

## Uncertainty Evaluation of DAC Time Response Parameters

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**Abstract-** The paper proposes a novel approach to evaluate the measurement uncertainty of time domain dynamic parameters in Digital to Analog Converters (DACs). In particular, DAC rise time and fall time are taken into account and analyzed experimentally by means of dedicated measurement stations. The uncertainty of such measures is evaluated using the bootstrap method in order to define confidence intervals from short acquisition records without needing hypotheses about the measurand distribution.

The final aim of this work is to give a contribution, in terms of uncertainty assessment, for the draft of IEEE Std.1658 concerning terminology and test methods for DACs.

### I. Introduction

Digital-to-Analog Converters (DACs) are essential components that provide a link between the digital and analog sections of a mixed-signal system. These components are crucial in fields such as optical networking, high-end medical imaging, cellular and smart phones, and digital video camcorders, requiring wide frequency band and high resolution. Therefore, there is an increasing interest in these devices, both from the scientific and industrial communities, resulting in a wider and deeper choice to the potential users.

On the other hand, the considerable improvement that characterized DACs in the last years produced burdensome problems due to the lack of standard terminology and test methods. DACs made by different manufacturers are often difficult to compare, due to the different ways of specifying and assessing the performance parameters, making hard the selection of the best suited device for a user needs. For this reason, the IEEE Instrumentation and Measurement Society TC-10 “Waveform generation, measurement and analysis” is working for developing a standard on DAC terminology and test methods [1].

Of course, removing the ambiguities existing in DAC terminology and testing methods cannot provide useful compatibility information without specifying the parameter measurement uncertainty. The measurement uncertainty is a quantitative indication of its quality, allowing the comparison of results coming from different sources or with reference values given in specifications or standards. Therefore, a given customer can correctly compare and choose the converter suitable to his/her particular needs only if the uncertainty about the parameters of interest has been specified in the manufacturer’s datasheet. Unfortunately, currently the evaluation of the measurement uncertainty is not yet included in any DAC standard [2].

This work is focused on the uncertainty evaluation of the main DAC time domain dynamic specifications as the rise time and fall time. Rise and fall time give information about the slew time of the DAC output waveforms digitized during a test session.

The meaning of the above quoted terms is quite considered universal. Basic textbooks on engineering themes describing the characterization of step response of linear systems define a rise time and a fall time. Unfortunately there is not an unique and unambiguous definition for the corresponding DAC characteristics, as shown in [3]. In particular, concerning rise and fall time, an agreement is missing about the output amplitude levels to be used as start and stop trigger for the time measurement, as well as about the transition range to be considered [3].

In this paper the definitions of rise and fall time are derived from the transition duration definition in IEEE Std.181 [4], that is: “the difference between the two reference level instants of the same transition. Unless otherwise specified, the two reference levels are the 10% and 90% reference levels”. Such definition can be specialized by adding the transition amplitude and its direction. Therefore, according to [3] and [4], the rise time

will be a step-up transition duration of a specified amplitude. On the contrary, the fall time will be a step-down transition duration of a specified amplitude.

The uncertainty of the considered DAC time domain specifications and their relative confidence intervals is evaluated using the bootstrap method [5-8], according to the "Guide to the expression of uncertainty in measurement" [9]. In particular, the bootstrap method is adopted in order to obtain a suitable confidence interval estimation by using only a few measures, in order to make the proposed method feasible in an industrial test environment without making a priori hypotheses on the measurement distribution. In the next Sections, the bootstrap proposed method is first introduced, then, its experimental validation, carried out on commercial generation and acquisition equipment, is described. Finally, the achieved results, by comparing the proposed method and the traditional one, are presented and discussed.

## II. Proposed confidence interval estimation method

Bootstrapping is a statistical method for estimating properties of an estimator from an approximating distribution, such as the empirical distribution of the observed values. The method can be used for deriving robust estimates of standard errors and confidence intervals of a population parameter like mean, median, proportion, odds ratio, correlation coefficient or regression coefficient. It can be implemented by resampling several times the original measurement set. Each new sample is obtained by random sampling with replacement from the original measurement set.

Bootstrapping may also be used for constructing hypothesis tests and it can be applied when the population is affected by outliers, too. The basic idea behind bootstrapping consists in drawing inferences from the observed sample rather than making potentially unrealistic assumptions about the unknown population the sample has been taken from. Therefore, the observed sample is processed as if it was the whole population.

In fact, the bootstrap method is often used as a robust alternative to inference based on parametric assumptions when those assumptions are in doubt and when parametric inference is impossible or requires very complicated formulas for the calculation of standard errors.

Moreover, the bootstrap doesn't require a big sample size: approximately 5÷10 samples are enough [5-6]. This is an important property, from the manufactures' point of view, considering the increase in testing time and equipment cost due to the exponential growth in DAC internal complexity.

A technical standard including easy test procedures that can be executed in the industrial environment, within a reasonable amount of time, would be more easily adopted by manufacturers. For such a reason, the paper proposes the adoption of such method in estimating the confidence levels for DAC time domain dynamic parameters.

As written above, the basic idea of the bootstrap method is to generate a large number of independent bootstrap samples by random resampling the original observed values,  $W_1, \dots, W_n$ . The bootstrap samples  $W_1^*, \dots, W_n^*$ , are defined to be a random sample of size  $n$  drawn randomly with replacement from the original observed values, with elements taken zero, once or multiple times. Provided that  $n$  is big enough, for example about 1000, the 95% confidence interval of the mean estimator can be obtained using the 0.025 percentile,  $\overline{X}_{0.025}^*$ , and 0.975 percentile,  $\overline{X}_{0.975}^*$ , of the bootstrap sample arithmetic means as endpoints.

## III. Experimental validation

Modern digital storage scopes (DSOs) and digital phosphor scopes (DPOs) are popular and offer an excellent solution for performing rise and fall time measurements. These scopes offer real-time sampling rates of several GHz and are much less sensitive to overdrive than older analog scopes or traditional sampling scopes. Even with modern DSOs and DPOs, overdrive should still be checked by changing the scope sensitivity by a known factor and making sure that all portions of the waveform change proportionally. The sensitivity of the scope should be sufficient to measure desired error bands.

The method described in the previous Section has been validated by measuring rise time and fall time of square waveforms produced by different equipment including DAC output sections, i.e. arbitrary waveform generators and data acquisition boards. The measures have been carried out by automating the preset measurement functions of two digital oscilloscopes. The number of generating and digitizing equipment has been chosen to put in evidence the potential influence of instrument-related biases on the correct estimation of the confidence levels.

On each generator-digitizer couple, ten groups of ten rise time and fall time measures have been taken, thus achieving 100 values for each parameter and transferred on a PC where the calculations have been carried out. The aim of the whole validation phase is to compare the uncertainty analysis results obtained by the proposed bootstrap method, for which only 10 samples are required, with the Student's *t* method, based on 100 samples, in order to prove the validity of the proposed approach.

#### A. Experimental validation on a data acquisition board

The first validation test phase has been carried out by using a data acquisition board as generator. In particular, the parameter values of the DAC embedded in the acquisition board NI PCI MIO 16E1, installed in and controlled by a PC, have been measured by means of the oscilloscope Tektronix TDS 5104 and an oscilloscope LeCroy SDA 6000 by changing the characteristics of the generated signals. A square wave having an amplitude of 10V and a frequency of 1kHz, has been first generated by the DAC of the acquisition board, using a LabVIEW virtual instrument, at an update rate of 100kHz and acquired by means of the Tektronix oscilloscope (signal S1). Then, the rise and fall time have been measured by the oscilloscope at the sampling frequency of 5GSa/s. The second signal, generated with the same data acquisition board, a square wave with an amplitude of 150mV, a frequency of 1 kHz, and an update rate of 1 MHz, has been acquired by the LeCroy oscilloscope at the sampling frequency of 20GSa/s (signal S2).

The uncertainty intervals, with a confidence level of 95%, for the rise and the fall time measurement are shown in Table I and compared with the results achieved by applying the traditional estimation method relying on Student's *t* distribution.

Table I. DAC time response parameters: statistical analysis.

DAC parameter	Bootstrap Method			<i>t</i> -Student method		
	Mean	Standard deviation	Confidence interval @ 95%	Mean	Standard deviation	Confidence interval @ 95%
S1 rise time [ns]	715	2	711 – 719	715	6	714 – 716
S1 fall time [ns]	825	3	820 – 829	825	8	823 – 826
S2 rise time [ns]	1.38	0.01	1.37 – 1.39	1.38	0.02	1.38 – 1.39
S2 fall time [ns]	1.44	0.01	1.43 – 1.45	1.44	0.02	1.43 – 1.44

By analyzing the first results, it's possible to see that both the methods provide similar uncertainty intervals. The main differences between the measures corresponding to S1 and S2 are due to the very different slew rates set for the tests. The comparison between the confidence intervals for the same figure of merit on the same signal shows the effectiveness of the bootstrap method and its advantages: (i) no assumptions about the samples distribution, (ii) a reduced number of measurements required, (iii) easier and faster acquisition sessions than the inference-based methods, and (iv) therefore, more suitable for industrial applications.

#### B. Experimental validation on arbitrary waveform generators

The second experimental validation phase has been carried out by changing the generation equipment while keeping the same digitizer. Both the second test phase configurations use the oscilloscope LeCroy SDA 6000 to measure 10 values of both rise time and fall time by setting a sampling frequency of 20GS/s.

In the first test setup the DAC is embedded in an Agilent 33220A arbitrary waveform generator. The rise time and fall time have been measured, by means of the oscilloscope, on a square wave having an amplitude of 1V and a frequency of 20 MHz (Signal S3).

In the second test setup the DAC is embedded in a Tektronix AWG 420 arbitrary waveform generator. The square wave signal used for the test has been generated with an amplitude of 1V and a frequency of 10MHz (Signal S4).

Also in this second measurement campaign, ten groups of ten rise time and fall time measures have been carried out, thus achieving 100 values for each parameter. The achieved confidence intervals, with a confidence level of 95%, for the rise and the fall time measurement are shown in Table II along with the results achieved by applying the traditional estimation method.

Table II. DAC time response parameters from second measurement campaign: statistical analysis.

DAC parameter	<i>Bootstrap Method</i>			<i>t-Student method</i>		
	Mean	Standard deviation	Confidence interval @ 95%	Mean	Standard deviation	Confidence interval @ 95%
S3 rise time [ns]	9.25	0.11	9.17 – 9.33	9.25	0.13	9.22 – 9.27
S3 fall time [ns]	9.24	0.09	9.17 – 9.31	9.24	0.11	9.22 – 9.26
S4 rise time [ns]	2.20	0.01	2.18 – 2.22	2.20	0.03	2.20 – 2.21
S4 fall time [ns]	2.02	0.01	2.01 – 2.04	2.02	0.03	2.02 – 2.03

By analyzing the results shown above, it's possible to see again that the uncertainty intervals are almost the same in both of the two methods. In this way a further confirmation of the effectiveness of the bootstrap method has been achieved.

#### IV. Conclusions

The paper proposes a new methodology to evaluate the measurement uncertainty of time domain dynamic parameters, in particular rise time and fall time in DACs relying on a bootstrap technique. This approach allows to define confidence intervals from short acquisition records without needing hypotheses about the measurand distribution. Moreover, other advantages consist in a reduced number of measurements required, easier and faster than the inference-based methods and, therefore, more interesting from an industrial point of view. In order to validate the proposed approach, several measurement set-ups have been used, with different instrumentation and configurations.

From the assessment of the experimental results, it has been possible to see the effectiveness of the proposed bootstrap technique, compared a traditional uncertainty estimation method such as the Student's *t* method. Therefore, the proposed method can give a contribution, in terms of uncertainty assessment, to the IEEE Std.1658 concerning terminology and test methods for DACs.

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