Abstract-. This paper presents a case study concerning the optimization of a DC ferrofluid actuator designed to transmit small pressures, using genetic algorithms. The optimization problem consists in finding the values of some mechanical parameters of the actuator as well as those of the magnetic permeabilities of the device components, that render a maximum equivalent force. The magnetic field inside the actuator is determined using the finite element method and the force acting on the flexible parts is calculated using Maxwell’s magnetic tensor.

I. Introduction

Actuators using magnetic fluids or ferrofluids (which are basically colloidal suspensions of small Fe₃O₄ particles dispersed in a dielectric carrier liquid) gain an increasing importance, their usage being rather extended for the transmission of small pressures and forces. One application of these small scale devices is based on the possibility to control the pressure exerted by a dielectric fluid with magnetic properties ($\mu_r>1$) by means of small variations of a D.C. current in an excitation coil that magnetizes the fluid.

Technical literature presents several types of magnetic fluid actuators that have metallic mobile parts, placed inside or outside the ferrofluid, or that have flexible parts, allowing for a distortion that depends on the fluid magnetization [1], [2]. If a D.C. field of moderate amplitude is applied the magnetic force will be quadratic in terms of the D.C. current producing the magnetic field.

In our previous papers [2], [3] several types of ferrofluid sensors and actuators were analyzed theoretically and tested experimentally. In [4] an optimum configuration of the actuator was sought for using the hill-climbing method.

The usage of genetic algorithms (GA’s) in electromagnetic optimization problems date back to 1990’s, a significant number of articles on this topic being published since, both for the optimization of D.C. or low frequency devices, as well as for high frequency ones [5]...[9]. Some of these studies concern the optimization of coupled electro-mechanical problems.

The present paper presents a study of a magnetic fluid actuator with two flexible membranes. The force transmitted by the actuator is determined by performing a numerical quadrature of the pressure exerted on the membranes, while the magnetic field (used in calculating the pressure) is obtained using the finite element method (FEM). Genetic algorithms are used to find the optimum configuration that maximizes the overall force transmitted by the device.

II. Physical model of the device

Figure 1 presents the physical model of a current controlled actuator with flexible membranes used to transmit small pressures. The actuator consists of a coil having $N$ turns, with a tubular iron core, coaxial with a cylindrical chamber filled with ferrofluid. The device is bounded at both ends by flexible diaphragms. An exterior iron tube is used to enhance the force that the actuator exerts on the flexible diaphragms. The system has axial symmetry and one half of the device is represented in the $(r,z)$ plane in Fig.1. When a current $I$ passes through the coil the ferromagnetic particles dispersed in the fluid are magnetized and the pressure on the two diaphragms increases.

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In the case of an axially symmetric system with known current density \( J = J_\theta e_\theta \) (tangential component), the functional used by the finite element method is formulated in terms of the modified scalar magnetic potential \( U(z,r) = r A_\theta \) where \( A = A_\theta e_\theta \) is the magnetic vector potential. The magnetic flux density may be obtained using the relation:

\[
B = B_r e_r + B_z k = -\frac{1}{r} \frac{\partial U}{\partial z} e_r + \frac{1}{r} \frac{\partial U}{\partial \theta} k. \tag{1}
\]

The pressure on the two membranes is determined using Maxwell’s magnetic tensor, \( T_{mn} \), having the expression [10]:

\[
T_{mn} = \frac{\mu_f - \mu_0}{2 \mu_0} \begin{pmatrix}
  B_r^2 & B_r B_z \\
  B_r B_z & B_z^2
\end{pmatrix}
\tag{2}
\]

and the equivalent force is calculated with the relation [4]:

\[
F_{\text{eqv}} = \pi \int_0^{r_m} \left( T_{mn}(r) \right)_{\text{upper}} - \left( T_{mn}(r) \right)_{\text{lower}} (r_m - r)^2 \, dr,
\tag{3}
\]

where \( r_m \) is the radius of the membrane.

**III. Optimization using GA**

Genetic algorithms are basically function optimizers, the search for an optimum being a stochastic process. Their large usage in the optimization of electromagnetic problems is due to the fact that they tend to converge to a global maximum (or minimum), the final result being thus less prone to the initial guess for the solution. Moreover, GA’s can handle easily nondifferentiable objective functions (which the deterministic optimization methods can not) and discrete search spaces for the design variables.

The goal of the optimization for the problem studied in this paper is to obtain the maximum equivalent force on the two diaphragms. The magnetic field was analyzed using the finite element method, in terms of the modified scalar magnetic potential \( U(r,z) \).

The design variables used in the optimization process were the distances \( z_3, z_4, r_3, d_z \), and the permeabilities \( \mu_m, \mu_0 \) (Figure 1). Two constraints were imposed to the resulting solutions:

a) the coil was considered to have an imposed window surface, \( S_w \);  
b) the geometric parameters \( z_4 \) and \( z_5 \) must satisfy the condition \( z_4 + z_5 < l_{\text{max}} (l_{\text{max}} = 34 \, \text{mm}) \).

Moreover, the search space for each design variable is a specified interval in the real number set.

The genetic algorithm uses a population of \( N_{\text{IND}} \) possible solutions (chromosomes). The initial population is randomly generated with a uniform distribution in the search space. The fitness of the individuals in each population is evaluated using the ranking method and considering as the objective function the modulus of the equivalent force produced by the actuator, calculated for the constructive parameters coded in the chromosome. The best solutions are encouraged to participate in the genetic crossover. The mating pool, containing \( N_{\text{IND}}/2 \) individuals, is filled using a stochastic universal sampling and the crossover operator is applied to the selected parents. In order to produce the children, GA’s use mechanisms that imitate the recombination of DNA biological chains. Random variations of small amplitude are then applied to the genetic material using the mutation operator. The children
solutions compete with the parent solutions and the fittest survive in the next generation. A
deterministic and elitist insertion is used in order to create the new population.

The evolution loop is implemented using the island model. This means that the population is divided
into SUBPOP subpopulations that evolve independently for NOMIGR generations, after which
migration is activated, ensuring an exchange of genetic material between subpopulations (20% of the
fittest individuals migrate to other subpopulations). This mechanism ensures a convenient diversity in
the genetic material of the population and allows for a better exploration of the search space.

Two genetic optimization methods that encode directly the design variables are used. The first
method is based on the canonic GA and is characterized by the fact that chromosomes are formed by a
concatenation of the binary strings associated with each design variable. The length of each sub-string
is selected according to the prescribed range and precision for the design variable (z4 and z5 use 10 bits,
r3, dz and μf use 5 bits, while μm uses 30 bits). In this case multiple point crossover and uniform
mutation are used.

The second method is based on floating point codification, in which case the length of the
chromosome is equal to the number of design variables. This method has the advantage that the
precision in obtaining the optimum does not depend on the chromosome length and that the
chromosome is represented more naturally. However, in the example studied in this paper the precision
is imposed by technical considerations. The roundups for the decision variables are made separately,
because they are not encoded in the GA. The genetic operators used in this case are intermediate
crossover and uniform mutation.

In both cases correction techniques are used in order to fulfill the condition z4+z5<lmax imposed by
the actuator design. In the case when the chromosome does not satisfy this constraint, the sub-chain
that encodes one of the two decision variables, randomly chosen, is reconfigured so that it encodes the
value (lmax-z)·ran where z is the unmodified value of the decision variable and ran is a random number
between 0 and 1 with uniform distribution. The method may present an advantage in the case when the
optimum is not close to the boundary z4+z5=lmax.

Several experiments were conducted in order to establish the optimum GA parameters. The
evolutionary process developed for a number of MAXGEN=20 generations using a crossover
probability PC=0.7 and a mutation probability PM=0.1. In all cases a small number of individuals was
employed (NIND) due to the large computer time needed for the objective function calculation. That is
why all the evolutionary mechanisms were set to ensure the best exploration of the search space: rank
fitness computation, subpopulation evolution, etc.

The algorithm performance is highly influenced by the number of subpopulations and by the time
interval set for migration. If NOMIGR is too small, the information exchange is inefficient since the
emigrants are not well adapted; if NOMIGR is too large the subpopulations are isolated for a large
period of time and the experience of the other groups is not well exploited. The variable SUBPOP must
be established in accordance with NIND, in order to ensure an adequate cardinal for each
subpopulation.

IV. Results and discussions

The ferrofluid actuator presented in Figure 1 was analyzed using the finite element method for field
computation and genetic algorithms for optimization with respect to Feqv. The search space for the
design variables was: r3 ∈ [3, 7]mm, z4 ∈ [5, 30]mm, z5 ∈ [0, 10]mm, dz ∈ [-4, 4]mm, μf ∈ [1.1, 4],
μm ∈ [600, 6000]. Some of the results obtained for the case NOMIGR=5, which gave the best
individuals, are presented in Table 1.

<table>
<thead>
<tr>
<th>No.</th>
<th>Codification</th>
<th>NIND</th>
<th>SUBPOP</th>
<th>Optimum equivalent force Feqv [N]</th>
<th>Design variables in optimum case</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>binary</td>
<td>16</td>
<td>2</td>
<td>1.5280</td>
<td>{3.5, 12.75, 3.25, 3.75, 3.90, 3341.53}</td>
</tr>
<tr>
<td>2.</td>
<td>binary</td>
<td>32</td>
<td>2</td>
<td>1.8808</td>
<td>{1.75, 10.75, 3.25, 4.0, 3.90, 4094.38}</td>
</tr>
<tr>
<td>3.</td>
<td>binary</td>
<td>32</td>
<td>4</td>
<td>1.8208</td>
<td>{1.5, 13.75, 3.6, 4.0, 2.970, 3085.587}</td>
</tr>
<tr>
<td>4.</td>
<td>real</td>
<td>32</td>
<td>2</td>
<td>1.7176</td>
<td>{0.75, 13.5, 3.75, 3.75, 3.89, 3999.56}</td>
</tr>
</tbody>
</table>

In all the analyzed examples the required precision was handled better by the GA’s using binary
encoding. Real floating point codification produces children with a precision larger than that imposed
by technical considerations, thus wasting exploration effort. The stochastic nature of the GA is also evident from the data in Table 1, the search direction being not always obvious. However, as expected, larger values of the ferrofluid and magnetic core permeabilities lead to a larger equivalent force. At the same time the results show that it is preferable to use a coil with a larger external radius and a smaller height $z_4$ (line 2- best individual, compared to the other lines in Table 1). As for the design variables $z_5$ and $dz$ both the coil and the tubular iron core must be placed close to the upper membrane (lines 2 and 3 in Table 1). The radius of the two iron tubes, $r_3$, must be confined to the lower boundary of its search space in order to achieve a larger transmitted force.

The pressure $T_{mn}$ exerted on the two membranes for the configuration presented in line 2, Table 1, which produced the optimum result, is plotted in Figure 2. As may be seen, the highest pressure is obtained on the upper membrane, in the region corresponding to the iron core. The values of Maxwell’s magnetic tensor on the lower membrane are much smaller due to the larger distance to the magnetic field source.

Figure 2. Magnetic pressure on the actuator flexible membranes

Figure 3 plots the performance of the fittest individual over the generations for the case that gave the best result. The results show important improvements of the equivalent force in the generations immediately following the migrations, proving the efficiency of the island model. The plot shows that in the analyzed cases a good result is obtained even after a small number of generations (12 generations), this being an important aspect due to the large amount of CPU time used in evaluating all the individuals in the subsequent populations.

Figure 3. Evolution of the best individual

Figure 4. Diversity of the final population
Figure 4 plots the performance of the individuals in the last population for the case corresponding to line 2 in Table 1. The two subpopulations are marked with the “+” and “o” signs, respectively, the results showing a high diversity of the chromosomes. The large differences that appear in the performance of the individuals belonging to the last generation indicate that genetic algorithms are volume oriented and work on the entire population.

The performed analysis shows that the results obtained using GA’s for the optimization of the ferrofluid actuator lead to better solutions than those obtained using the conventional hill-climbing method [4], requiring a smaller participation of the designer during the optimization process. The number of design variables may be increased, if other geometric parameters of the actuator can be modified, but with greater computer time consumption.

IV. Conclusions

A stochastic optimization based on genetic algorithms was carried out for optimum design of an electro-pneumatic device used to transmit small pressures and forces. The electro-mechanical coupled problem was solved numerically using the finite element method for stationary axially symmetric magnetic problems and by using Maxwell’s magnetic tensor for pressure and force computation.

The GA parameters (number of individuals, number of subpopulations, crossover and mutation probability, codification type, etc.) were firstly established for optimum performance.

The results proved satisfactory in terms of the best individual (best design parameters for maximum equivalent force), showing a good behaviour of the genetic algorithm in optimizing the analyzed coupled problem.

Further studies are envisaged for the optimization of other types of ferrofluid actuators also aiming to decrease the overall computation time.

References