed to a new coordinate system whose *z*-axis coincides with the axis of the LS cylinder of the 3D data found by the method described in [7]. This step is important to achieve a coaxial positioning of the 3D data sets for the piston and the cylinder bore. The effective area of the piston-cylinder assemblies is calculated by the Dadson theory [1], taking into account all possible combinations of the piston's and cylinder bore's dimensions along a section containing the assembly axis.

4. Results and discussion

The new approach was applied to three gas-operated piston-cylinder assemblies, two with a nominal effective area of 10 cm^2, identified by the Nos. 288 and 290, and one of 5 cm^2, identified by the No. 6222, which are used at PTB as primary gas pressure standards for the range up to 2 MPa. The 3D data of assembly 6222, generated by the new approach from the dimensional measurement results obtained in 2006, are shown in Figure 1. This figure demonstrates an agreement within a few nanometres between the S, R and D data linked by the new approach. It is interesting to note that the anomaly of one of the generatrices, which might have been produced by an impurity on the cylinder surface or by another disturbance and is evidently not a real property of the cylinder, would lead to wrong data for this generatrix and other traces if the anomaly lied in the level of the reference diameters and the data were evaluated by the old successive procedure.

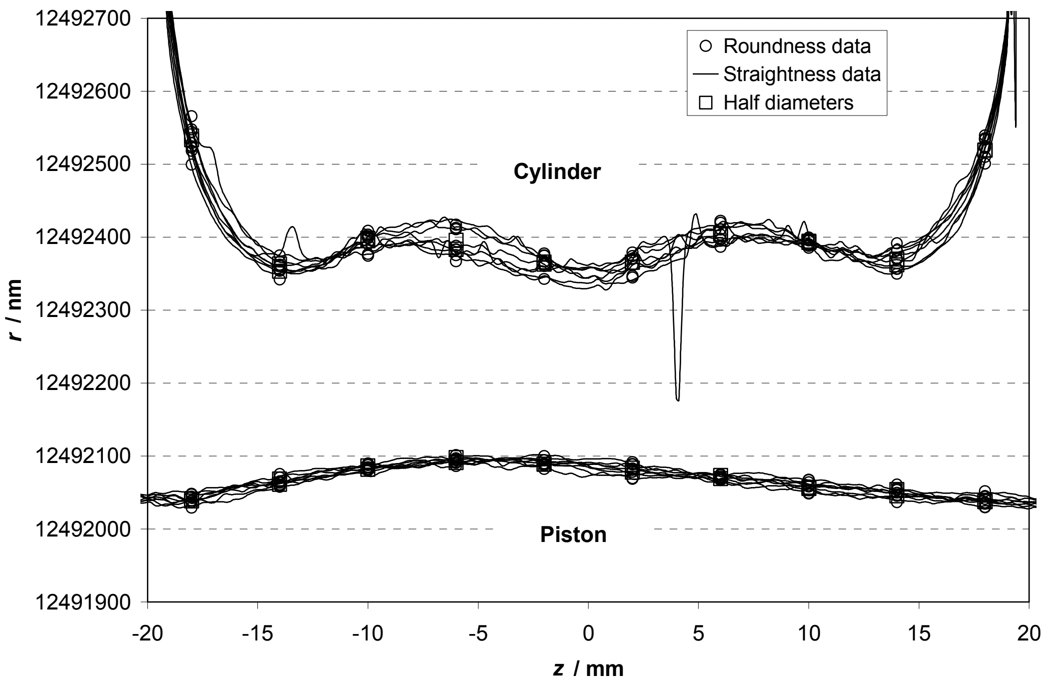

Figure 1 Dimensional properties of the piston and the cylinder bore of piston-cylinder assembly 6222, characterised by roundness-, straightness- and diameter measurements

Details of the dimensional measurements and the evaluation of the dimensional data for the three piston-cylinder assemblies are given in Table 1.

Unit	Arte-fact	X	Instrument	u_X in nm	Y	$\delta(r_Y)$ in nm	$u(r_X)$ in nm	$u_{r,R}(A_0)/A_0$, $u_{r,S}(A_0)/A_0$, in 10^{-6}	$u(A_0)/A_0$ in 10^{-6}
288	c	D	KOMF	10	D-R	16			
		R	MFU-8	30	D-S	10	19	2.0	
		S	MFU-8	40	R-S	9	14		2.4
	p	D	KOMF	5	D-R	14			
		R	MFU-8	30	D-S	11	16	1.6	
		S	MFU-8	40	R-S	8	14		
290	c	D	MFU-8	20	D-R	10			
		R	MFU-8	30	D-S	15	16	1.6	
		S	MFU-8	40	R-S	8	20		2.3
	p	D	MFU-8	20	D-R	4			
		R	MFU-8	30	D-S	6	11	1.8	
		S	MFU-8	40	R-S	4	12		
6222	c	D	MFU-8	20	D-R	4			
		R	Talyrond 73	20	D-S	4	12	1.9	
		S	MFU-8	40	R-S	5	12		2.2
	p	D	MFU-8	20	D-R	5			
		R	Talyrond 73	20	D-S	3	12	1.9	
		S	MFU-8	40	R-S	5	11		

Table 1 Summary of the dimensional measurements and evaluation for the 3 piston-cylinder units Nos. 288, 290 and 6222: artefact measured – cylinder (c) and piston (p); property (X) – diameter (D), roundness (R) and straightness (S); instrument applied for dimensional measurement; standard measurement uncertainty of property X (u_X); differences between half diameters, R and S radii (Y); standard deviations of these differences [$\delta(r_Y)$]; standard uncertainties calculated for R and S radii [$u(r_X)$]; their contributions to the effective area uncertainties [$u_{r,R}(A_0)/A_0$] and [$u_{r,S}(A_0)/A_0$]; and combined standard uncertainties of the effective areas, determined from the dimensional data [$u(A_0)/A_0$].

The residual differences between the half diameters and the radii [$\delta(r_Y)$] determined by the new linking approach are always smaller than 16 nm and support the uncertainties of the dimensional measurements (u_X) claimed for each of the measuring instruments. The uncertainties of the radial values [$u(r_X)$] were calculated for R and S traces using the following relationships:

$$u(r_R) = \left\{[u(D)/2]^2 + \delta^2(r_{D-R}) + \delta^2(r_{R-S})\right\}^{0.5}, \quad (17)$$

$$u(r_S) = \left\{[u(D)/2]^2 + \delta^2(r_{D-S}) + \delta^2(r_{R-S})\right\}^{0.5}. \quad (18)$$

The uncertainties $u_{r,R}(A_0)/A_0$ and $u_{r,S}(A_0)/A_0$ in Table 1 present contributions of $u(r_R)$ and $u(r_S)$ to A_0 when the latter is calculated either from r_R or r_S data. The uncertainty in the last column, $u(A_0)/A_0$, is a combined uncertainty which takes into account the difference between A_0 obtained from R and S radii as well as variations of A_0 with a change in the angular coordinate, angular position of the piston in the cylinder and pressure.

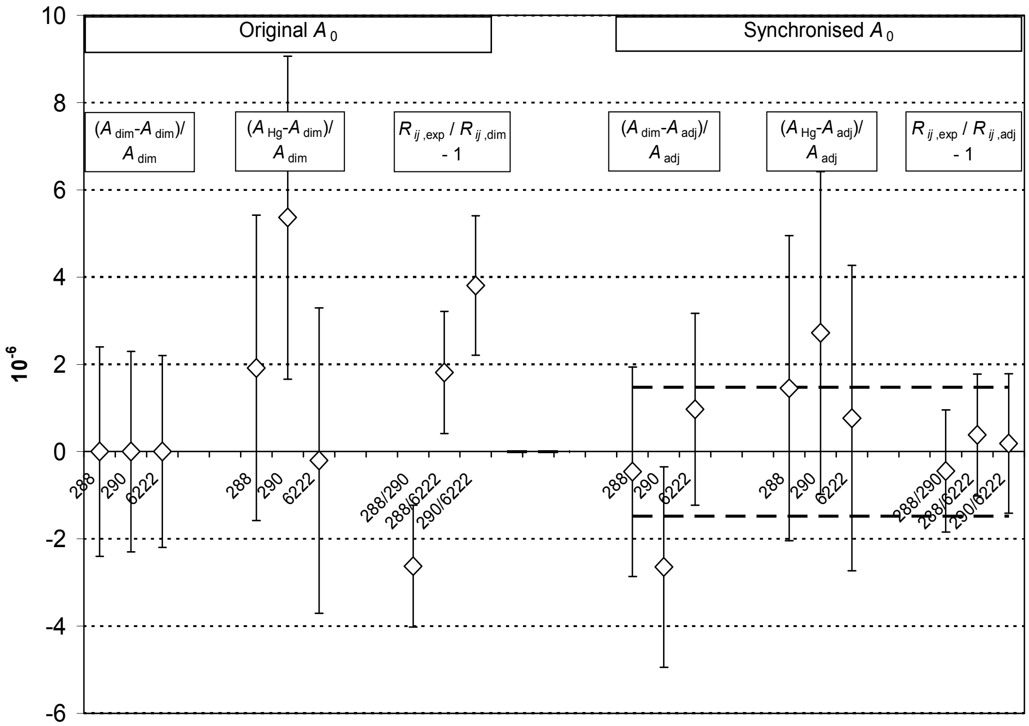

Figure 2 Relative deviations of the experimental effective areas or their ratios from the dimensional values (Original A_0) and of the dimensional and experimental effective areas or their ratios from the synchronised ones (Synchronised A_0).

The results for the dimensional and experimental effective areas are presented in Figure 2. The experimental data include effective areas obtained using a mercury manometer (A_{Hg}) as a reference [8] and ratios of the effective areas observed in cross-float experiments. The left part of the figure shows these results in reference to the dimensional values. From the dimensional and the experimental results, adjusted effective areas (A_{Adj}) have been derived which present the best compromise for all data. The points in the right part of the figure show differences between the original and the adjusted effective areas. The adjusted effective areas demonstrate good agreement with the cross-float A_0 ratios and, due to the independence of A_{dim} and A_{Hg}, have combined standard uncertainties (shown by the dashed lines) smaller than $u(A_{dim})$ and $u(A_{Hg})$.

Conclusions

The new approach allows diameter, straightness and roundness measurement data to be optimally linked and their uncertainties to be estimated. Discrepancies in the dimensional data sets produced by the new approach are typically smaller than 16 nm, which agrees with the uncertainties claimed for each kind of the dimensional measurements. Using the new approach, the effective areas of three primary gas-operated piston-cylinder assemblies could be determined with relative standard uncertainties smaller than $2.5 \cdot 10^{-6}$. They agree well with the experimental results obtained using a primary mercury manometer and from cross-float experiments, and allow combined relative standard uncertainties of smaller than $2 \cdot 10^{-6}$ to be obtained.

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