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Uncertainty-based combination of signal processing techniques for the identification of rotor imbalance

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***Abstract* – This paper describes a method for the uncertainty-based combination of signal processing techniques for the identification of rotor imbalance. The main idea of the proposed method is to compute the imbalance with different algorithms and to average their results. The method is based on the data fusion at feature level and uses the measurement uncertainty of the imbalance as a figure of merit for the weight computation. A static and a dynamic implementation are presented. In the static one, the weights are computed in a dedicated training phase, in which four algorithms (Fourier transform and quasi-harmonic fitting of signal denoised with Hilbert-Huang Transform, Hilbert Vibration decomposition, and Wavelet Packet decomposition) have been used to estimate the known imbalance of car wheels. In the dynamic one, the weights are computed at runtime by estimating the difference between each predictor and the actual signal. Experimental results evidenced the validity of the proposed method, with uncertainty reductions between 10 and 40%, with larger benefits in presence of localized disturbances.**

I. INTRODUCTION

In many cases there are different signal processing techniques that can be used to extract specific features from a signal. The different information can be combined with the information fusion technique, which can be implemented at data-level, at feature-level and at decision-level [1]. The combination of different sensors or features always allows reaching a superior accuracy than that achievable using a single information [2-4]. Consequently, several literature studies focused on the combination of different techniques for the identification of parameters related with mechanical systems. Niu et al. [5] proposed a faults diagnosis mechanism using the wavelet analysis and decision-level fusion technique for motors fault diagnosis. In another study, Niu et al. [6] proposed a decision fusion system for fault diagnosis integrating data sources from different sensors and decisions of multiple classifiers; the use of multi-agent classifiers as the core of the fault diagnosis system allowed increasing the accuracy of the fault detection. At the current state of the art, the data fusion has never been adopted to increase the accuracy of rotors' balancing, the procedure in which the mass distribution of a rotor is measured and, if necessary, adjusted to ensure given tolerances. The rigid rotors' imbalance can be identified at constant rotation speed using the influence coefficient method [7, 8] and corrected on two arbitrary planes [9] starting from the vibrations V_1 and V_2 measured at two planes of motion. The advantages provided by data fusion in a simple and traditionally successful application such as the balancing may appear limited, since the influence coefficient method already provides for satisfying results when the signal to noise ratio is favourable. In presence of measurement noise and mechanical disturbances, however, the influence coefficient method requires a large number of averages in order to provide for reliable results (i.e. a stable imbalance phase). Since a reduced balancing time is a key factor in the commercial success of a balancing machine, we have investigated the possibility of obtaining more reliable imbalance estimation using innovative data analysis procedures. With this approach, the measurement accuracy can be improved with the same measurement hardware, increasing the measurements reliability without any hardware modification. In a recent study, performances of four different algorithms for the identification of car wheel imbalance were compared [10]. The imbalance amplitude and phase were derived from transient signals using four different numerical methods: the computed order tracking and Fourier Transform (FT-COT) and the quasi-harmonic regression on the signal denoised using the Hilbert Vibration Decomposition (HVD), the Hilbert Huang Transform (HHT) and the Wavelet Packet Decomposition (WPD). The main idea behind the use of HVD, HHT and WPD is that, since their errors should be uncorrelated with those of the FT, the combination of different algorithms provides for a better accuracy with respect to the use of a single method. In ref [10], performances of the four algorithms were analysed using the Design of Experiments and results allowed evidencing the accuracy of each method. Performances of the HHT were penalized by the simplistic method used for the 1X identification, while performances of HVD, WPD and FT-COT were comparable; the choice of the best method was not straightforward.

In this paper, we propose different approaches that can be adopted to merge the indications of different algorithms using the measurement uncertainty (evaluated in accordance with the ISO GUM) as a figure of merit for the identification of the algorithms' weights. The approaches are similar to the variance-based weighting for data fusion [13], but weights are computed by with different expressions derived from the measurement uncertainty.

II. METHOD

If a measure *Y* (a parameter that can be extracted by a signal *X*(t) as, for instance, the rotor imbalance) can be obtained using *M* different approaches, it is possible to compute *Y* by averaging the different estimates Y_i (being *i* an index that varies between 1 and *M*). The arithmetic average is, in general, a non-optimal choice, given that the same relevance is given to all the Y_i independently on their performances [13]. Let us consider the estimation of *Y* with two quantities Y_1 and Y_2, each one characterized by a measurement uncertainty U_1 and U_2. *Y* can be obtained as a linear combination of Y_1 and Y_2 using two weights α and β.

$$\begin{cases} Y = \alpha Y_1 + \beta Y_2 \\ \alpha + \beta = 1 \end{cases} \tag{1}$$

In the case of rotor balancing, the amplitude of the vibration component synchronous with the rotation (1X) can be estimated from the signal spectrum (Y_1) or in the time domain (Y_2, best quasi-harmonic signal fitting the de-noised experimental data in a least square sense). The amplitude can be computed with one of the two methods (α or $\beta = 0$) or as the average between Y_1 and Y_2 ($\alpha = \beta = 0.5$). The best estimation is the one in which α and β minimize the uncertainty of Y, U_Y. Under the hypothesis of non-correlated uncertainties, U_Y can be computed according to the ISO GUM as:

$$U_Y = \sqrt{\left(\frac{\partial Y}{\partial Y_1} U_{Y_1}\right)^2 + \left(\frac{\partial Y}{\partial Y_2} U_{Y_2}\right)^2} \tag{2}$$

Given that the sum of coefficients is 1, equation (2) becomes

$$U_Y = \sqrt{\left(\alpha U_{Y_1}\right)^2 + \left((1-\alpha) U_{Y_2}\right)^2} \tag{3}$$

U_Y can be minimized setting $dU_Y/d\alpha=0$, obtaining

$$2\alpha {U_{Y_1}}^2 + 2\alpha {U_{Y_2}}^2 - 2{U_{Y_2}}^2 = 0 \tag{4}$$

The coefficient α minimizing the measurement uncertainty is:

$$\alpha = \frac{{U_{Y_2}}^2}{{U_{Y_1}}^2 + {U_{Y_2}}^2} \tag{5}$$

The above expression is equivalent to the variance averaging proposed by Taniguchi and Tresp [13] if the uncertainty is computed with the ISO GUM type A approach; in this case, the weights are proportional to the inverse of the variance, exactly as in the variance approach. More generally, a measurement Y can be expressed as the combination between *n* estimations:

$$\begin{cases} Y = \alpha_1 Y_1 + \alpha_2 Y_2 + \ldots + \alpha_n Y_n \\ \sum_{i=1}^{n} \alpha_i = 1 \end{cases} \tag{6}$$

An alternative to the use of the variance-based weighting, is given by the normalization criterion with the weights α_i computed as follows:

$$\alpha_i = \frac{\left(\sum_{j=1}^{n} {U_{Yj}}^2\right) - {U_{Yi}}^2}{(n-1)\sum_{j=1}^{n} {U_{Yj}}^2} \tag{7}$$

Equation (7) states that, if *Y* has *n* uncorrelated estimators Y_j, each estimator has a weight that is proportional to the "residual variance", i.e. the variance introduced from the other estimators. This approach, as later explained, is more robust versus mismatches between the training phase and the actual usage.

A. Computation of weighting coefficients

Weighting coefficients α_i can be computed with different approaches, depending on the evaluation of the measurement uncertainty.

1 - If the parameter Y is estimated by a single model, uncertainty can be estimated at runtime using the measurements repeatability and the difference between the model prediction and the experimental data.

2 - If the parameter Y is estimated by *M* different models, uncertainty can be estimated using the results of a training phase in which the measurand is known.

3 - If a parameter Y is estimated by *M* models, uncertainty can be initially estimated by single models (approach 1) and the *M* results can be averaged with the approach 2.

With the approach 1 (hereinafter referred to as dynamic), uncertainty is estimated with a unique model, repeating the estimation on N different signal subsets [14]. In each subset j, it is possible to evaluate the method uncertainty U_{Yj} starting from a the difference E_j between the model prediction (X_j) and the measured signal ($X_{m,j}$), as shown in Figure 1.

$$E_j(t) = X_j(t) - X_{m,\mathrm{j}}(t) \tag{8}$$

An indication of the uncertainty U_{Yi} is provided by the root mean square of E_i(t):

$$U_{Yj}^{(1)} = \frac{1}{T}\sqrt{\sum_{t=0}^{T}\left[E_j(t)\right]^2} \tag{9}$$

If the measured signal and the model prediction are similar, the error E_j(t) and its RMS U_{Yj} are small. In presence of localized disturbances occurring, for instance, in the k-th buffer, the measured signal $X_{m,k}$ is different from the model prediction, and U_{Yk} is larger than the uncertainties U_{Yj} in the other buffers. The weighting coefficients α_i are therefore larger than α_k and the k-th buffer has a small relevance in the computation of Y.

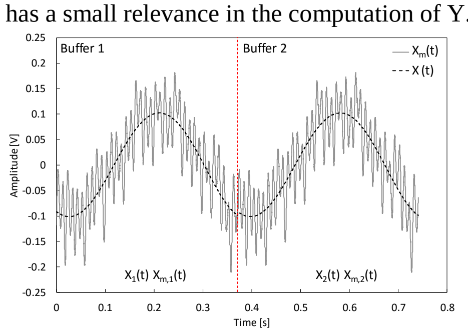

Figure 1 Measured signal $X_m(t)$ and modelled signal $X(t)$

With the approach 2 (hereinafter referred to as static), uncertainties of M different models are estimated in a training phase where the measurand is known (Y_j) and the performances of each model are evaluated from the differences E_j between the measured parameter $Y_{m,j}$ and Y_j. The method uncertainty can be assessed from the errors E_j affecting the M measurement models; the total uncertainty is:

$$U_{Yj}^{(2)} = \sqrt{\sum_{j=1}^{M} E_j^2} \tag{10}$$

In this case, the method gives more relevance to the models that, in a training phase, provided for a best approximation of Y_i in a least square sense. The approach 3 (hereinafter referred to as hybrid) simply consists in the application of the dynamic approach of different algorithm to obtain M different estimations, which are eventually weighted using the weighting coefficients as in the static approach.

III. RIGID ROTOR BALANCING

The proposed method has been used for the rotor balancing operation. The easiest way to balance a rotor is the influence coefficient method: if the rotor angular velocity is constant, the imbalances on the two planes of correction are the product of the measured vibration vector times by the influence coefficients matrix, identified through machine calibration. With a matrix notation:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{11}$$

The imbalance vector **I** has two elements I_1 and I_2 that are the complex imbalances on the correction planes Π_1 and Π_2. The imbalance moduli are the product of the balancing masses m_1 and m_2 times their distances from the rotation axis r_1 and r_2. The imbalance phases are respectively φ_1 and φ_2. In principle, the identification of imbalance modulus is straightforward, given that the vibration signals measured by the two transducers should be harmonic; the force signals are therefore:

$$\begin{aligned} V_1 &= A_1\,\omega^2\,\sin(\omega t + \gamma_1) \\ V_2 &= A_2\,\omega^2\,\sin(\omega t + \gamma_2) \end{aligned} \tag{12}$$

The dependence between the imbalances moduli and phases (mr and φ) and the amplitudes and phases of the measured signals (A and γ) is expressed by the influence coefficients of equation (11). In real applications, the measured signals differ from the harmonic ones because of the spurious vibration generated by the machine components, because of mechanical and electrical disturbances and because of possible rotation speed variations. In real conditions under the hypothesis of linear system – a condition always satisfied in balancing machines in their working range – it is possible to fit the experimental data in a least square sense with the quasi-harmonic signals of equation (12), where the angular velocity ω is instantaneously measured by an encoder [10]. Since the radii of correction are known, the results of the regression A_1, A_2, γ_1 and γ_2 can be used to compute the balancing masses and phases using the influence coefficients as per equation (11) also in case of transient signals.

B. Experimental setup

Experiments were performed on two prototypes of machines for car wheel balancing, operating between 120 and 160 RPM. The influence coefficients were identified with the machine calibration, i.e. measuring the known imbalance in two different conditions. The charge outputs of two piezoelectric force sensors were conditioned using a Bruel & Kjaer Nexus charge amplifier. The output voltage was A/D converted (together with the 128 pulses encoder signals measuring the shaft speed) by two National Instruments 9234 DAQ boards in a Compact DAQ chassis. The sampling rate was 12 kHz and the signal included the run-up and coast-down transients.

C. Signal Processing

An example of the signals measured by two load cells located on the hub supports is shown in Figure 2. The figure shows that, although the harmonic component is visible between 3 and 7 s, the spurious components are relevant.

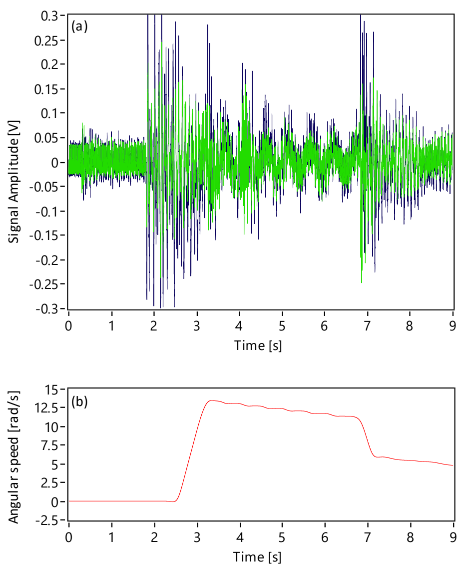

Figure 2 Vibration signals (a) and angular speed (b) measured during a typical balancing machine cycle.

The dynamic and the static approaches were tested in different ways: uncertainty of FT-COT was computed inline dividing the time history in non-overlapped subsets (containing an integer multiple of wheel revolutions) in order to reduce the effect of impulsive events. The weighted averaging technique has been compared with the arithmetic averaging technique. Tests were performed with two machines operating at respectively at 130 RPM (tests *a* and *b*) and at 160 RPM (test *c*). Two wheel diameters were tested, with imbalances ranging from 0 to 60 g applied on the wheel rim. The robustness of the dynamic approach has been tested generating impulsive disturbances on three different parts of the machine (basement, cover and shaft). The weighted averaging and the arithmetic averaging have been compared using the mean residual imbalance, defined in the next pages. The dynamic approach has been used in specific tests with the aim of understanding the effect of impulsive disturbances. In the static tests, the training phase was based on the 216 tests described in ref. [10]. The wheels' imbalance has been evaluated with the three methods (HVD, WPD and FT-COT) using a full factorial design of experiments performed upon varying the balancing machine, the rotor size, the rotor imbalance the presence and the nature of mechanical disturbances. Uncertainties were used to compute the weighting coefficients α_i multiplying the imbalances estimated by the HVD, WPD and FT-COT. Optimal coefficients were then used to identify the MRI in a new series of tests, performed with the two machines, using the small wheel with imbalances of 20 and 60 g applied on both the internal and the external side of the rim with phases of 0 and 90°. Mechanical disturbances (slow mechanical actions on the cover and on the mandrel, different from the impulsive ones used in the experiments) perturbed the machine during the measurements. 12 tests for each machine were performed.

The hybrid approach is currently under evaluation and will not be described in this paper. The methods' efficiency has been evaluated using the mean residual imbalance (MRI). In each test j, the error $\mathbf{e}_j$, has been computed as the difference between the measured imbalance $\mathbf{I}^*$ and the known imbalance $\mathbf{I}$.

$$\mathbf{e}_j = \mathbf{I}_j - \mathbf{I}_j^* \qquad (13)$$

The vector $\mathbf{e}_j$ components are the inner and outer balancing planes errors. These two quantities have been summarized by the mean residual imbalance MRI_j:

$$MRI_j = \frac{\mathrm{mod}\left(e_{1,j}\right) + \mathrm{mod}\left(e_{2,j}\right)}{2} \qquad (14)$$

IV. RESULTS AND DISCUSSION

D. Dynamic Approach

With the dynamic approach, the measurement uncertainty is estimated by the difference between the prediction of a single model and the experimental data. The estimation is repeated on different signal subsets with the aim of giving more relevance to the ones in which the model prediction and the experimental data are similar. Experimental results are summarized in Figure 3: the MRI using the proposed method decreases from 1.7 to 1.4 g (average values on the entire experimental set) and the MRI_j standard deviation decreased from 3 to 2.3 g. The maximum error decreased from 18.2 to 15 g. The benefits deriving from the adoption of the approach 1 are more pronounced in conditions *c* at 160 RPM in presence of mechanical disturbances acting on the shaft. The benefits deriving from the use of the weighted averaging technique in undisturbed conditions are minor If the disturbances are stationary, the four uncertainties are almost equal and the weighted average is very close to the arithmetic average.

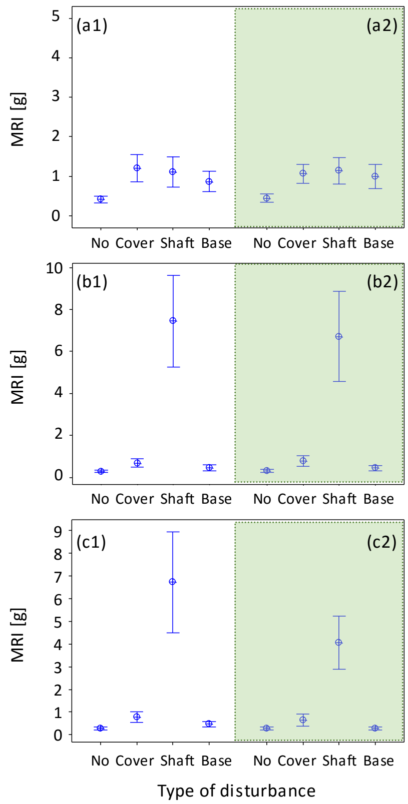

Figure 3 Comparison of the MRI (95 % CI) obtained with different types of disturbances. on different machines (a, b and c) using the standard FT-COT (a1, b1 and c1) and the static approach (a2, b2 and c2).

The same consideration applies to the disturbances on the cover and on the basement, where the uncertainty $U_{Yi}^{(1)}$ in the buffer with the disturbance is comparable to the uncertainty in the undisturbed buffers. From this point of view, the approach 1 is more efficient if the SNR is favourable: increasing the rotation of the same machine from 130 to 160 RPM (i.e. passing from condition *b* to condition *c* of Figure 3) increased the accuracy benefit from 10 % to 40% when disturbances are relevant.

Results presented in this section may look not particularly encouraging; the differences between the dynamic approach and the COT-FT are generally small and the advantage may look negligible in comparison with the more complex data processing required by the method. However, when the COT-FT performances are already satisfying (good SNR) the proposed method does not introduce any meaningful advantage; conversely, when the COT-FT method has limitations deriving from the presence of impulsive disturbances (as in Figure 3 – C1), the dynamic approach significantly reduces the measurement error. Such a reduction, however, is strongly dependent on the SNR.

E. Static approach

The static approach consists in the weighted average of measurements obtained using different methods, with weights identified in a preliminary (training) phase. The weights of the HVD, WPD, and FT-COT have been identified from the results of experiments described in ref. [10]. The measurement uncertainties were computed summarizing the results of 216 balancing tests using the type A ISO GUM approach. Standard uncertainties and were 3.5 g (HVD), 2.1 g (WPD) and 2 g (FT-COT). The relative weights of the three computation methods, computed with equation (7), are 20.4% (HVD), 39.3% (WPD), and 40.3% (FT-COT). As a term of comparison, the weights computed with the variance approach described in ref. [13] were 14.6% (HVD), 40.6% (WPD) and 44.8% (FT-COT).

The validity of the static approach has been verified with 12 tests performed on machines a and b. MRI are summarized in Figure 4, which compares the balancing error obtained with the static approach with the balancing errors obtained with the HVD, WPD, and FT-COT.

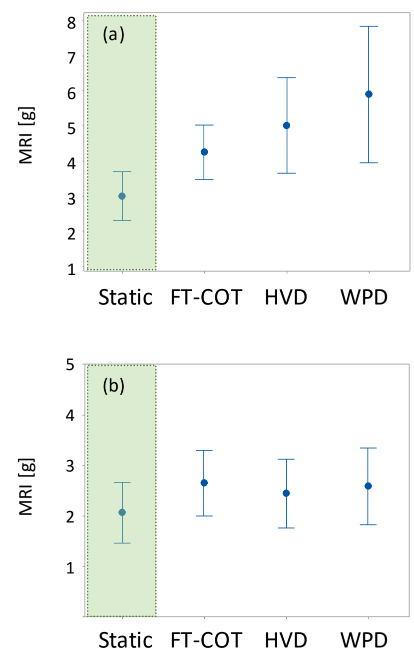

Figure 4 Comparison between the MRI (95% CI of the mean) measured by the static method (green) and the "raw" algorithms with machines (a) and (b).

The plot shows that on both machines the balancing error obtained as a weighted average of the three methods is lower than the balancing error obtained by the best method, coherently with the uncertainty minimization described in section 2. The plots also show that in tests performed with the machine (a) the best method is the FT-COT, while in tests performed with the machine (b) best results are obtained using the HVD. Although the accuracy of the different methods may vary depending on the experimental condition, if the training phase covered a wide set of possible test configurations, the combination of different methods provides for better results than the single methods.

Additional analyses were performed considering only the WPD and FT-COT methods with weights of 0.48 and 0.52 respectively. Results evidenced that the MRI obtained with only two methods is averagely larger (+30%) than the MRI obtained using three methods. The analyses performed using the weight identified with the variance method described by Taniguchi and Tresp [13] lead to results similar to those obtained with the static method; this outcome was expected, since coefficients of the static method and of the variance method were comparable, as evidenced in the previous section. The variance method can be used also in the dynamic method; the comparison performed on a limited number of tests evidenced that the benefits are limited, given that differences were small (<5%) in comparison with the results variability.

V. DISCUSSION AND CONCLUSIONS

Experimental results showed that both the static and the dynamic approaches reduce the uncertainty in the identification of rotors' imbalance. The proposed data processing methods allow reducing the effect of disturbances by using a weighted average of multiple measurements of the same phenomenon.

The first aspect deserving to be discussed is that equation (2) is based on the hypothesis of non-correlated errors. If the static approach is based on similar methods (e.g. spectral estimation with two different windows) measurement errors might be correlated. In this case, the proposed approach is still valid, but the weights must be computed including the errors' covariances that should be determined along with the uncertainties in the characterization phase. Experimental results evidenced that benefits of the dynamic method are dependent on the SNR; consequently, the absence of spurious harmonic components in the measurement signals and the quality of the measurement chain are crucial.

The efficiency of the static approach is based on the similarities between the tests performed in the training phase and the real ones; it is therefore important for the training phase to include the majority of the events that can be found in the real operating conditions with their actual percentages of occurrence. The comparison of the regularization criterion described in this paper and the variance-based method proposed by Taniguchi and Tresp outlined similar performances, with differences that are small in comparison with the tests repeatability. In general, the variance criterion is more selective and gives more relevance to the methods that, during the training phase, lead to better results. Nevertheless, in presence of mismatches between the training conditions and the actual experimental conditions, the high selectivity of the variance-based method can be counterproductive. Independently from the adopted normalization criterion, the use of the data fusion techniques is very promising and allow reducing the measurement uncertainty with respect to the traditionally used COT-FT method without any modification to the measurement hardware.

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