XVIII IMEKO WORLD CONGRESS
Metrology for a Sustainable Development
September, 17 – 22, 2006, Rio de Janeiro, Brazil

A NEW VECTORIAL MODEL FOR THE ESTIMATION OF UNCERTAINTY IN NANO CO-ORDINATE MEASURING MACHINES

R. Füßl [1], *G. Jäger* [2], *R. Grünwald* [3], I. Schmidt[4]

[1] TU Ilmenau, Ilmenau, Germany, roland.fuessl@tu-ilmenau.de
[2] TU Ilmenau, Ilmenau, Germany, gerd.jaeger@tu-ilmenau.de
[3] TU Ilmenau, Ilmenau, Germany, gruenwald@tu-ilmenau.de
[4] TU Ilmenau, Ilmenau, Germany, ingomar.schmidt@tu-ilmenau.de

Abstract: Nano coordinate measuring machines (NCMMs) are technological devices enabling the positioning, touching and measuring of centimetre-sized objects with nanometre precision. When using such measuring machines the specification of measurement results requires the expression of uncertainty of measurement.
This paper describes a concept for the expression of three-dimensional uncertainty of NCMMs based on a vectorial metrological model. By means of a modular model approach submodels can be easily included in the metrological main model. Furthermore, cross-coupling effects arising between the measuring axes can be taken into account. The described model provides a basis for the expression of uncertainty according to the Guide to the Expression of Uncertainty in Measurement (GUM) or by means of the Monte-Carlo-Method. The results of the uncertainty analysis are shown for a special example of a vectorial model.

Keywords: Nanometrology, uncertainty of nano coordinate measuring machines, vectorial metrological model.

1. INTRODUCTION

Nano coordinate measuring machines are high-precision three-dimensional measuring systems with a resolution of less than 0.1 nm over entire ranges of 25 mm x 25 mm x5 mm and larger. Figure 1 shows such a device.

Those high-tech instruments consisting of precision 3D-guides, interferometers, a 3D-reference mirror and nano-probes, connected by a stable frame are the subject of research in nanometrology [1],[2],[3]. Their metrological accuracy can be shown by uncertainty estimation [4]. Most of the hitherto described uncertainty budgets for such machines were focused on the measurement of one-dimensional objects, such as step-height standards or pitch standards. For the validation of two- or three-dimensional measurements a three-dimensional uncertainty budget is necessary. This means that such a measurement is a result of two ore more touches by means of a nanoprobe. The uncertainty budget has to consider the errors of all touches. Furthermore, the correlation of the errors of the touches is to be taken into account. An error- or uncertainty budget of a three-dimensional measurement requires a three-dimensional metrological model of the nano coordinate measuring machine. The paper presents the new approach of such a model in vectorial form.

Fig. 1: Nano coordinate measuring machine (SIOS Company)

2. VECTORIAL METROLOGICAL MODEL

For measuring the distance between two positions, A and B on an object, it is necessary to touch the object twice in point A and in point B (Fig. 2). In the present case, the object to be measured is firmly attached to the mirror, with the laser beam of the interferometer virtually pointing to the tip of the probe. This measurement method is a precondition to avoiding Abbe errors. For each measuring point, a closed metrological chain exists:

$$x_{Aa} + x_{Ia} - x_{FRa} = 0 \quad (1)$$

$$x_{Bb} + x_{Ib} - x_{FRb} = 0 \quad (2)$$

with x_{Aa} - coordinate of point A at time a
x_{Bb} - coordinate of point B at time b
x_I - distance from the mirror surface to the interferometer centre
x_{FR} - length of the metrological frame

The distance between the two points A and B can be calculated by the difference of the chains (1) and (2):

$$x_M = x_{Bb} - x_{Aa} = x_{Ia} - x_{Ib} - x_{FRa} + x_{FRb} \quad (3)$$

The difference $x_{Ia} - x_{Ib}$ represents the interferometrically measured displacement of the object including the mirror. If the frame does not change its length between the two probe touches, its value has no influence on the measuring result. However, in reality the frame is not invariant.

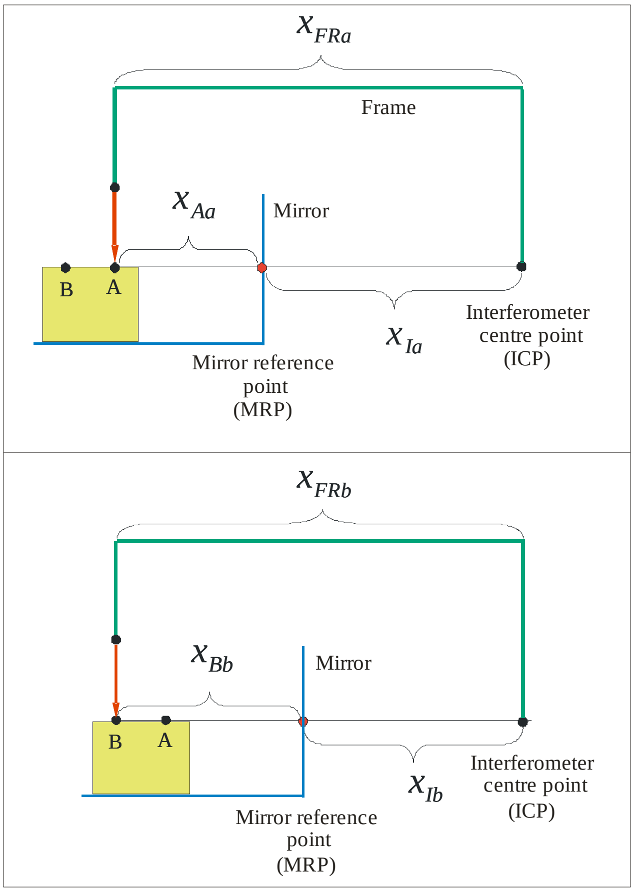

Fig. 2: Metrological chains for two measuring points

If we consider the NCMM, the characterization of the metrological chains for the three measuring axes requires a vectorial model as shown in Fig. 3. In the same ways as illustrated in Fig. 2, two vector chains can be created for touching point A and B.

The connected vectors in each vector chain represent metrological submodels (e.g. the probe model, the frame model, the interferometer model, the 3D-reference mirror model, the Abbe-error model, etc.).

The main vectorial model is created by the difference of the two vector chains:

$$\vec{r}_{Bb} - \vec{r}_{Aa} = \vec{r}_{Pb} - \vec{r}_{Pa} + \sum_i \vec{r}_{FRai} - \sum_i \vec{r}_{FRbi} - \vec{r}_{IRa} + \vec{r}_{IRb}$$
$$- \vec{r}_{Ima} + \vec{r}_{Imb} - \vec{r}_{cosa} + \vec{r}_{cosb} - \vec{r}_{Abbe\text{-}a} + \vec{r}_{Abbe\text{-}b} - \vec{r}_{MIea} + \vec{r}_{MIeb} \quad (4)$$

The distance vector between points A and B at time b results from the following equation:

$$\vec{r}_M = \vec{r}_{Bb} - \vec{r}_{Aa} - \vec{r}_{ab} \quad (5)$$

The vector $\vec{r}_{ab}$ considers the possible shift of the distance vector between time a and time b.

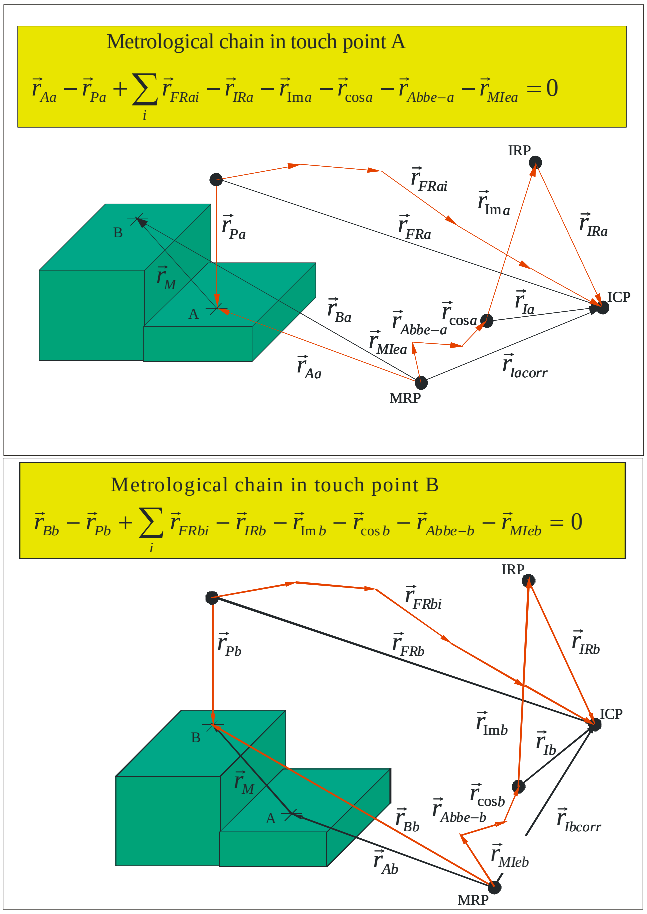

Fig. 3: Metrological vectorial chains

The benefit of the vectorial consideration is the modularity and the possibility of easily expanding the model. Furthermore, cross-coupling effects arising between separate coordinates can also be effectively included in the uncertainty budget.

3. SUBMODELS

Abbe error submodel

The vectorial Abbe error submodel can be described by the Cartesian product of the tilt angle error vector of the guides, and the misalignment vector considering the probe tip is not exactly placed in the virtual point of intersection of the three laser beams:

$$\vec{r}_{Abbe} = \vec{\varphi}_t \times \vec{r}_{mis} \quad (6)$$

This error vector is an example for cross coupling between the coordinates.

Cosine error submodel

The cosine error vector includes the geometrical cosine error of misalignment occurring in the case of angle tilting of the guides and the wave front error at the diaphragm of the interferometer:

$$\vec{r}_{\cos} = \begin{pmatrix} \frac{x_{PD} + x_{MD}}{2} \cdot (\varphi_y{}^2 + \varphi_z{}^2) \\ \frac{y_{PD} + y_{MD}}{2} \cdot (\varphi_x{}^2 + \varphi_z{}^2) \\ \frac{z_{PD} + z_{MD}}{2} \cdot (\varphi_x{}^2 + \varphi_y{}^2) \end{pmatrix} \quad (7)$$

In Eq. (7), the index *PD* of *x*, *y*, *z* denotes the distance between the probe tip and the diaphragm of the interferometer, the index *MD* denotes the distance between the mirror and the diaphragm, and φ describes the tilt of the guides.

Mirror error submodel

The 3D-mirror of the NCMM does not have ideal surfaces. Its topography can be measured by means of a phase shift interferometer. The data set obtained is associated with measurement uncertainty.

Figure 4 shows the consideration of mirror errors.

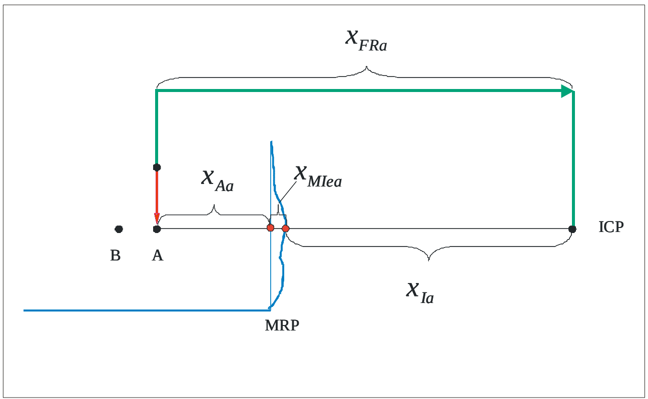

Fig. 4: Mirror error

In the NCMM a mirror error vector $\vec{r}_{MIe}$ is assigned to each point of the measuring space.

Frame model

The frame of an NCMM is the carrier and the fixed connection of the interferometer and the probe. It consists of several parts made of various materials. Needless to say that the goal is to realise a stable frame without any length shift due to environmental, aging or other effects. However, in reality especially temperature influences affect the NCMM. The length shift of the frame parts can be described by the following terms:

$$\vec{r}_{FRi} = \vec{r}_{FR20i}(1 + \alpha_{thi} \cdot (\vartheta_i - 20°C)) \quad (8)$$

$$\vec{r}_{FR} = \sum_i \vec{r}_{FRi} \quad (9)$$

Interferometer model

The translational displacement of the object and the mirror while touching point A and B is measured by means of three interferometers. The interferometer measures the difference between its measurement path and its reference path. In Fig. 5, the measurement path is the distance between the reflecting surface of the mirror and the interferometer centre point ICP. The reference path is formed by the distance between the interferometer centre point and the interferometer reference point IRP situated on the surface of the reference mirror.

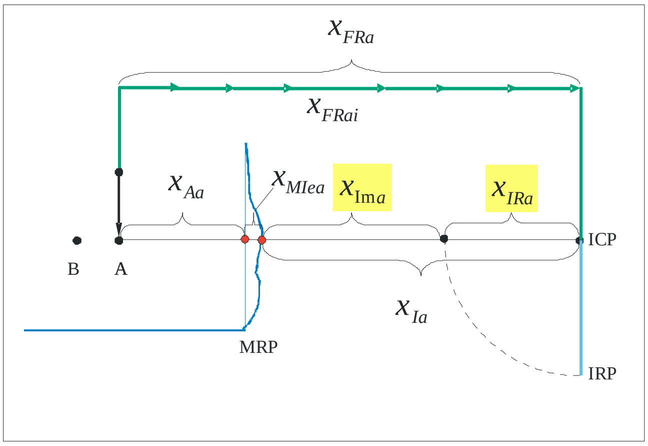

Fig. 5: Interferometer model

The interferometer output x_{Im} is given by:

$$x_{\mathrm{Im}} = x_I - x_{IR} \quad (10)$$

In the 3D-space equation (10) is represented by virtual vectors:

$$\vec{r}_{\mathrm{Im}} = \vec{r}_I - \vec{r}_{IR} \quad (11)$$

The interferometer output vectors themselves, $\vec{r}_{\mathrm{Im}\,a}$ and $\vec{r}_{\mathrm{Im}\,b}$, for the touch points A and B are formed by subvectors embodying the laser wavelength, the index of refraction, the Edlen formula including environmental values, the counted ordinal number and the interpolation.

Thus, all subvectors in the main vectorial model (4) are described.

4. COORDINATE TRANSFORMATION

The norm of the distance vector (5) between points A and B is calculated by:

$$|\vec{r}_M| = \sqrt{x_M^2 + y_M^2 + z_M^2} \quad (12)$$

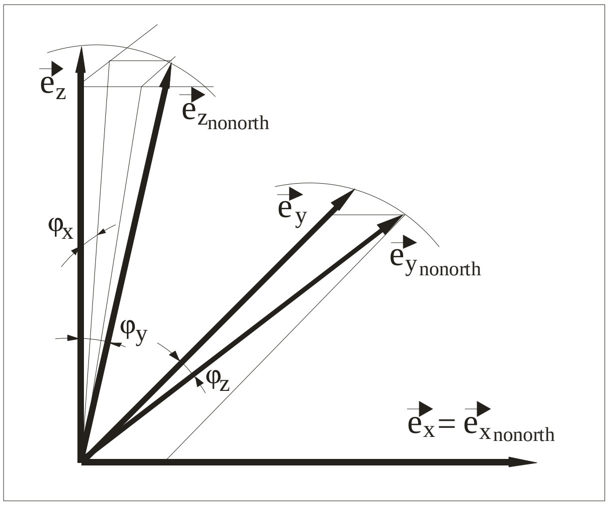

Fig. 6: Coordinate systems

Equation (12) is valid only for an orthogonal coordinate system. In fact, due to angle errors of the 3D-mirror and adjustment errors of the interferometer beams, we have to deal with a non-orthogonal system (Fig.6).

The angles $\varphi_x, \varphi_y, \varphi_z$ represent the deviations from orthogonality. This means we have to transform the non-orthogonal coordinates into orthogonal ones:

$$\begin{bmatrix} x_{Mo} \\ y_{Mo} \\ z_{Mo} \end{bmatrix} = A * \begin{bmatrix} x_{Mno} \\ y_{Mno} \\ z_{Mno} \end{bmatrix} \qquad (13)$$

with

$$A = \begin{bmatrix} 1 & -\sin\varphi_z & \frac{\tan\varphi_y}{\sqrt{(\tan\varphi_y)^2 + (\tan\varphi_x)^2 + 1}} \\ 0 & \cos\varphi_z & \frac{-\tan\varphi_x}{\sqrt{(\tan\varphi_y)^2 + (\tan\varphi_x)^2 + 1}} \\ 0 & 0 & \frac{1}{\sqrt{(\tan\varphi_y)^2 + (\tan\varphi_x)^2 + 1}} \end{bmatrix} \qquad (14)$$

Equations (13) and (14) represent the coordinate transformation model. The norm of the distance vector can now be calculated by:

$$\left|\vec{r}_M\right| = \sqrt{x_{Mo}^2 + y_{Mo}^2 + z_{Mo}^2} \qquad (15)$$

5. CALCULATION OF MEASUREMENT UNCERTAINTY

The combined uncertainty of the distance vector $\vec{r}_M$ can be calculated according to the GUM

$$\vec{u}_c(\vec{r}_M) = \sqrt{\sum_i \left(\frac{\partial f}{\partial \vec{r}_i}\right)^2 \cdot u^2(\vec{r}_i) + 2\sum_i \sum_j \frac{\partial f}{\partial \vec{r}_i}\frac{\partial f}{\partial \vec{r}_j} \cdot u(\vec{r}_i, \vec{r}_j)} \qquad (16)$$

with the correlation between the input values having to be considered. The differentiated model function in Eq. (16) is established by the combination of the model equations (4) to (11) and the coordinate transformation (13) and (14). The uncertainty of the norm of the distance vector is given by

$$u_c(\left|\vec{r}_M\right|) = \sqrt{\frac{x_M^2 \cdot u^2(x_M) + y_M^2 \cdot u^2(y_M) + z_M^2 \cdot u^2(z_M)}{x_M^2 + y_M^2 + z_M^2}} \qquad (17).$$

The uncertainties $u(x_M)$, $u(y_M)$ and $u(z_M)$ in Eq. (17) are derived from the uncertainty vector (16):

$$\vec{u}_c(\vec{r}_M) = \begin{bmatrix} u(x_M) \\ u(y_M) \\ u(z_M) \end{bmatrix} \qquad (18)$$

For an NCMM with a measuring range of 200 x 200 x 5 mm^3 and a distance vector directed from the centre to the coordinates {100; 100; 2,5} the uncertainty budget was simulated [5]. Under vacuum conditions an expanded uncertainty (coverage factor 2) of less than 35 nm was achieved. In the case of the assumed small input uncertainty values the calculation by means of the Monte-Carlo-Method produced the same results. The uncertainty budget shows that especially the mirror errors including its deviation from orthogonality which is not exactly known exert a major influence on the 3D-uncertainty.

6. CONCLUSION

The presented method of estimating the uncertainty of nano coordinate measuring machines permits an effective way of analysing the metrological efficiency of such precision instruments. The advantage of the method is its modularity for including submodels as well as the possibility of taking the coupling effects between the coordinates into account in an easy way.

It was shown how a concrete 3D- measurement uncertainty of a nano coordinate measuring machine can be calculated. Based on the vectorial model, the calculation was accomplished according to the “Guide to the Expression of Uncertainty in Measurement” [4]. Another possibility is to calculate measurement uncertainty using the Monte Carlo method, which offers the advantage of taking nonlinearities in the metrological model into account.

REFERENCES

[1] A. Weckenmann: Micro and Nano Coordinate Measuring Technique. 50th International Scientific Colloquium, invited paper, Ilmenau, Germany

[2] Jäger, G.; Grünwald, R.; Manske, E.; Hausotte, T.; Füßl, R.: Nanopositioning and Nanomeasuring Machine: Operation-Measured Results; Nanotechnology and Precision Engineering, Vol. 2, No. 2, Juni 2004

[3] Jäger, G.; Manske, E.; Hausotte, T.; Füßl, R.; Grünwald, R.; Büchner, H.-J.: Optical Fibre Coupled Miniature Interferometers Designed for Application in Micro and Nano Devices. Proceedings of 19. Annual Meeting of the American Society of Precision Engineering, October 24-29, 2004. Orlando, Florida, pp. 145-148

[4] Guide to the Expression of Uncertainty in Measurement. BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, IUPAP, OIML, 1993

[5] Kreutzer Ph.: Monte-Carlo-Simulation for metrological models of NCMM; diploma thesis, TU Ilmenau, 2006