

JOULE INSTEAD OF KELVIN IN A SET OF BASE UNITS

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Abstract: The joule is proposed to replace the kelvin in the set of SI base units. Arguments in favour of such replacement are presented, including improved standard accuracy and unit system coherence. The joule is also proposed to be added to the quantum metrological triangle (which couples the volt, the ampere and the hertz) to transform the latter into a system called quantum metrological pyramid, that would couple four units rather than three, and allow comparison of the respective standards.

Keywords: International System of Units, base units, quantum standards

1. INTRODUCTION

Though it seems obvious today, the arrangement of different units of measure into a system was a very long process. The first step in the development of the presently accepted system of units was the deposition of two standards, representing the metre and the kilogram, in the Archives of the Republic in Paris on 22 June 1799. The assumed metre standard represented 10^{-7} or one ten-millionth of the length of the section of the meridian passing through Paris from the North Pole to the equator. A kilogram was assumed to be the mass of 10^{-3} m^3 of pure water in its state of the greatest density (at 4°C) [1].

In 1832 Carl Gauss, in his dissertation *Die erdmagnetische Kraft auf ein absolutes Maß zurückgeführt*, proposed a coherent system of units comprising three units of measure for three respective quantities: length, mass, and time. The second, defined as a part of a solar day, was the time unit assumed in Gauss's metric system. The millimetre was chosen for the unit of length, and the gram for the unit of mass. The subsequent addition of units of electrical quantities resulted in an extended system proposed by Gauss and Weber. The fundamental requirements for a coherent system of units composed of base units and derived units were defined by James Clarke Maxwell and William Thomson through the British Association for the Advancement of Science (BAAS). The role of base units in a system of units is fundamental, as the accuracy of their representation affects that of the representation of the derived units. The coherent CGS system of units, developed according to the above-mentioned requirements and comprising three base units, was introduced by BAAS in 1874. The base units in this system were the centimetre, the

gram and the second, for three mechanical quantities: length, mass and time, respectively.

The beginning of an international measurement system was marked by the Metre Convention, signed by 17 countries in 1875, and sanctioning a system of units with metre, kilogram and second assumed as base units. Following the French example, the standards representing the metre and the kilogram were made of an alloy of 90 per cent platinum with 10 per cent iridium. Thirty prototypes of the meter were made in years 1878-1889; however, tests and standard comparisons required the temperature scale to be defined as well. The standards were kept at controlled temperature, as its effect on the standard parameters was already known. Let us note that the temperature was then regarded as a merely auxiliary physical quantity. The physical standards of the metre and the kilogram were approved in 1889 by the 1st General Conference of Weights and Measures (CGPM), along with the adoption of the so-called normal hydrogen scale, which is the centigrade scale of the constant-volume hydrogen thermometer, having for fixed points the temperature of pure melting ice (0°C) and that of the vapour of boiling distilled water (100°C).

The system of units developed further into the MKSA system, recommended by the International Committee for Weights and Measures (CIPM) and approved in 1946, with the ampere (the unit of electric current) added to the metre, the kilogram and the second, and increasing the number of base units to four. A further extension of the set of base units by the kelvin (the unit of thermodynamic temperature) and the candela (the unit of luminous intensity), increasing the number of base units to six, resulted in a system of units formally accepted by the 10th CGPM in 1954.

The system of units in force today is the International System of Units (SI) accepted by the 11th CGPM in 1960. At its starting point, the system comprised six base units (the set approved in 1954), two auxiliary units (the radian and the steradian), and derived units. The number of SI base units was increased to seven by adding the mole – the unit for amount of substance – at the 14th CGPM in 1971. From that time on, the SI base units include the metre, the kilogram, the second, the ampere, the kelvin, the candela, and the mole.

Let us stress once again the fundamental role of the base units in a system of units. All other quantities are called

derived quantities, defined in terms of the base quantities by the quantity relations. The corresponding derived units are then defined as products of powers of the base units, using the same relations as those used to define the derived quantities in terms of the base quantities. All the derived units for geometric, mechanical, electrical, thermal, magnetic, light and acoustic quantities, as well as those for ionizing radiation, can be defined by means of base units only. Obviously, metrological institutions strive to minimize the uncertainty of practical realizations of the base units, which always involves substantial costs. Among the seven base units of the SI only three are independent: the kilogram, the second and the kelvin. The four remaining base units – the metre, the ampere, the candela and the mole – are derived from the kilogram and the second (Fig. 1).

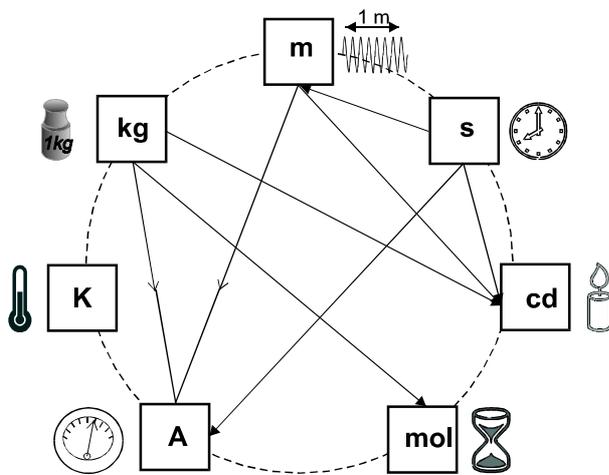


Fig. 1. The SI base units.

The derived units are described as coherent if no numerical factors are introduced in their defining relations in terms of base units, even though there may often be numerical factors in the corresponding quantity relations. It is an important property of the SI that there is only one coherent derived unit for each quantity. Coherence is an important advantage of a system of units.

In a system of units, a unit for different physical quantity can be expressed through units for other quantities covered by the system. For example, the unit for electric current, the ampere, can be found through measurements of force and length. The only SI base unit unlinked to other SI base units is kelvin. The corresponding quantity, temperature, is quite exceptional as well.

2. TEMPERATURE

According to James Clark Maxwell, “temperature of a body is its thermal state, regarded as a measure of its ability to transfer heat to other bodies”. Heat (energy) can be transferred only from a body of higher temperature to a body of lower temperature, and never vice versa. Terry J. Quinn begins his monograph *Temperature* [2] with the following definition: “Temperature is a quantity which takes the same value in two systems that are brought into thermal contact

with one another and allowed to come to thermal equilibrium”. Note the necessity of bringing the bodies (or systems) into thermal contact. This can be realized with or without physical contact between the bodies; thermal contact without physical one is possible through radiation. To meet the conditions of the temperature definitions quoted above, in the case of contact temperature measurement, the body and the sensor should be in the state of thermodynamic equilibrium. In non-contact temperature measurements, the temperature of a body is determined through measuring the density of thermal radiation emitted by the body.

In measurements of the temperature of a body or a system, beside the assumed thermodynamic equilibrium between the body and the thermometer sensor, also the parts of the body (system) are assumed to be in thermodynamic equilibrium. The latter assumption, however, is not satisfied in measurements of extreme temperatures, such as those of plasma. The reason is the difference between the electron energy and the ion energy. Artificial notions of “electron temperature” and “ion temperature” are introduced to overcome this problem. In ultra-low temperatures, the energy of electrons is differentiated from that of atomic nuclei, leading to a shocking result: negative absolute temperature. Negative absolute temperature was found to be reached by electrons in silver atoms in experiments carried out in Espoo, Finland [3].

As suggested by the above examples, the use of the notion of temperature should be limited to macroscopic objects with non-extreme internal energy. The temperature of a body is a measure of its internal energy. Richard P. Feynman explicitly writes that to describe the state of a gas, it would be best to name “temperature” the average kinetic energy of the gas [4]. Though temperature is an important physical quantity, it is only a parameter related to one form of energy. Energy itself, along with work and heat, is a much wider notion.

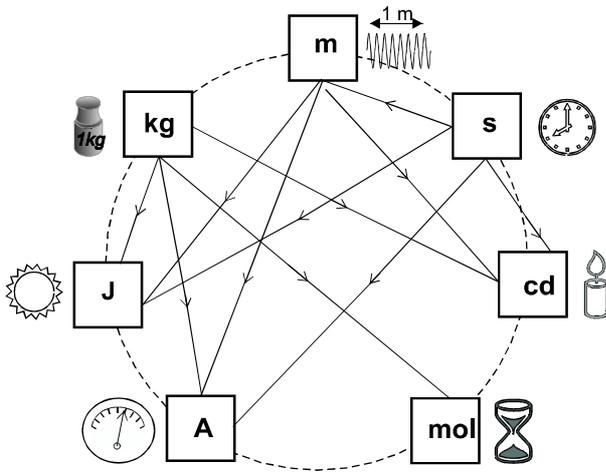
3. THE JOULE AS REPLACEMENT OF THE KELVIN

I propose to replace the kelvin, the unit for temperature, with the joule, the unit for energy (work, heat), in the set of base units of the SI. The kelvin would consequently become a derived unit. The proposed set of base units, and the links between them, are shown in Fig. 2.

The following arguments have been put forward to support the idea of replacing the kelvin with the joule in the set of SI base units:

1. The set of base units should comprise units for physical quantities which are the most important to people. Accordingly, the SI base units include the metre, which is the unit for length, and the kilogram, a unit for mass, used e.g. in trade. As energy is also a common object of trade on a large scale, its unit, the joule, and the accuracy of its standard are of great importance for commercial and economic activity.
2. Energy is perhaps the most universal of all physical quantities in nature. One of the formulations of the first

law of thermodynamics is: “The total quantity of energy in the universe remains constant”.



3. Fig. 2. The proposed set of SI base units, with the joule instead of the kelvin.

4. As a derived unit of the SI, the joule is expressed as a function of the same base units as the ampere:

$$J = N \times m = m^2 \times kg \times s^{-2}.$$

The standard of the joule can be realized more accurately than that of the kelvin, by means of the quantum standards of voltage and electrical resistance, and the time unit standard.

$$J = V^2 \times s \times \Omega^{-1}.$$

The standard uncertainties of these units are: $\delta = 10^{-9}$ for the volt, $\delta = 10^{-10}$ for the ohm and $\delta = 3 \times 10^{-15}$ for the second, while the standard uncertainty of the best practical realization of the thermodynamic temperature unit, the kelvin, is as high as 3×10^{-7} [5].

5. Energy changes affecting the sensor must usually be taken into account in estimating the measurement resolution limits. Energy is one of the four physical quantities which figure in Heisenberg’s inequalities describing the quantum limits of measurement resolution, according to the uncertainty principle:

$$\begin{aligned} \Delta x \times \Delta p &\leq h, \\ \Delta E \times \Delta t &\leq h. \end{aligned} \quad (1)$$

This is another argument for regarding energy as a more universal physical quantity than temperature.

6. Another advantage of energy over temperature is the additive character of the former. Being an additive quantity, energy can be measured by a series of partial measurements. Temperature is not an additive quantity.

7. Different forms of energy: mechanical energy or work, thermal energy, electric energy and nuclear energy, allow to compare standards of mechanical, thermal and electrical quantities as well as mass standards to one another, and to the standard of energy. With the unit for energy in the set of base units, the system of units will

become more coherent. Here are some of the different forms of energy;

– mechanical work

$$dW = d\vec{F} d\vec{l}, \quad (2)$$

where, F is the force acting on the body, and l is displacement (path);

– kinetic energy

$$E = m v^2 / 2, \quad (2)$$

where, m denoting the mass of the body, and v its velocity;

– potential mechanical energy U in a field of force

$$\vec{F} = -\nabla U, \quad (3)$$

where, ∇U , is the gradient of potential mechanical energy;

– mass-energy equivalence

$$E = m c^2, \quad (4)$$

where, c denoting the speed of light in vacuum;

– electric energy

$$E = V I t, \quad (5)$$

where, V is the voltage, I the electric current, and t denotes time;

– heat

$$Q = c_h m \Delta T, \quad (6)$$

where c_h is the specific heat, m is the mass, and ΔT is the temperature difference;

– thermal energy

$$E = k_B T, \quad (7)$$

where, k_B denoting the Boltzmann constant, and T being the absolute temperature;

– a quantum of electromagnetic radiation energy

$$E = h \nu, \quad (8)$$

where, h is Planck’s constant, and ν denotes the radiation frequency.

However, beside its many advantages, introducing the joule into the set of SI base units would involve some problems as well. First, many legal acts and standards concerning the SI system would need to be revised. Also, standardization of the energy unit with the best possible uncertainty and quantum resolution would involve the development of appropriate experimental setup. Moreover, energy measurement has a disadvantage of being based on energy conversion, which occurs with efficiency less than 100%, resulting in measurement irreproducibility.

4. BASE UNIT STANDARDS DERIVED FROM FUNDAMENTAL PHYSICAL CONSTANTS

Being realized by means of “artefacts”, classical standards of units are conditioned on specific material properties. For example, the Weston cell, used for the

standard of electromotive force, consists of two electrodes, one mercuric and the other mercuric-cadmium, with a cadmium sulphate (CdSO_4) solution as the electrolyte. The electromotive force generated by the Weston cell depends on the technology of its realization (the cell can be saturated or non-saturated), as well as on the temperature, the discharge ratio, mechanic shocks and other factors. In general, the quality of classical standards strongly depends on the component materials used, the technology used for the production of the standard (e.g. the type of heat treatment of the metal alloys), and on the temperature. The standards used require periodic comparisons to those deposited in other metrological laboratories. The uncertainty of reproduction of the volt and the ohm is increased by the effect of transport on the parameters of the classical standard. For these reasons, there is a tendency to replace classical standards with their quantum equivalents, as the accuracy of the latter is affected by neither the material properties nor the environmental conditions such as temperature oscillations or transport shocks.

Realizations of standards dependent only on fundamental physical constants have been proposed for many years. Already in 1870 J. Clark Maxwell in Cambridge anticipated the use of atomic properties for the realization of natural standards of units [5]. Richard Feynman observed that in principle, only two physical constants are used in quantum electrodynamics, and the majority of the other constants should be deducible from those two [4]. Both of those constants are parameters of the electron: its electric charge, e , and its rest mass, m_e . Quantum standards based on the former constant, $e = 1.602 \times 10^{-19}$ C, already exist for units of electrical quantities: voltage and electric resistance. Attempts are made to develop a quantum standard of electric current on the basis of single electron tunnelling. A hypothetical development of the system of units based on fundamental physical constants was presented by T. Quinn [6].

Three quantum standards of electrical quantities (voltage, resistance and current) can be used for the construction of a compact system that would allow comparison between the measured quantities, thanks to their interrelations. The so-called quantum metrological triangle, corresponding to such system and proposed by K. Likharev and A. Zorin in 1985 [7], is shown in Fig. 3. The vertices of the quantum metrological triangle are physical quantities: voltage (V), electric current (I), and frequency (f), coupled by three quantum effects, which represent the sides of the triangle. These quantum effects include Josephson effect, single electron tunnelling (SET) and quantum Hall effect (QHE). Quantization of electric resistance (conductance) in QHE occurs with steps $R_K = h/e^2$. Note that according to Landauer theory of electric conductance [8], conductance quantization takes place also in a conductor sample of nanometre dimensions, depending on its width and thickness. In this quantization effect, quite different from QHE, the conductance step is $G_0 = 2e^2/h = 2/R_K$.

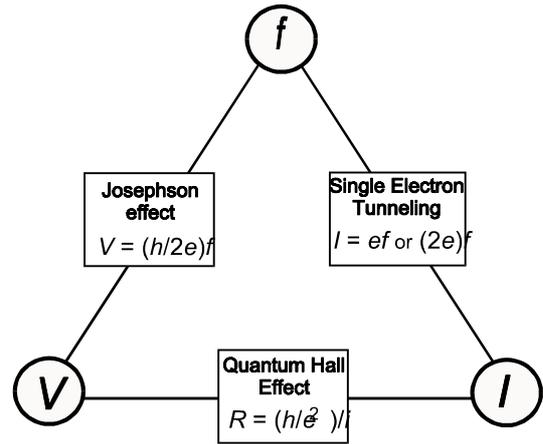


Fig. 3. Quantum metrological triangle for realization of electrical standards.

The experimental setup for quantum metrological triangle standardization is described by Piquemal and Geneves [9]. The main idea of their experiment was to measure the coherence of values of fundamental physical constants occurring in the Josephson effect, QHE and SET with uncertainty better than 10^{-8} .

Through adding the standard of energy, the quantum metrological triangle can be transformed into the quantum metrological pyramid, shown in Fig. 4.

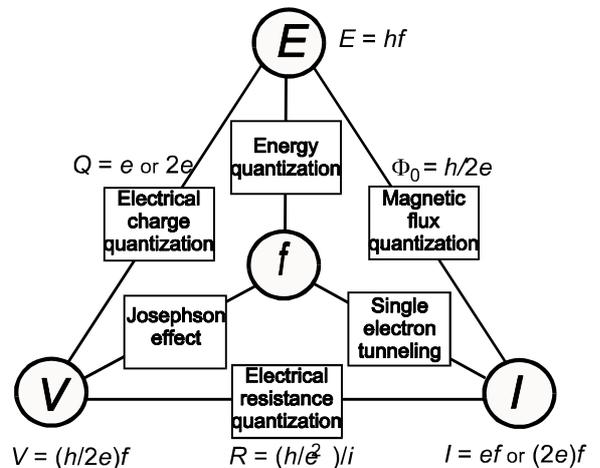


Fig. 4. Quantum metrological pyramid for realization of electrical standards.

Three quantization effects add to the above-mentioned four in the quantum metrological pyramid:

- quantization of energy transported by electromagnetic radiation with energy step $E = hf$, an effect anticipated by Planck;
- magnetic flux quantization with steps $\Phi_0 = h/2e$, detectable by SQUID;
- electric charge quantization, observed in many physical phenomena, e.g. in single electron tunnelling.

Note that each of the three electrical quantum standards (of V , I and E) is realized by means of standard frequency f .

All the quantum effects in the system involve only two fundamental physical constants: e or h .

The links between the fundamental physical and atomic constants and the proposed SI base units (including the energy unit) are shown in Fig. 5.

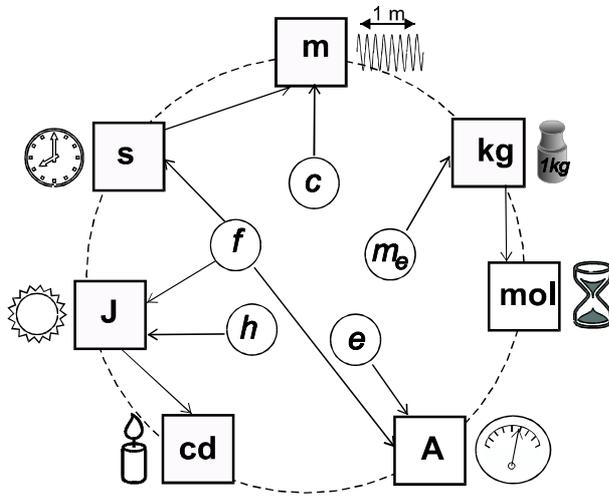


Fig. 5. The proposed set of base units and their connections with fundamental constants.

The base units in this set depend only on four fundamental constants (c , h , e , m_e) and the standard frequency, f . Let us stress that a quantum standard of energy can be realized much more accurately than a standard of temperature using the Boltzmann constant, k_B .

5. CONCLUSIONS

This paper proposes to replace the base unit for temperature, the kelvin, with the energy unit, the joule. The joule is also proposed to be added to the quantum metrological triangle (which couples the volt, the ampere and the hertz) to transform the latter into a system called quantum metrological pyramid, that would couple four units rather than three, and allow comparison of the respective standards.

As the evolution of the system of units into a system of quantum standards based on fundamental constants seems inevitable, incorporating the joule into the set of base units in this new system could be a worthwhile effort.

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