UNCERTAINTY ANALYSIS OF AVERAGE FLOW DURATION CURVES USED IN HYDRPOWER PLANTS FEASABILITY STUDIES

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Abstract:

This study applies a Monte Carlo method (MCM) to estimate the uncertainty related to the determination of the annual flow duration curve determined at a river weir, considering the dynamic behaviour of processes and the intrinsic uncertainty contributions to the raw data. The computation of the curve uses a time series of average daily flow data collected at a hydrometric station at an upstream river section. This curve was used for a hydropower plant feasibility study.

Results show how model uncertainty can be estimated from the measurement uncertainty of raw data and mathematical formulation applied.

Keywords: Uncertainty; time series; mathematical models; Monte Carlo method; data.

1. INTRODUCTION

The growing awareness of the impact of climate change and the importance of the United Nations' sustainable development goals [1] emphasizes the need for reliable measurements of quantities to support urban and water resources management [2].

The increasing concern of the water supply and wastewater management utilities with the quality of the measurement of quantities is leading to a growing need for specific knowledge and skills in Metrology. Measurement quality depends on multiple aspects that can significantly affect the quality and accuracy of the quantity to be measured, namely the acquisition and processing of data and the transformation into valuable information for decision-making.

The knowledge regarding measurement data reliability and associated uncertainty is essential for managing water supply and wastewater networks [3, 4]. The development of good measurement practices, including the evaluation of uncertainty, is needed to improve the quality of data and increase confidence in results.

The measurement uncertainty is an informative indicator for understanding the factors affecting measurements and their impact on models' results.

With time series, data are based either on individual readings or processed values (e.g., average values), implying that the respective uncertainty can be only the instrumental uncertainty or its combination with the uncertainty resulting from the processing adopted. Propagation of the measurement uncertainty allows evaluating the uncertainty of the model parameters, thus providing traceability to the mathematical models themselves and to their predictions.

Measurement uncertainty is composed of components depending on its origin: instrumental uncertainty, calibration uncertainty, data acquisition and processing uncertainty, and method uncertainty [5]. For the present study, traceability information was incomplete, thus requiring an estimate of the accuracy based on previous knowledge and the typical magnitude of the measurement uncertainty for the type of instrumentation used.

This study is based on previous work [6], a hydropower plant feasibility study using an annual flow duration curve where the evaluation of measurement uncertainty was not calculated. However, this is needed to enrich the analysis of the physical problem and attain more accurate and reliable results. The computation of the curve uses a time series of average daily flow data collected at a hydrometric station at an upstream river section. The method presented in this paper contemplates the definition of a process to calculate the measurement uncertainties associated with the annual flow duration curve.

In this study, the uncertainty of the mathematical model used to calculate the flow duration curve, relating input and output quantities, accounts for measurement and model formulation uncertainties and applies a Monte Carlo method. To propagate the uncertainties, this method requires probability distribution functions for the uncertainties of the input quantities.

2. MATERIALS AND METHODS

2.1. Study case and data

Flow data used are from a hydropower plant feasibility study for installing an Archimedes screw turbine in an existing weir in the Tâmega river, in the North of Portugal [6]. The primary objective of the present study is to incorporate the evaluation of measurement uncertainties in the determination of the annual flow duration curve, based on average daily flows, used as the basis for the referred feasibility study.

A data series of average daily flow rates was used, comprising six complete hydrological years, namely 1994/95, 1995/96 and between 2005/06 and 2008/09 (see Figure 1).

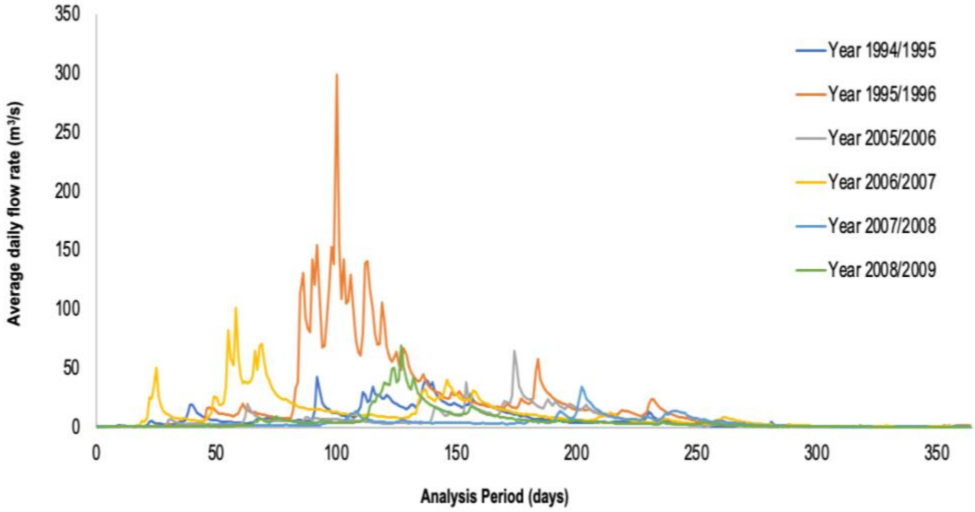

Figure 1: Average daily flow rate series for six hydrological years.

The hydrological year from October 1995 to September 1996 presents abnormally higher flow rate values when compared with the other years, so these data were not included in the study [6]. The final measurement series has 1826 values (*n*=1826) of average daily flows. Figure 2 shows the experimental data series utilised in the study, $F_{\text{ori,n}}$.

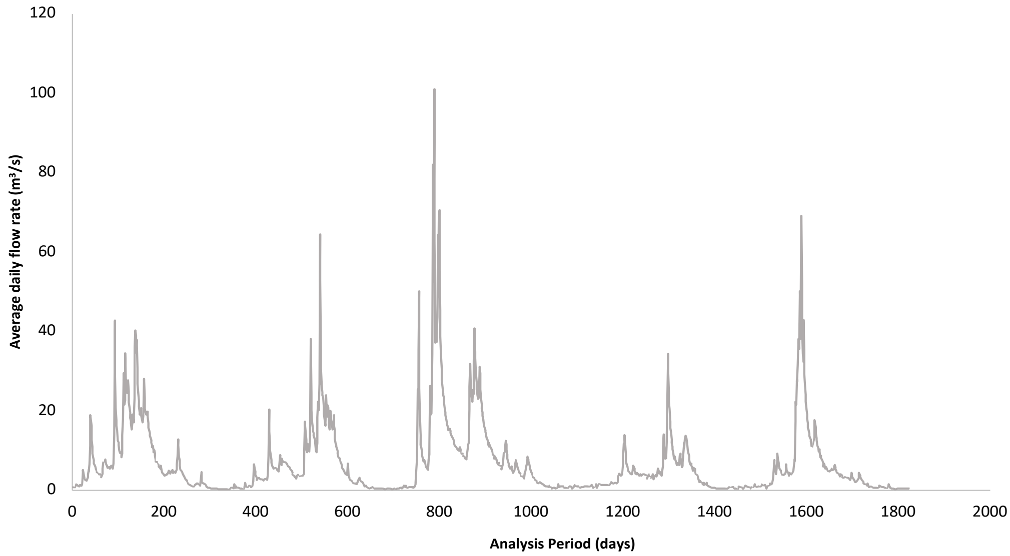

Figure 2: Average daily flow rate series excluding the hydrological year of 1995/96.

2.2. Mathematical model

This study is divided into two parts. The first part is related to the correction of the average daily flow rate series; the second part corresponds to the determination of the respective flow duration curve. Since the hydrometric station with collected flow rate data is not at the weir section, it is necessary to correct the series using the relationship between the drained basin areas. This correction is carried out using the Myer formula, applied to the whole sample, *n*:

$$\frac{F_{\text{ori,n1}}}{F_{\text{ori,n2}}} = \left(\frac{A_1}{A_2}\right)^{0.7} \quad (1)$$

in which $F_{\text{ori,n,1}}$ (m^3/s) and A_1 (km^2) are the average daily flow rate and the drained area associated with station 1, respectively, and $F_{\text{ori,n,2}}$ (m^3/s) and A_2 (km^2) are the average daily flow rate and the drained area associated with station 2, respectively. Applying equation (1), the series of corrected average daily flows rates, $F_{\text{corr,n}}$ (m^3/s), is given by equation (2):

$$F_{\text{corr,n}} = F_{\text{ori,n1}} \left(\frac{A_1}{A_2}\right)^{0.7}. \quad (2)$$

In the second part of the study, to determine the series of average daily flow rates, an ecological flow rate, F_{ec} (m^3/s), is set as 5% of the average daily flow rates (see equation 4) of the corrected average daily flow rate, $F_{\text{corr,av}}$, calculated with equation 3.

$$F_{\text{corr,av}} = \frac{\sum_{i=1}^{n} F_{\text{corr,n}}}{n} \quad (3)$$

Thus, the ecological flow rate, F_{ec}, is given by:

$$F_{\text{ec}} = 0.05 F_{\text{corr,av}} \quad (4)$$

The final series of average daily flow rates, $F_{\text{d,n}}$ (m^3/s) to use in the feasibility study corresponds to the difference between the corrected average daily flow rates, $F_{\text{corr,n}}$ (m^3/s), and the ecological flow rate, F_{ec} (m^3/s). This series is determined by applying equation (5).

$$F_{\text{d,n}} = F_{\text{corr,n}} - F_{\text{ec}}. \quad (5)$$

2.3. Evaluation of measurement uncertainties in the estimation of average flow rates

The common method used for the evaluation of measurement uncertainty is presented in [7], known as the GUM (Guide to the Expression of Uncertainty in Measurement), first published by ISO, IEC, and other organizations in 1993. It states that for a functional relation *f* of the type:

$$y = f(x_1, \cdots, x_n) \quad (6)$$

being *y* the output quantity calculated from *n* input quantities, x_i, using the development of the function as a 1st order Taylor series, a formulation for the measurement standard uncertainty of the output quantity, *u(y)*, given by the Law of Propagation of Uncertainties:

$$u^2(y) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \left(\frac{\partial f}{\partial x_i}\right)\left(\frac{\partial f}{\partial x_j}\right) u(x_i, x_j). \quad (7)$$

Supplement 1 of GUM [8] introduced the use of Monte Carlo methods to non-linear and more complex functional relations, which was considered more adequate in the current circumstances of the mathematical models studied.

The Monte Carlo method is implemented using Rstudio© tools, by making numerical simulation, using the software intrinsic function (*rnorm*) to generate a set of gaussian pseudo-random data series (typically of 10^6 random numbers in each one) and validated built-in statistical tools to have as a result the expanded measurement uncertainty interval.

The steps for implementing the numerical simulation using a Monte Carlo method (using Rstudio© tools), for the first part, are:

1. The estimates for each input quantity (A_1, A_2, $F_{\text{ori,n1}}$) and their measurement uncertainties are found by propagating the probability distribution functions using numerical simulation, generating data series from 10^4 runs. This option implies a long time for processing; the resulting output limits of the expanded uncertainty have enough accuracy to define the intervals. For each input quantity mentioned before, present in equation (2). Publication [8] states that the number of observations in the model needs to be defined, recommending 10^6 observations because this value can provide a 95% confidence interval for the sample considered. However, a lower number can be used if the accuracy of percentiles is tested. The measurement uncertainty for each quantity is estimated from previous knowledge applied to the type of instrumentation used. The estimated values [9], as well as the estimated uncertainty, are presented in Table 1. The uncertainties of the measuring instrument and calibration were not considered in this evaluation due to the lack of existing information.

Table 1: Input quantities and output quantity estimates and measurement uncertainty related to the evaluation of the data series.

Quantity	Estimate	Standard Uncertainty	Probability Distributions Functions
$F_{\text{corr,n}}$ /(m^3/s)	Data series	0.1	Gaussian
A_1 /(km^2)	989.43	0.01	Gaussian
A_2 /(km^2)	1070.46	0.01	Gaussian

2. The calculation of the corrected average daily flow series, $F_{\text{corr,n,}}$ is carried out by using equation (2). Because of the non-linear nature of the mathematical function, the calculation of the corrected average daily flow rate series is based on the propagation of probability distribution functions using numerical simulation, as already mentioned.
3. Each parameter of the vector series, $F_{\text{corr,n}}$, is ordered using the function *sort*;
4. The 0.025 and 0.975 percentiles of the ordered series are evaluated, allowing to obtain the (centred) limits of the expanded measurement uncertainty. Figure 3 shows the experimental data series considered, $F_{\text{ori,n}}$, and the corrected series, $F_{\text{corr,n}}$.

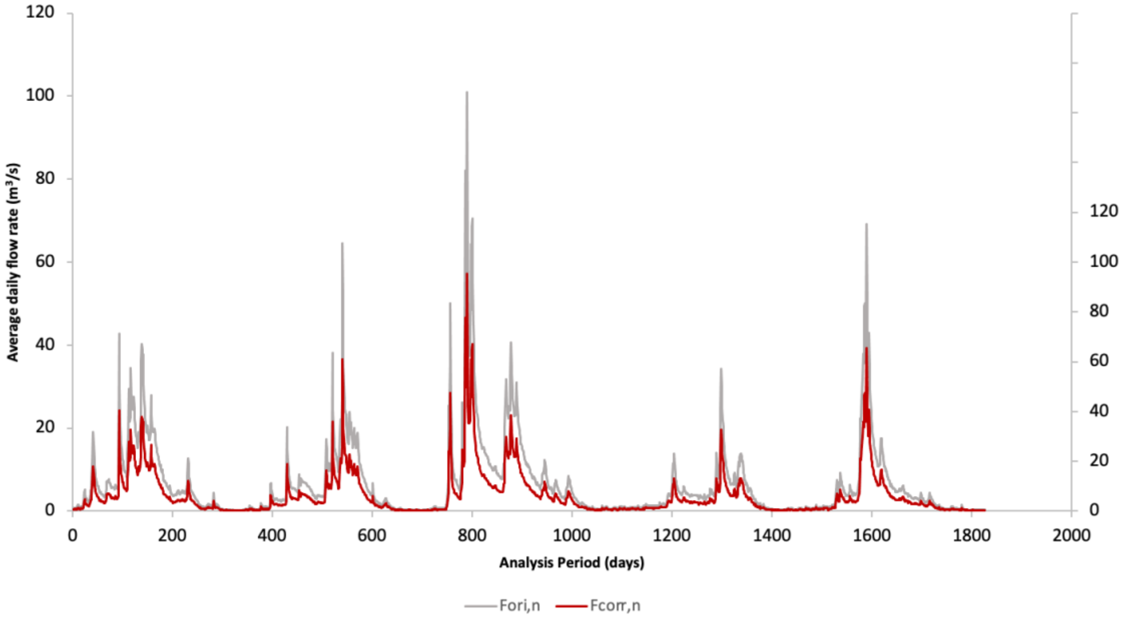

Figure 3: Average daily flow rate data series (excluding the year 1995/96): original data ($F_{ori,n}$) and corrected data ($F_{corr,n}$).

Subsequently, equation (3) is applied to calculate the average daily flow rate, $F_{corr,av}$ of 6.35 m^3/s with expanded measurement uncertainty, $U_{95}(F_{corr,av})$, of 0.19 m^3/s. Note that the expanded uncertainty is given by the 0.025 and 0.975 percentiles of the numerical output series.

The estimated relative uncertainty (with 95% confidence level) [7] of standard uncertainty $u(F_{corr,n})$ of the input estimate $F_{corr,n}$ is obtained and is represented in Figure 4.

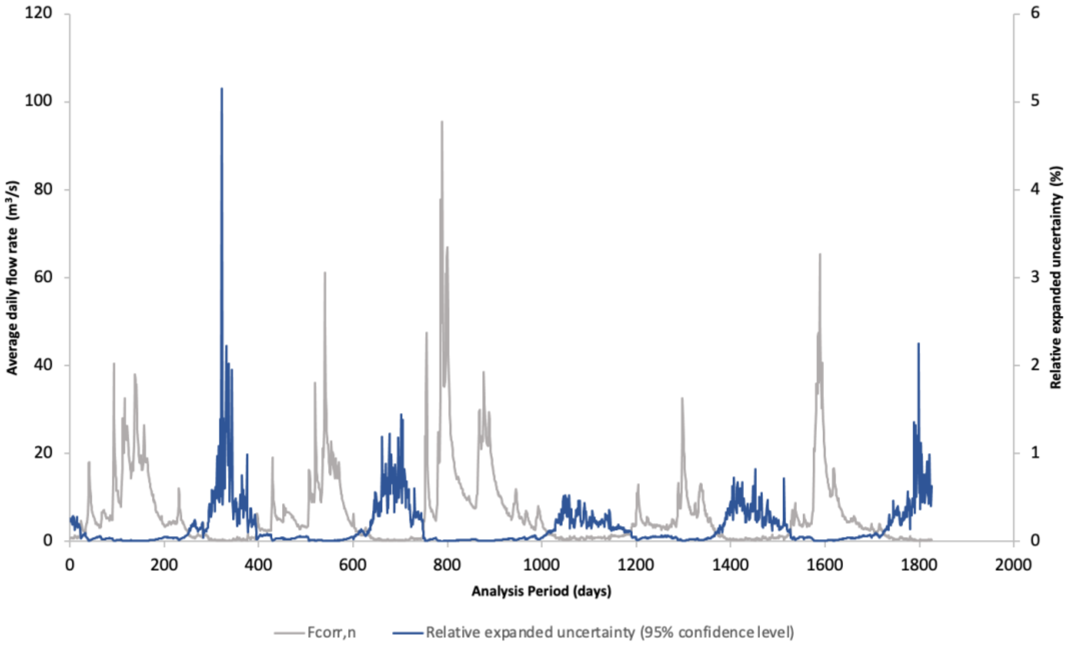

Figure 4: Relative uncertainty of the corrected average daily flow rate series.

In the second part of this study, the series of average daily flow rates is determined. The ecological flow rate, F_{ec} (m^3/s), is calculated using the corrected average daily flow rate value and applying equation (4) as F_{ec} = 0.318 m^3/s. Using this value, the series of average daily flow rates is determined and used to obtain the annual flow duration curve of average daily flows. As mentioned in the first part of the study, there is no information about the uncertainty of the ecological flow rate.

The ecological flow rate estimate results directly from the application of equation (4). The propagation of probability distribution functions is carried out using the Monte Carlo method numerical simulation, generating data series of 10^4 runs to calculate the measurement uncertainty, and using the constant value for the ecological flow using the software RStudio©.

The ecological flow is subtracted from the corrected average daily flow rates series, to have the average daily flow rates series.

This series is sorted, using the *sort* function, of the Rstudio© software. Then the following parameter and curve are determined:

– the 0.025 and 0.975 percentiles to determine the limits (centred) of the expanded measurement uncertainty (see Figure 5 and detail in Figure 6); and
– the annual flow duration curve (see Figure 5).

The average flow rate, F_{av} (m^3/s), to be considered as the average flow rate for the hydroelectric power station design is 6.04 m^3/s and the expanded measurement uncertainty, $U_{95}(F_{av})$ is 0.01 m^3/s.

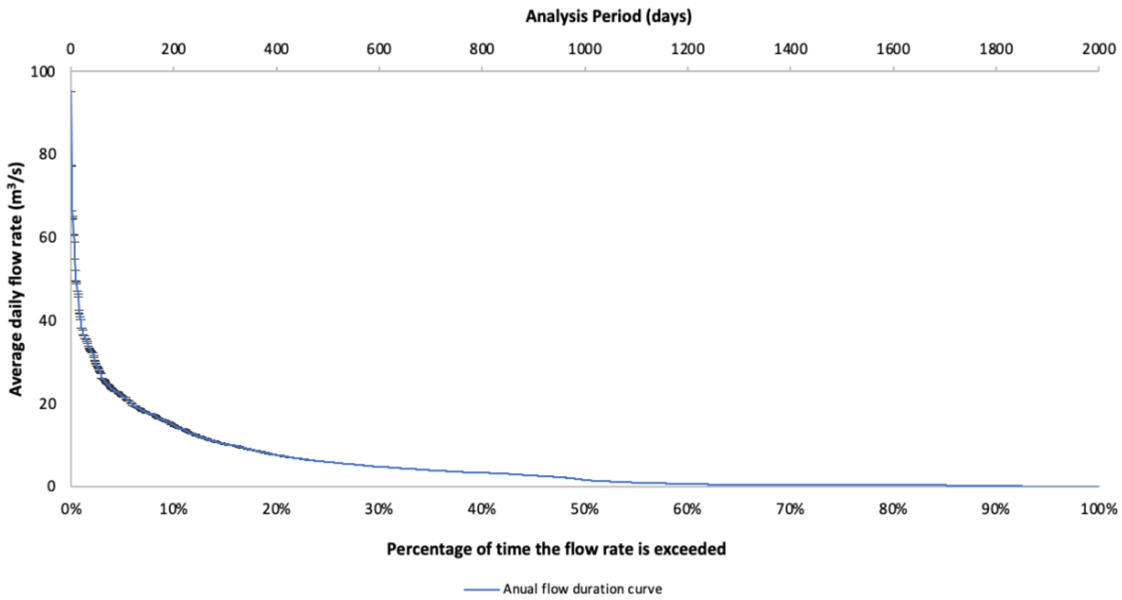

Figure 5: Annual flow duration curve based on average daily flow rates and expanded measurement uncertainty.

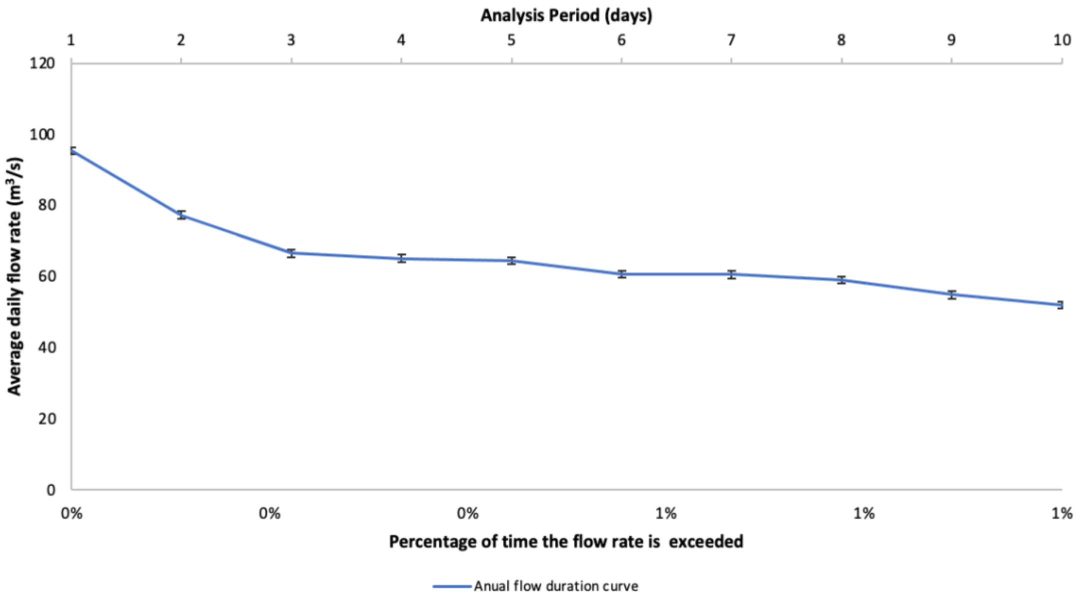

Figure 6: Detail (for the 10 days of the sample) of the annual flow duration curve based on average daily rates and expanded measurement uncertainty.

3. CONCLUSIONS AND FURTHER WORK

Observation of time-varying phenomena under conditions of a cyclic or permanent nature is common in many branches of science and is known in Hydrology [3].

In this context, the primary objective of this study was to show how to calculate model uncertainties by considering measurement uncertainties and model formulation in a simplified way. This study aimed at the evaluation of the uncertainty associated with the estimate of the annual flow duration curve of average daily flow rates to be used in hydropower feasibility studies.

The approach used is feasible and sound, allowing for the calculation of the flow duration curve and associated uncertainty. In summary, for the data available, the results are:

– the average daily flow rate of the series, $F_{\mathrm{corr,av}}$, of 6.35 m^3/s with an expanded uncertainty (95% confidence) of 0.19 m^3/s;
– the ecological flow rate, F_{ec}, was estimated from a set percentage of the mean flow rate and deduced from the series of corrected average daily flow rates, to get the annual flow duration curve of average daily flow rate;
– the average flow rate, F_{av}, from the annual flow duration curve, is 6.04 m^3/s with an expanded uncertainty (95% confidence) of 0.01 m^3/s.

Further work is necessary to validate the method developed by using daily flow rate series together with precipitation series and investigate whether these results are improved. Additionally, it is worth investigating the application in more complex models, using software available in the market, and evaluating the impact of uncertainty in these cases.

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