CONFORMITY ASSESSMENT USING MONTE CARLO METHODS

Jos G.M. van der Grinten[1] and Alex M. van der Spek[2]

[1] *NMi EuroLoop, Petroleumweg 36, 3196 KD Rotterdam, The Netherlands, jos.vandergrinten@nmi-euroloop.nl*
[2] *ZDoor B.V., Sleedoornlaan 23, 3053 ZN Rotterdam, The Netherlands, zdoor@xs4all.nl*

Abstract
Conformity assessment is the activity to determine whether specified requirements relating to a product, process, system, person or body are fulfilled. Often measurements are used to show that the measurand is within (legal) tolerances. Currently analytical methods are available to test whether tolerances are met with a preset level of confidence, e.g. 95%. The test requires the availability of the overall measurement uncertainty and the statistical distribution of the measurand. In absence of better information this distribution is assumed to be Gaussian.
The new point in this paper is that Monte Carlo methods can be applied directly to perform the conformity assessment. The reason is that the Monte Carlo process generates the cumulative distribution, whereby the (legal) tolerances can be compared directly. The advantage of this process is that the type of distribution does not need to be known and the (worst case) assumption of the distribution being Gaussian can be avoided. Consequently, for a Monte Carlo method the difference between tolerances and acceptance criteria is slightly smaller than for analytical methods.
A test of the Monte Carlo method applied to a calibration of a high-pressure gasmeter meeting MID tolerances demonstrates the applicability of the method in practice.

Introduction
Conformity assessment is the activity to determine whether specified requirements relating to a product, process, system, person or body are fulfilled. Often measurements are used to show that the measurand falls within (legal) tolerances. Currently analytical methods [4] – [8] are available to test whether tolerances are met with a preset level of confidence, e.g. 95%. The test requires the availability of the overall measurement uncertainty and the associated probability density function.
The overall uncertainty can be evaluated by analytical methods [1], [2] or Monte Carlo simulation [3]. The probability density is often assumed to be Gaussian. If no exact knowledge of the statistical distribution of the measurand is available, the choice for a Gaussian distribution is the worst case approximation [4], [5].
Monte Carlo methods for uncertainty evaluation prove to be especially useful in cases where a non-linear relationship exists between input quantities and the measurand, where the uncertainty is large compared to the value of the quantity, or where input quantities can only be evaluated numerically by software code.

During verification a measuring instrument is approved or rejected on the basis of legal tolerance limits. The confidence level of these metrological decisions is affected by the uncertainty of the measurand. The statistical methods in these publications [4] - [8] have in common that substantial knowledge of statistical testing is required to perform the tests.
At this point the new Monte Carlo Method offers an interesting opportunity. At the same time the uncertainty analysis is made, the statistical testing can be performed. Only a few lines of software code need to be added to the implementation of the Monte Carlo Method.
NMi has developed a Monte Carlo software tool in which this additional feature was implemented. This tool was used to apply the Monte Carlo method on the example of the verification of a high-pressure gasmeter that has to meet MID tolerances [9], [10].

Calibration model
The calibration of volume flow gasmeters with high-pressure natural gas is based on the integral formulation of the mass conservation law applied to a fixed volume V with surface F. The increase in mass per unit time in V equals the mass flow through the surface F

$$\frac{\partial}{\partial t}\iiint_V \rho\, dV = -\iint_F \rho\,(\mathbf{v}\cdot\mathbf{n})\, dF \qquad (1)$$

in which $\mathbf{v}$ is the fluid velocity vector and $\mathbf{n}$ a normal vector of length 1 perpendicular to the surface F pointing outside of V. $(\mathbf{v}\cdot\mathbf{n})$ is the dot product of $\mathbf{v}$ and $\mathbf{n}$, which means that a fluid flow entering V gets a negative sign and a fluid flow leaving V gets a positive sign.
For the calibration V is the volume between the master meters and the meter under test (MuT). The fluid enters V via the master meters and leaves V via the MuT. Both masters and MuT measure gas in volume units. The volume V having thick steel walls is assumed to be constant in time. As the fluid flow through closed conduits the fluid velocity times the cross section is the volume flow rate Q. With n parallel master meters equation (1) can now be written as

$$V\frac{\partial \rho_V}{\partial t} = -\rho_2 Q_2 + \sum_{i=1}^{n} \rho_{1i} Q_{1i} \qquad (2)$$

where the index 1 refers to the cross section at the master meters and 2 refers to the MuT.
The density ρ is a function of pressure p temperature t and the gas composition x:

$$\rho = \rho(p, t, x) \qquad (3)$$

The equation of state used to compute ρ from p, t and the gas composition x is in this example the AGA-8 model[11], which can handle 21 gas components. The AGA-8 algorithm will be evaluated numerically.

The flow rate Q is computed from an integer number of pulses N collected during a interval τ:

$$Q_{si} = \frac{N_{si}}{I_{si}\,\tau_{si}}, \quad Q_m = \frac{N_m}{I_m\,\tau_m} \tag{4}$$

in which I is the impulse factor of the meter [pulse / m^3].
The objective of the calibration is to determine the deviation e_m of the meter under test as a function of the flowrate Q_m indicated by the MuT

$$e_m = \frac{Q_m}{Q_2} - 1 \tag{5}$$

In the calibration process corrections are applied for all known deviations. For the master meter i the deviation e_{si} is depending on calibration pressure and flowrate Q_{si} indicated by master. The correction for this deviation leads to

$$Q_{1i} = \frac{Q_{si}}{1 + e_{si}} \tag{6}$$

The mass accumulated in V during calibration time interval τ_V is determined from the volume V and the densities $\rho_{start} = \rho(p_{start}, t_{start}, x)$ at the start and $\rho_{end} = \rho(p_{end}, t_{end}, x$ at the end of the calibration. The gas composition is assumed to be constant during the calibration time interval τ_V.
Successive substitution of equations (3-6) in equation (2) leads to

$$V\,\frac{\rho_{end} - \rho_{start}}{\tau_V} = -\frac{\rho_2\,Q_m}{1 + e_m} + \sum_{i=1}^{n} \frac{\rho_{1i}\,Q_{si}}{1 + e_{si}} \tag{7}$$

from which the deviation e_m can be solved:

$$e_m = \frac{\rho_2\,{N_m}/{I_m\,\tau_m}}{\frac{(\rho_{start} - \rho_{end})\,V}{\tau_V} + \sum_{i=1}^{n} \frac{\rho_{1i}\,N_{si}}{I_{si}\tau_{si}(1 + e_{si})}} - 1 \tag{8}$$

Statistical testing

All statistical tests start with the formulation of a null hypothesis H_0, which in our example is that the gasmeter will meet the MID tolerances. If the measurand with its associated uncertainty is well within tolerances, H_0 will be accepted. This corresponds to the green points in the figure 1 below. If the measurand equals the tolerance, there is an equal probability that the measurand will and will not meet the tolerances. In that case a decision cannot be made. This corresponds to the blue points.
If the null hypothesis H_0 cannot be accepted it will be possible to test and alternative hypothesis H_1, which states that the measurand is outside the tolerances. If the red points are observed, H_1 will be accepted. If the blue points are found, hypothesis H_1 has to be rejected.
Figure 1 below shows that there is a region (blue points) in which both H_0 and H_1 cannot be accepted.
H_0 is typical for conformity assessment where we need to show that meters are conforming. The alternative hypothesis H_1 is typical to inspections where the inspector looks for non-conforming products or the police enforcing the speed limit. A more elaborate discussion on this topic can be found in [4] and [5]. For the case of our gasmeter we will only test H_0.

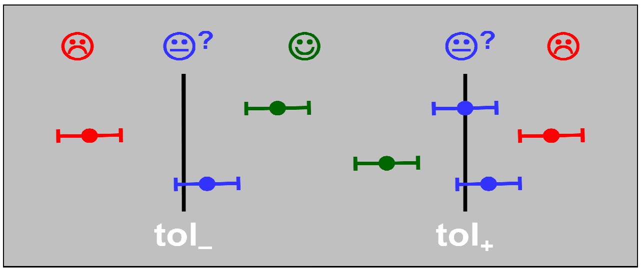

Figure 1: Conforming (green) and non-conforming (red) measurands. The blue points are not conforming and not non-conforming.

Monte Carlo Method

The Monte Carlo process is schematically depicted in Figure 2. The output quantity Y is a function of a vector X of N input quantities X_j, each with its specific probability density function (PDF-j). Now M trials are chosen, e.g. 100000. Input estimates $x_{j,k}$ are generated for $j = 1..N$ and the output estimate $y = f(x_j)$ is calculated. This process is repeated for each trial $k = 1..M$. The result is a bin with M values of y. The values y_k are sorted in ascending order, which gives the cumulative distribution function (CDF-Y). The probability density function of y (PDF-Y) is obtained by differentiation of the CDF-Y.
The estimate of Y is $\bar{y}$ the average of all y_k. The associated standard uncertainty is the experimental standard deviation of all y_k.

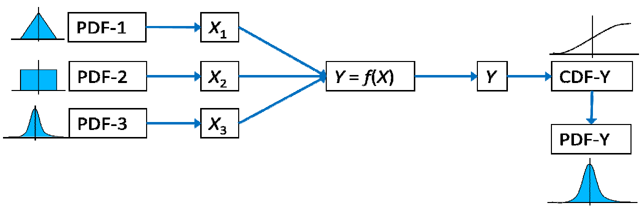

Figure 2: Schematic of the Monte Carlo process.

In the next step the 95% coverage interval will be obtained. Figure 3 shows the sorted output estimates. For any value y_q the probability that the value Y will be smaller than the q^{th} estimate of y is $P(Y \le y_q) = q/M$. For $q = 0$ $P(Y \le y_q) = 0$ and for $= M$ $P(Y \le y_q) = 1$. An interval covering 95% of the output estimates is $[y_q, y_{q+0.95 \cdot M}]$. This corresponds to the blue cells in figure 3. The index q can now be chosen such that the interval is symmetric around $\bar{y}$. It is also possible that to choose q such that the length of the interval $[y_q, y_{q+0.95 \cdot M}]$ around $\bar{y}$ is minimal, which may result in a non-symmetrical uncertainty interval.
In general the coverage interval between y_q and y_k is $P(y_q \le Y \le y_k) = (q - k)/M$.

y_1		y_q			...	$\bar{y}$	....			$y_{q+0.95 \cdot M}$		y_M

Figure 3: Output estimates y_k, $k = 1..M$ in ascending order. The blue cells represent the 95% coverage interval.

Statistical tests using Monte Carlo

Now the coverage interval has been established it is a straightforward procedure to compare the coverage interval with a series of preset tolerances. This is schematically shown in figure 4. In the upper and middle rows the coverage interval is within tolerance and the null hypothesis H_0 is accepted. In the lower row the lower tolerance is in the coverage interval and H_0 is rejected.

Based on the acceptance of H_0 the decision is made that the meter complies with the requirements. The use of a 95% coverage interval means that this decision is made with a confidence level of at least 95%. By changing the length of the coverage interval the confidence level of the decision changes accordingly.

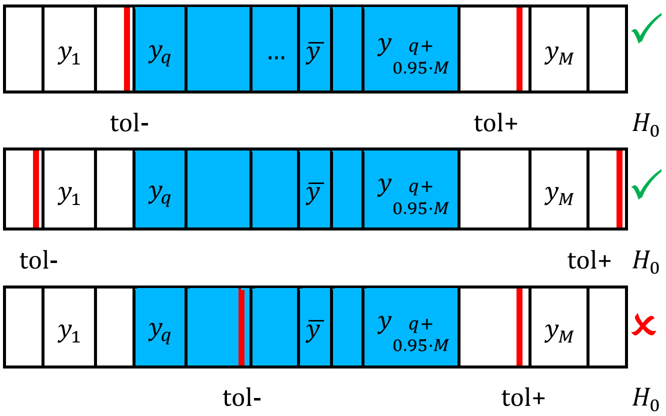

Figure 4: Tolerances (red lines) and coverage intervals around $\bar{y}$ (blue cells) while testing the null hypothesis H_0. In the upper and middle rows the coverage interval is within tolerance and H_0 is accepted. In the lower row the lower tolerance is in the coverage interval and H_0 is rejected.

In the same way the alternative hypothesis H_1 can be tested, i.e. the tolerances are exceeded. This process is schematically shown in figure 5. If the 95% coverage interval is entirely exceeding the upper tolerance (tol+), H_1 is accepted. If not H_1 is rejected.

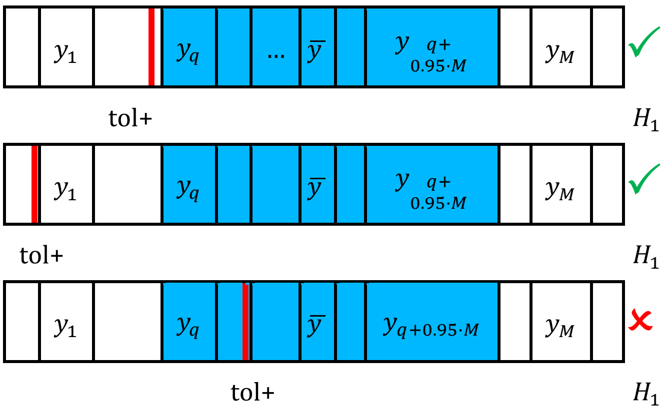

Figure 5: Tolerances (red lines) and coverage intervals around $\bar{y}$ (blue cells) while testing the alternative hypothesis H_1. In the upper and middle rows the coverage interval is within tolerance and H_1 is accepted. In the lower row the lower tolerance is in the coverage interval and H_1 is rejected.

For quality control purposes we created the possibility of testing using additional criteria at the same time.

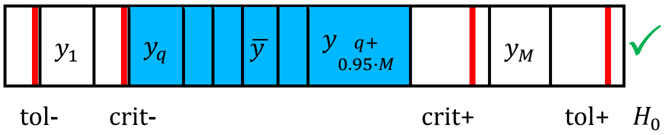

Figure 6: Statistical test with additional criteria crit- and crit+. In the above figure the result $\bar{y}$ meets both the tolerances tol- and tol+ and the additional criteria crit- and crit+ with 95% confidence.

Monte Carlo software tool

Thanks to the fact that PCs and laptops have become more and more powerful, Monte Carlo methods have become easily accessible for metrologists. The Monte Carlo software tool developed at NMi was initially an experiment to see whether the method would be feasible in the daily metrology practice.

The design choices for the Monte Carlo Method (MCM) software were, from the outset, heavily biased towards the user. It was thought to be essential that the MCM could be used by metrological workers with varying skill levels. In effect widespread use of the MCM was deemed more important than speed, security or implementation details of varying nature. This has led us to develop the MCM tool entirely within Microsoft Excel.

A user can run a Monte Carlo simulation in code without having to program a single line of code. This is enabled by a feature in VBA not commonly found in other programming languages. VBA allows code to modify itself at runtime. In this case a Set command runs a small piece of dedicated code that replaces the previous code line holding the old model formula with the new model formula.

Validation

Software validation is imperative when developing software tools. The validation of the Monte Carlo Simulator was performed by using a series of reference cases:

- 10 examples of Monte Carlo analysis discussed in the GUM supplement [7]. These resulted in uncertainty values that were in mutual agreement within 1%.
- Cases that have been analyzed analytically. These are the master meter method for calibrations using air for which the analytical uncertainty analysis was described in [12]. The second case was the bending of an aluminum bar due to a force exerted in the middle of the bar.
- The last part was a comparison with NPL [14]. Despite entirely different Monte Carlo implementations, different random number generators and different seeding (i.e. the base number for the random number generator), uncertainty results were comparable within 0.5%.

Verifications using Monte Carlo

The calibration model was implemented in the Monte Carlo Simulator. Table I shows the input variables and

Table I: Input quantities (first 7 columns) and results (right most column) of Monte Carlo Simulation

Quantity	unit	distribution	mu	sigma	a	b	process	traceability	$u(y)$
p_11	bar	Gaussian	59.875	0.00316			0.001	0.003	3.1E-05
t_11	°C	Gaussian	14.12	0.06185			0.015	0.06	1.8E-04
N_s1		Rectangular	14243		14242	14244			2.5E-05
I_s1	m-3	Constant	206.5						0
tau_s1	s	Gaussian	100.5035	0.00014			0.0001	0.0001	7.2E-07
e_s1		Gaussian	0.95%	0.00200				0.20%	1.0E-03
p_12	bar	Gaussian	59.884	0.00316			0.001	0.003	3.0E-05
t_12	°C	Gaussian	14.06	0.06083			0.01	0.06	1.7E-04
N_s2		Rectangular	13452		13451	13453			2.6E-05
I_s2	m-3	Constant	206.5						1.95E-18
tau_s2	s	Gaussian	100.5041	0.00014			0.0001	0.0001	6.8E-07
e_s2		Gaussian	0.69%	0.00200				0.20%	9.7E-04
p_start	bar	Gaussian	59.875	0.00361			0.002	0.003	2.2E-09
t_start	°C	Gaussian	14.06	0.06500			0.025	0.06	1.2E-08
p_end	bar	Gaussian	59.865	0.00361			0.002	0.003	2.2E-09
t_end	°C	Gaussian	14.12	0.06500			0.025	0.06	1.2E-08
V	m3	Gaussian	15.2	0.00000					3.2E-10
tau_V	s	Gaussian	100.5	0.00014			0.0001	0.0001	2.4E-14
p_2	bar	Gaussian	59.844	0.00316			0.001	0.003	6.1E-05
t_2	°C	Gaussian	14.08	0.06083			0.01	0.06	3.5E-04
N_m		Rectangular	53146		53145	53147			1.3E-05
I_m	m-3	Constant	400						0
tau_m	s	Gaussian	100.4992	0.00014			0.0001	0.0001	1.4E-06
X1_C1	molfrac	Gaussian	0.90327	0.00163			0.00157	0.00045	2.7E-08
X2_C2	molfrac	Gaussian	0.04791	0.00050			0.00044	0.00024	9.2E-08
X3_C3	molfrac	Gaussian	0.01297	0.00075			0.00074	0.00006	2.5E-07
X4_iC4	molfrac	Gaussian	0.00191	0.00011			0.00011	0.00001	5.5E-08
X5_nC4	molfrac	Gaussian	0.00276	0.00016			0.00016	0.00002	7.4E-08
X6_iC5	molfrac	Gaussian	0.00063	0.00004			0.00004	0.00000	2.4E-08
X7_nC5	molfrac	Gaussian	0.00054	0.00005			0.00005	0.00000	3.0E-08
X8_C6	molfrac	Gaussian	0.00067	0.00091			0.00091	0.00000	6.7E-07
X9_C7	molfrac	Constant	0						0
X10_C8	molfrac	Constant	0						0
X11_C9	molfrac	Constant	0						0
X12_C10	molfrac	Constant	0						0
X13_CO2	molfrac	Gaussian	0.01099	0.00018			0.00017	0.00006	1.6E-08
X14_N2	molfrac	Gaussian	0.01835	0.00030			0.00029	0.00009	4.2E-08
X15_H2S	molfrac	Constant	0						0
X16_He	molfrac	Constant	0						0
X17_H2O	molfrac	Constant	0						0
X18_O2	molfrac	Constant	0						0
X19_Ar	molfrac	Constant	0						0
X20_H2	molfrac	Constant	0						0
X21_CO	molfrac	Constant	0						0

their associated standard uncertainties. The uncertainties listed in the column sigma are the root sum square of the uncertainties from the traceability and the process conditions. For a rectangular distribution the lower and upper limits are given in columns a and b. The colum on the right hand side of Table I gives the uncertainty contribution of each input quantity to the uncertainty of the output estimate. The root sum square of all these uncertainties is equal to the overall uncertainty shown in Figure 7. The cells are colored: the higher the uncertainty contribution the darker the color. The most important uncertainty

Monte Carlo Simulation tool **v 2.00**

Validated software for internal use in NMi Euroloop © NMi 2013

Project information						
Client	NMi Euroloop				*Reference*	Flomeko 2013
Problem description	High pressure gas flow calibration using full gas composition and AGA-8 Equation of State				*Date of analysis*	woensdag 17 juli 2013
Model description	Instationary integral mass balance				*Output file*	Flomeko
Project/Engineer	Jos van der Grinten	*Dept.*	Metrology		*Tag number*	41472814813784

Environmental and other parameters								
Quantity	**Unit**	**Value**	**Quantity**	**Unit**	**Value**	**Quantity**	**Unit**	**Value**
Barometric pressure	mbar	1015	*Base pressure*	bar	1.01325	*Gravitational accelerati*	m/s^2	9.812184
Room temperature	°C	15	*Base temperature*	°C	0			
Relative humidity	%	45	*Combustion temperatu*	°C	25			

Output request									
Quantity	**Value**	**Unit**	**Quantity**	**Value**	**Statistical testing**				
Output quantity y	y	-	*Bins for histogram*	101	*Criterion*	*lower tol.*	*lower crit.*	*upper crit.*	*upper tol.*
Number of Trials	100 000		*Random seed*	- 654 321	*Value*	-1.00%	-0.25%	0.25%	1.00%
Coverage interval	95.0%		*Detailed output*	TRUE	*CDF*	0.00%	30.46%	99.82%	100.00%

Output results for uncertainty												
Quantity	y	u(y) (k=1)	CI1 95.0%	CI2 95.0%	Min	Q1	Median	Q3	Max		Kolmogorov Smirnov	
Unit	-	-	-	-	-	-	-	-	-		D value	P Value
Value	-0.175%	0.146%	-0.461%	0.111%	-0.778%	-0.274%	-0.175%	-0.077%	0.474%		0.0015	0.9738
Nominal			-0.461%	0.111%	*symmetric*							
Relative			-0.462%	0.111%	*Gaussian*			*Conf. level*	**95.0%**	Result meets tolerances		

Model equation	**Number of ReF**	**Number of MuT**	**Number of VoL**
AGA8	2	0.01	1

Figure 7: Dashboard of the Monte Carlo Simulator. The left red box contains the output estimate and the associated standard uncertainty. The top right hand box contains in the green cells the criteria and tolerances, the purple cells give the associated values of the CDF. The lower right hand box gives the result of the test based on the stated confidence level.

source is the traceability of the reference meters, which is fairly common in gas flow metrology.

Figure 7 shows the dashboard of the Monte Carlo Simulator. The green cells are input to the simulator, the purple cells are the output results. The left red box contains the output estimate and the associated standard uncertainty, so the deviation of the meter under test e_m=−0.18%. The expanded uncertainty is twice the standard uncertainty, i.e. 0.29%. The top right hand box contains in the green cells the criteria (±0.25%) and tolerances (±1%). The purple cells give the associated values of the CDF. For the lower criterion 30.46% of the generated output values is less than -0.25%. The area between the lower and the upper criterion cover 69.36% of the observed values. The lower right hand box gives the result of the test based on the stated confidence level. The calibration result

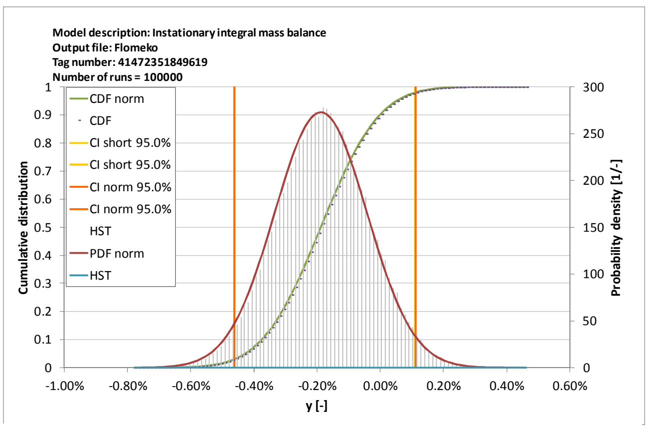

Figure 8: CDF and PDF as a function of the output estimate. The left vertical axis applies to the CDF, the right vertical axis to the PDF. The 95% coverage interval is marked by two vertical lines.

meets the tolerances with at least 95% confidence. The value of the confidence level can easily be changed without the necessity to re-run the simulation. For this case the tolerances are also met at a confidence level of 99.7%. If a confidence level of less than 69.36% is specified the result meets the criteria and consequently also the tolerances.

The Cumulative Distribution Function (CDF) and the probability density function are graphically depicted in Figure 8. The 95% coverage interval is [-0.46%; +0.11%]. The same values are obtained applying by analytical means assuming a normal distribution with a mean of -0.175% and a standard deviation of 0.146%. A logical result as most of the input distributions were assumed to be Gaussian.

Conclusions

The new point in this paper is that Monte Carlo methods can be applied directly to perform the conformity assessment. The reason is that the Monte Carlo process generates the cumulative distribution, with which the (legal) tolerances can be compared directly. The advantage of this process is that the type of distribution does not to be known and the (worst case) assumption of the distribution being Gaussian can be avoided. Consequently, for a Monte Carlo method the difference between tolerances and acceptance criteria is slightly smaller than for analytical methods.

A test of the Monte Carlo method applied to a calibration of a high-pressure gasmeter meeting MID tolerances demonstrates the applicability of the method in practice.

Abbreviations and symbols

CDF	Cumulative Distribution Function or Distribution Function	
MuT	meter under test	
PDF	Probability Density Function	
tol+	upper tolerance	
tol-	lower tolerance	
H_0	null hypothesis	
H_1	alternative hypothesis	

symbols

e	deviation	[%]
I	impulse factor	[1/m^3]
N	number of pulses counted during a calibration run	[-]
$\mathbf{n}$	normal vector	[-]
P	probability	[-]
p	absolute pressure	[bar]
Q	volume flowrate	[m^3/h]
t	Celsius temperature	[°C]
V	volume between master meters and MuT	[m^3]
v	fluid velocity	[m/s]
Y	output quantity	
y	output estimate	
$\bar{y}$	average of output estimates	
ρ	mass density	[kg/m^3]
τ	time interval corresponding to an integer number of pulses	[s]

Indices

1	at the position of the master meter
2	at the position of the MuT
i	rank number of the master meter
k	rank number of output estimates y
M	number of trials in the Monte Carlo simulation
m	indicated by the MuT
n	number of parallel master meters
q	rank number of the start of the coverage interval
V	at the volume between masters and MuT

References

[1] BIPM/IEC/IFCC/ISO/IUPAC/IUPAP/OIML (1993): Guide to the expression of uncertainty in measurement, first edition, ISO 1993

[2] JCGM 101 (2008): Evaluation of measurement data - Supplement 1 to the "Guide to the expression of uncertainty in measurement" - Propagation of distributions using a Monte Carlo method, Guide JCGM 101

[3] OIML G1-101 (2008): Evaluation of measurement data - Supplement 1 to the "Guide to the expression of uncertainty in measurement" - Propagation of distributions using a Monte Carlo method, Guide OIML G 1-101

[4] J.G.M. van der Grinten (2003): Confidence levels of measurement based decisions, OIML Bulletin Volume XLIV • Number 3 • July 2003, pp. 5 – 11

[5] J.G.M. van der Grinten (2003): Confidence levels of measurement based decisions, Proceedings of Flomeko XIII, 13-15 May 2003, Groningen, The Netherlands

[6] Sommer, K.D. and M. Kochsiek (2002): Role of measurement uncertainty in deciding conformance in legal metrology, OIML Bulletin Vol. 18, No. 2, pp. 19–24

[7] JCGM 106 (2012): Evaluation of measurement data – The role of measurement uncertainty in conformity assessment, Guide JCGM 106

[8] OIML TC3 / SC5 (2009): The role of measurement uncertainty in conformity assessment decisions in legal metrology, Committee Draft CD1, OIML TC3 / SC5 / N1

[9] MID (2004): Directive 2004/22/EC of the European Parliament and of the Council of 31 March 2004 on measuring instruments, Official Journal, No L 135, pp. 1-80. http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2004:135:0001:0080:EN:PDF

[10] MID amendment (2009): Directive 2009/137/EC of 10 November 2009 amending Directive 2004/22/EC of the European Parliament and of the Council on measuring instruments in respect of exploitation of the maximum permissible errors, as regards the instrument-specific annexes MI-001 to MI-005, Official Journal, No L 294, pp. 7-9. http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2009:294:0007:0009:EN:PDF

[11] AGA 8 (1992): Compressibility factors of natural gas and other related hydrocarbon fluids, by K.E. Starling and J.L. Savidge, AGA Transmission Measurement Committee Report No. 8, American Petroleum Institute MPMS Chapter 14.2, 2nd edition 1992, 2nd printing 1994, Catalog No. XQ9212

[12] J.G.M. van der Grinten (2008): Validation of the Monte Carlo Simulation tool, NMi internal report, November 2008.

[13] J.G.M. van der Grinten (1994): A comparison of the methods for uncertainty analysis based on ISO 5168 and the guide prepared by ISO/TAG4/WG3, in: proceedings of the Flomeko '94, Flow Measurement in the mid 90's, June 1994, NEL, East Kilbride, Scotland.

[14] Peter Harris (2008): Oral communications, NPL, Teddington, September 2008.