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Covariance in 3D measurement uncertainty evaluation

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***Abstract* – The accepted way of expressing measurement results is through an estimated value and its associated measurement uncertainty. Part of the uncertainty evaluation is also to consider mutual sources of uncertainties, called covariance.**

This paper will describe a CMM 3D measurement and its uncertainty evaluation. A 3D measurement considered here is a length measurement in three orthogonal directions along the *x*, *y*, *z* axes of a Cartesian coordinate system. When a point in 3D space is located it is considered to be a direct method of measurement. However, when it comes to geometrical features such as the radius of a circle (hole), sphere, cylinder, cone etc. the measurement should be considered as an indirect method of measurement. Therefore, for both the above-mentioned methods, the procedure given by ISO/IEC Guide 98-3:2008 is to apply the law of propagation in uncertainty.
Standards such as ISO 10360 and VDI/VDE 2617 supply guidelines for the understanding of uncertainty contributions to 3D measurement uncertainty, and standard ISO 15530-3 describes techniques for determining uncertainty. However, these standards do not consider uncertainties having a mutual source to be dealt with as covariance.

This study presents a new approach to evaluating measurement uncertainty for 3D measurement, which helps to conclude where in 3D space the measurement is most affected by uncertainties.

Keywords – uncertainty, covariance, error, CMM, 3D measurement, straightness, squareness

I. INTRODUCTION

The first CMM machines were built and used around in 1956, and for many years linear and squareness of the machine were checked. American and German standards ASME B89.1.12-1985 and VDI/VDE 2617-1986 an were the first attempt to standardize CMM machine performance testing and informed the first international standard ISO 10360:1990 for testing CMM's.

CMM machine performance testing is about:

- Length test E_0: applies to the entire volume of 3D space and testing distance between any two points.
- Repeatability: the range of the length measurement error, R_0
- Length measurement error with the ram axis stylus tip offset of 150 mm, E_{150}
- Probing test: Probe testing in combination with the CMM.
- Testing in different operational modes: trigger probing or scanning.

At the time the above standards were created the ISO/IEC Guide 98:1993 "Guide to the expression of uncertainty in measurement" (known as ISO GUM) [1] was not yet issued. In 1993 when the standard was starting to be applied, mainly by national laboratories who were involved with the standard. However, in industry the term "uncertainty", even today is often miscommunicated as "error." A lack of knowledge and understanding regarding measurement uncertainties has led to consideration only of errors, which is at best misleading.
In addition, there is a constant drive in industry to reduce the time and cost of testing, verification, acceptance test of CMM's which can result in neglecting good practice such as testing squareness and replacing it with a quick and cheaper sphere measurement test. Furthermore, the above mentioned CMM standards even after revision do not present a robustly applicable approach to calculating measurement uncertainties.

However, there are two standards, which are trying to fill the gap and provide some recommendations regarding what to consider for uncertainty analysis. ISO 15530-3 "Technique for determining the uncertainty of measurement - Part 3: Use of calibrated workpieces or measurement standards" and ISO 23165 "Guidelines for the evaluation of coordinate measuring machine (CMM) test uncertainty." Unfortunately, their compliance with the ISO GUM [1] is arguably limited.

The ISO GUM [1] clearly states procedures to evaluate an uncertainty, which are:

- Define the measurand $Y = f\,(X_1,\ X_2, \ldots, X_N)$ where $X_1 \ldots X_n$ are input quantities, identified contributing factors
- Determine numerical representation of an input quantity (x_i)
- Evaluate the standard uncertainty $u(x_i)$
- Evaluate the covariance
- Calculate measurand (output quantity) $y = f\,(x_1,\ x_2, \ldots, x_N)$
- Evaluate the combined standard uncertainty $u_c(y)$
- Evaluate the expanded uncertainty U
- Report y, $u_c(y)$, $U(y)$ with level of confidence

The Standards ISO 15530-3[3] and ISO 23165[4] do not consider covariance. In terms of understanding the machine as a measuring system standard VDI/VDE 2617 [5] appears to be more practical and clearer. Nonetheless these standards do not give attention to the covariance and modelling measurement uncertainties to be consistent with ISO GUM. Therefore, when it comes to dimensional measurement, it is worthwhile to consider the following cases to understand modelling and uncertainty evaluation of a 3D measurement.

Measurement models

Case 1: Distance (Length) measurement of two points on a line in 1D Euclidean space. This can be direct or indirect depending on the method of measurement. If it is a tape measure, or linear it is a direct measurement, if it is based on a principle such as time of flight, the distance (length) D is calculated by using mathematical formula: $D = v \cdot t$, (v – speed of source, photon emitted and travelling till it received, t – time) it is considered to be indirect, unless we ignore the measurement principle and consider the measurement equipment as a black box, and therefore use the indicated value as a direct measurement indication.
Case 2: Position (x, y) of a point in 2D Euclidean space. To locate a point in 2D we need horizontal and vertical measurements of a point projected on each line (axis). The distance between the origin and the point is calculated as $d^2 = x^2 + y^2$. Distance (output) from the origin is a result of two input quantities (x, y), and is therefore an indirect measurement.
Case 3: The position of a point (x, y, z) in 3D Euclidean space. Similarly, a point in 3D is located as a length measurement of a projected point onto each axis. The distance from the origin is calculated as $d^2 = x^2 + y^2 + z^2$.
Case 4: The measurement of geometrical features such as the diameter of a circle, a sphere, a cone also should be considered as an indirect measurement, as it is calculated using formula from analytical geometry.
Case 5: The measurement of free form lines and surfaces in 3D. Indicators of free form lines and surfaces are expressed through form errors. Form error is a difference calculated between a reference point and measured point. However, interpolation is often used to estimate the function of a form. Using functions means that the measurements are indirect.

All these cases are present when we are performing 3D measurement. Although sources of uncertainties are known, evaluation of a 3D measurement uncertainty needs to be hierarchical and task specific.

Sources of uncertainties of a 3D measurement [7]

CMM Geometry
- Rigid body errors, construction, maintenance
- Scale resolution
- Quasi static errors
- Dynamic error

Sensor System
- Probe type
- Calibration strategy
- Stylus bending
- Scanning force and speed
- Stylus configuration
- Stylus bending
- Filtering

Data Analysis
- Fitting algorithm

Environment
- Thermal effects
- Humidity
- Vibration

Work piece
- Distortion by fixture
- Systematic form error

Sampling Strategy
- Number of locations and sampling points

II. METHODOLOGY

In the following analysis we will focus on CMM measurement with a contact probe head. The CMM measures length in three different directions along axes x, y, z, where the reference datum frame is a Cartesian coordinate system. The origin point of the 3D coordinate system is an intersection of three lines, axes X, Y, Z. In reality a CMM machine is a physical representation of a Cartesian Coordinate system.

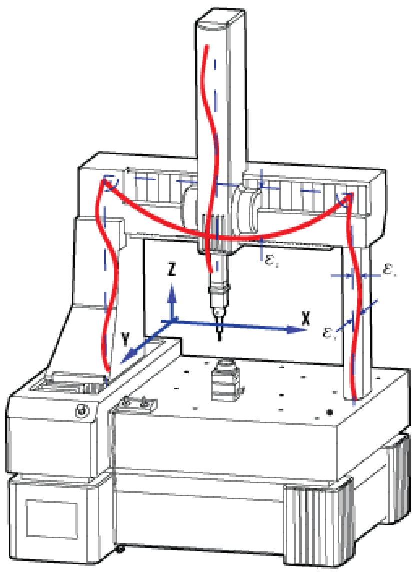
The standard ISO 15530 divide influencing factors into three main categories:

- **Instrumentation factors**
- **Measurement plan factors**: are the selected configuration of probe and stylus, point, number of repetitions and point sampling strategy. It is recommended to have these variables constant to get better transparency of the measurement.

- **Extrinsic factors**: These are factors related to the object of measurement, such as surface roughness, geometrical features to be measured, material properties of the workpiece, fixturing solution and thermal distortion.

Instrumentation factors

The length measured by CMM is measured with a built in linear scale. The best practice for determining error of linear scale by comparison against a Laser interferometer standard, measured in forward and backward direction (Case 1). With this method is possible to determine hysteresis and position accuracy of the machine on each



axis.

However, to determine measurement uncertainty of a point in 3D space (Case 3, 4, 5 above). The most critical instrument factors are straightness of the axis and squareness (perpendicularity) between each axis.

Fig.1.1 CMM's straightness and squareness errors

Straightness and squareness can be also measured by laser interferometer using a Wollaston polarizing prism. Standard methods for CMM acceptance and reverification test is also include use of a step gauge.

For testing and monitoring machines condition a ring gauge or sphere is employed. Although it became a common practice, to rely only on sphere measurement (called also probe error, probe offset test), which could be misleading or simply wrong.

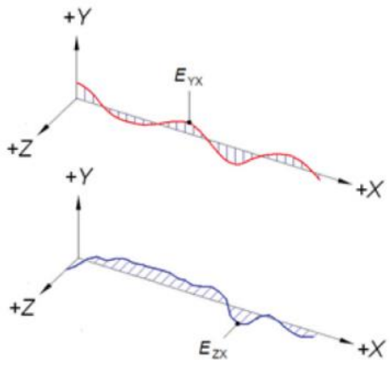


Fig.1.2 Error curve of the straightness of the axis X in YX and ZX plane

Sphere measurement shows cumulative effects of all sources of errors and uncertainties and is meant to be an indicator to see any issues with CMM.

It is advisable to check a set of located measuring lines in 3D space (work zone), to get a thorough understanding of our 3D measurement space and their measurement uncertainties. Measurement results from these tests will be the data source to fill up our Covariance matrix. The covariance matrix will look like this:

Covariance matrix will look like this (Eq.1.0)

$$\boldsymbol{U}_W = \begin{pmatrix} u(x)^2 & u(x,y) & u(x,z) \\ u(x,y) & u(y)^2 & u(y,z) \\ u(x,z) & u(y,z) & u(z)^2 \end{pmatrix}_{(3x3)} \qquad (1.0)$$

Where covariance between axes are calculated

$$\begin{aligned} u(x,y) &= r_{x,y} \cdot u(x) \cdot u(y) \\ u(x,z) &= r_{x,z} \cdot u(x) \cdot u(z) \\ u(y,z) &= r_{y,z} \cdot u(y) \cdot u(z) \end{aligned} \qquad (1.1)$$

$r_{x,y}$; $r_{x,z}$; $r_{y,z}$ are correlation coefficients

From GUM [1 page 55 (E.3) law of propagation of uncertainty in matrix notation is written as

$$\boldsymbol{U}_y = \boldsymbol{J} \cdot \boldsymbol{U}_W \cdot \boldsymbol{J}^{\mathrm{T}} \qquad (1.2)$$

Where ***J*** vector is Jacobian vector

$$\boldsymbol{J} = \begin{pmatrix} \frac{\partial l}{\partial x} & \frac{\partial l}{\partial y} & \frac{\partial l}{\partial z} \end{pmatrix} \qquad (1.3)$$

In its expanded form the law of propagation uncertainty for 3D measurement will be

(1.4)

$$u(y) = \left(\frac{\partial l}{\partial x}\cdot u(x)\right)^2 + \left(\frac{\partial l}{\partial y}\cdot u(y)\right)^2 + + \left(\frac{\partial l}{\partial z}\cdot u(z)\right)^2 + \left(2\cdot\frac{\partial l}{\partial x\partial y}\cdot u(x,y)\right) + \cdots \cdots + \left(2\cdot\frac{\partial l}{\partial x\partial z}\cdot u(x,z)\right) + \left(2\cdot\frac{\partial l}{\partial y\partial z}\cdot u(y,z)\right)$$

The equation (1.4) will be used to determine uncertainty of the length measurement in 3D space. Thus the sensitivity coefficients will be

$$\frac{\partial l}{\partial x} = \frac{x}{x^2+y^2+z^2}; \quad \frac{\partial l}{\partial y} = \frac{y}{x^2+y^2+z^2}; \quad \frac{\partial l}{\partial z} = \frac{z}{x^2+y^2+z^2} \tag{1.5}$$

Here, we need to determine correlation coefficient using method A and B. If the measurement is indirect, such as cases 2, 3, 4, to evaluate distance between the point and the origin, the input quantities are mutually dependent and are random by nature, then method A will be used for the evaluation an expression given by GUM [1 page 22 sec.: (5.2.3, 17)]. Method B, will follow either empirical approach or more qualified estimation by running finite element method analysis (FEM) on the mechanical structure of the CMM (such as gantry, column etc.). FEM analysis reflects mechanical, material (such as rigidity) and kinematical properties and other imperfections of the CMM machine on point displacement (Fig.1.1).

III. RESULTS AND DISCUSSIONS

There are two methods of performing a measurement:

- Absolute

Absolute method will result an estimated value with known amount in standard units.

$$l_x = l_i + \Delta\varepsilon$$

- Relative

Relative measurement estimates values comparing to a reference value (can be nominal value or conventionally true value) and result is expressed as error or deviation.

$$E_x = (l_i - l_s) + \Delta\varepsilon$$

l_x – value obtained after applying corrections
l_i – value indicated by measurement instrument
$\Delta\varepsilon$ – error (correction)
E_x – error (deviation) obtained from comparison

When it comes to an absolute measurement, for most of the cases the uncertainty value is attached to the estimate. When measurement is relative, in practice it is rarely seen to be expressed estimated value (error, deviation) with uncertainty. Same issue we can experience by studying standard ISO 10360. Performance test focus on determining E_0, E_{150}, (which is an error value), but there is no information about uncertainty. For verification or acceptance checks, the Maximum Permissible Error (MPE) is expressed in equation form E_0 = a+b·L. and using ISO 14253 "Decision rules for verifying conformity or nonconformity with specifications" an uncertainty value has to be attached to the estimated error value. In an uncertainty value the covariance contribution has to be involved, otherwise uncertainty value might be better as it stated.

Results for 3D measurement should be expressed as
$(x_l \pm u(x_l)), (y_l \pm u(y_l)), (z_l \pm u(z_l))$.

IV. CONCLUSIONS AND OUTLOOK

The above designed method of evaluation of a 3D measurement uncertainty reflects knowledge of uncertainty evaluation procedure described in GUM and trying to align it with the knowledge and skills gained from 3D measurement. Important is to keep in mind, each geometrical feature measured with a CMM is an individual measurement case and that how has to be approached when it comes to uncertainty modelling. During calibration, it is important to check CMM's straightness and squareness as well, which are the main sources of covariance and use law of propagation uncertainty when distance of point to point is calculated in 3D Euclidean space.

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