

## The achievable uncertainty for balance-based force standard machines in the range from micronewton to newton

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### Abstract

At PTB, a force standard machine (FSM) has been investigated [1-2] which uses electromagnetically compensated balances (ECBs) with a full range of 10 N or 2 N respectively, for reaction force compensation of force transducers. This should enable the calibration and measurement capability of force transducers with a range from 1 mN to 10 N. The facility has been completely redesigned to allow the measurement with the load button upwards in compression direction.

In this contribution, the uncertainty budget for the measurement of an exemplary transducer with this new FSM will be elaborated. The estimation is done in compliance with the GUM [3] and ISO 376 [4] for different measurement procedure scenarios. Measurement results of this transducer gained before and after the redesign of the facility prove their reliability. The observed hysteresis and zero value residual are explained with simulated values which are based on the measured step-response function of the transducer.

*Key words:* Force standard machine, traceability, small forces, force transducer, ISO 376, electromagnetically compensated balance

### Introduction

Up to now, no “calibration and measurement capabilities” (CMC) for force transducers in steps smaller than 0.5 N have been available at the national metrology laboratories [5]. Facilities provided for the traceable calibration of force transducers are “force standard machines”. For high accuracy, they commonly rely on the principle of “direct mass loading” (deadweight). To cover the range between 1 mN and 10 N, PTB has proposed the use of electromagnetically compensated balances (ECBs) [1, 2]. They serve as linear scale; the traceability of the unit Newton is guaranteed by the traceability of a deadweight using the principle of substitution. The traceability of the deadweight to SI units is done by mass artefacts and measurement of the local gravity [6].

The use of ECBs allows the calibration of transducers with arbitrary load steps, especially sub-ranges. Complete systems are available, easy to use and highly linear. Over the full range, they reach a maximum deviation from linearity of less than one part per million. For mass determination, substitution methods are used which eliminate most of the contributions to the uncertainty, especially the zero-point-drift of the balance. Contrary to this common use of ECBs, here

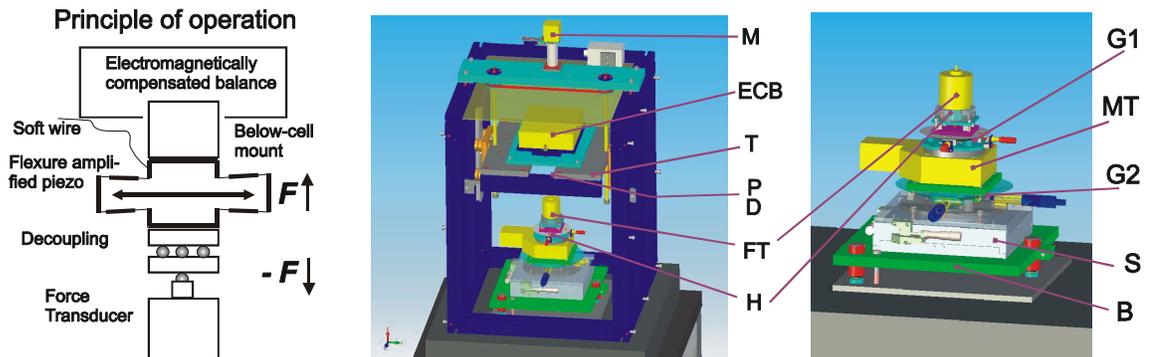
the timing conditions for calibrations are defined by standards which account for the requirements for (strain-gauge-based) transducers. Moreover, the load represents a restriction of the degrees of freedom. As force has a vector character, and as even excellent force transducers have a multicomponent sensitivity, the facility should be adjustable in different axes and should decouple parasitic components. In general, the 3 components of force and the 3 components of torque act on the balance, and on the transducer under test respectively. The sensitivity of the scalar signal  $S$  up to the square order can be described by zero value  $S_0$ , 6 sensitivity coefficients  $k_i$  and a matrix  $k_{ij}$  with 21 independent values, see [7].

$$S = S_0 + \sum_{i=1}^6 k_i X_i + \sum_{i,j=1}^6 k_{ij} X_i X_j, \quad \vec{X} = (F_x, F_y, F_z, N_x, N_y, N_z) \quad (1)$$

When constructing a FSM, the skill is to avoid the coupling of bending moments  $N_i$  and the reduction of lateral forces  $F_y, F_z$ . In the following three sections, first the concept and construction of the new FSM will be introduced. In the second section, a complete budget is established for the measurement uncertainty of a typical transducer. The third section shows measurements and calibration results of a transducer which are intended to prove the reliability of the budget.

## 1. Concept and Construction

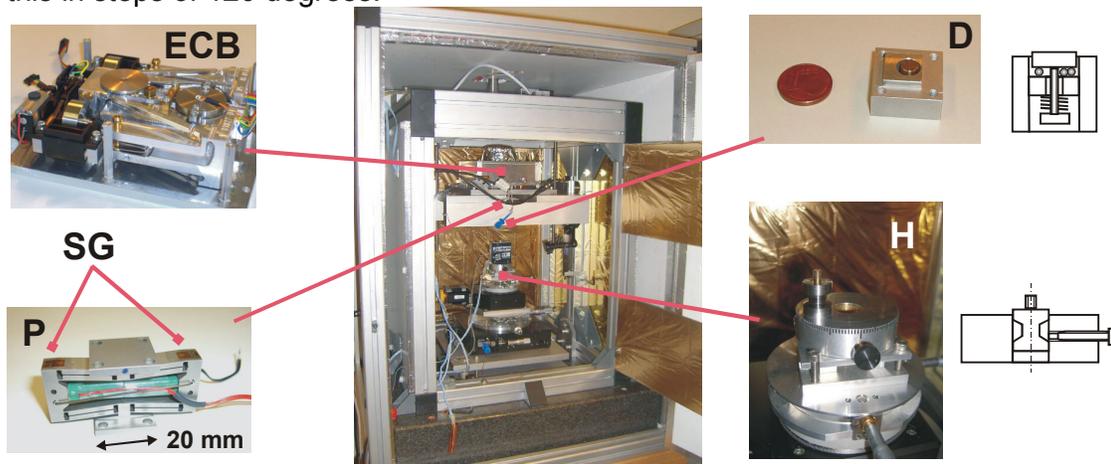
As published earlier [1, 2], it is possible to measure the characteristic of a force transducer under test (TUT) by use of an ECB. The force transducer was pressed onto the ECB with the load button side downwards, while the reaction force was measured. The “virtual” static stiffness of an ECB for the 2 N range can be higher than 500 kN / m. Insofar as the stiffness of the TUT is much lower (or reduced by an additional spring) and no resonances with other systems occur, the mechatronical system of the ECB stays undisturbed. There are two disadvantages: First, the mounting direction with the load button downwards is unusual. Second, it is necessary to shift the TUT physically, straightly and frictionless. This might become a severe practical problem, as the TUT might become heavy and connected with a rugged cable.



**Figure 1:** Concept and design. M: stepper motor, T: traverse, P: piezo unit, D: decoupler, FT: force transducer (under test), H: holder, G1,G2: goniometer, MT: motorised turntable, S: XY-  
The proposed solution is that the position of the TUT and of the ECB stays fixed. The TUT stays on the base and the ECB stays on a (motorised) traverse above. The necessary force is generated between them by a spreader (fig. 1).

As the reaction force equals the negative of the force acting on the ECB, both show corresponding signals every time. Actually, a piezo-actuator unit (PAU) is used. It can thus be supplied by two thin wires, with a small and constant spurious force. This PAU is mounted stiff and with defined direction to the below-cell mount. The system with the ECB and the TUT staying fixed is overdetermined. This requires a decoupling of the two lateral force components, all torques and a well-defined “point” of force introduction. These spurious components arise from residual alignment errors. From this, the reactive forces are obtained through multiplication by the stiffness of the TUT. Decoupling of the bending moments is realized by bringing a spherical load button into contact with a flat surface; the reduction of the lateral force components is realised by a reduction of the corresponding stiffness components.

To facilitate the future investigation of the positional and directional dependence of the signal, the base is adjustable in many axes (fig. 1, right). At the bottom, there is a tilting platform to adjust the direction of motion of an XY-stage. On its top, a 2-axes goniometer (180 mm pivot height) is mounted for adjustment of the TUT’s main axis. In addition it carries a motorised turntable with a second goniometer (100 mm pivot height). This enables the future automatic (oblique) rotation. On top, there is a holder with a traction bolt with centring function and angle graduation. The experimental treatment of the alignment matter according to ISO 376 [4] requires the rotation of the TUT directly on this in steps of 120 degrees.



**Figure 2:** Current set-up. P: piezo actuator with glued SG: strain gauges, D: decoupler, H: holder. The rack is built from aluminium profiles on a granite base. There is a thermal shield inside the rack and an outside freestanding thermal insulation without thermal bridges. The stepper motor is mounted outside, connected by a thin heat-insulating tube. Inside there are two thermometers for air and transducer temperature.

Figure 2 shows the final construction. The ECB and its properties were described earlier [2]. The PAU is clamped to the below-cell mount. It is a commercially available system such as that used for focus mirrors of telescopes. Its blocking force is 125 N, maximum travel path is 500  $\mu\text{m}$  (-30 V to 150 V) and resonance frequency is 410 Hz. The mass of about 50 g represents a tara for the ECB. On top of the balance, an additional tara mass of about 150 g is placed. This fits the operational range of the ECB for a TUT in compression direction and a full range of 2 N. The PAU can be driven in free-running mode by voltage or in fed back mode to the strain gauge sensors, glued on the flexure joints. As the existing DC sensor amplifier produces excess noise for frequencies  $<10$  Hz, it is abandoned for the following.

The piezo voltage might lead to an electrostatic force between the live end and the dead end of the balance. Additionally, an electrostatic force between the cables might lead to bending effects and to additional force. The wires used have an outer diameter of 75  $\mu\text{m}$ , and they are insulated by means of a coating. Four wires read out the strain gauge full bridge, two wires supply the piezo voltage. The strain gauge wires are in the form of helices. The ground potential piezo wire is also formed as a helix in whose inside the phase voltage wire is screened.

The force generation is fed back to the ECB's signal and reaches the set-point within 30 s with a residual regulation deviation of  $10^{-4}$  of the step height and within 60 s with  $10^{-5}$ . There is a residual regulation noise of  $\pm 10 \mu\text{N}$  for a stiffness of the TUT of 10 kN/m. This value depends on the stiffness, on the regulation parameters and on the ambient mechanical noise. Decoupling is realised by a planar ball bearing, preloaded by a tension spring. The lateral stiffness measured is 100 N/m. The holder features a finely turned and lapped surface, a brazen fitting and a drag anchor with thread.

The ambient conditions are optimized. The facility is located in an air-conditioned lab, with a continuous data logging of the air pressure, the humidity and the air temperature. The mounting volume for the transducer and the balance is heat-shielded twofold. The air temperature and the transducer temperature inside are logged. Inside the facility, mainly granite, aluminium, brass and stainless steel components are used. The components of the guide and drive of the traverse are demagnetized.

## 2. Uncertainty Budget of the Measurement

The following section describes the contributions to the measurement uncertainty of the sensitivity of force transducers. It considers the calibration of a typical strain-gauge-based force transducer according to ISO 376, as shown in the third section of this work. The "sensitivity" corresponds to the "average value of the deflections with rotation" at full load. The usually achievable uncertainty for the calibration of force transducers from 5 N to 2 MN is  $2 \cdot 10^{-5}$  of the actual load [5]. As this is a first publication, the budget is held conservative: possible corrections (e.g. for linear drifts, buoyancy, etc.) are not carried out, but added.

### 2.1. Modelling the Measurement

The measurement consists of two parts: first, the generation of the force; secondly the application and comparison of the force by means of a transducer. In the latter, parasitic (lateral and torque) components of the realisation or timing inaccuracy might masquerade as an excessive signal. The simplified model for the generation of force is:

$$F(t_0) = m_{cal} \cdot g_{loc} \cdot \left(1 - \frac{\rho_{air}}{\rho_m}\right) \cdot s + s_{lin}(s) + s_{hist}(t < t_0) + s_0(t_0) + r(t < t_0) + v(t_0)$$

$m_{cal}$  : mass of calibration weight

$g_{loc}$  : local gravitational acceleration (2)

$\rho_{air}, \rho_m$  : density of air / calibration weight during calibration

$s = m_{equ} / m_{cal}$  : extrapolation factor with force equivalent mass  $m_{equ}$

$s_0(t)$  : zero - drift after period  $T_{Zero}$  without zero setting, approx.:  $s_0(t) = const \cdot T_{Zero}$

The first three factors are well known from deadweight machines. The fourth factor  $s$  is the scale factor of the ECB. The second and the third summand represent a scale function which corresponds to the main property of an ECB as a graduation. The function  $s_{lin}$  represents the static nonlinear deviation,  $s_{hist}$  represents the dependence of the span on the preload history. For  $s$  equals one,  $s_{lin}$  is zero. The zero-drift of the balance-actuator system  $s_0$  has to be treated taking the measurement procedure's timing into consideration. It dominates for smaller forces. The generation of the force is realised by regulation. Hence  $r(t)$  displays the regulation oscillations and noise. As the actual value of force is measured by the ECB, this effect can be corrected. It remains a smaller residuum. Finally,  $v(t)$  represents parasitic electrostatic forces of the supply voltage.

The model for the calibration measurement must take the interaction of a real transducer with the parasitic components of the FSM and axis tilts into account, i.e. spurious torque and the vector character of force and sensitivity. Equation (1) shows a model of the signal of the TUT. In reality, the constants  $S_0$ ,  $k_i$ , etc., are time-dependent and include the history of applied forces. The signal contributions due to these effects, divided by the estimated sensitivity  $k_x$ , can be treated as an equivalent apparent force. Assuming that all quantities are small, except for  $F_x$  and  $k_x$ , and disregarding all terms higher than second order, yields:

$$F^{app} = \sum_{i=2}^6 \frac{k_i}{k_x} X_i + 2 \cdot F_x \sum_{i=2}^6 \frac{k_{1i}}{k_x} X_i + \frac{c(t)}{k_x}, \quad \bar{X} = (F_x, F_y, F_z, N_x, N_y, N_z), k_{ij} = k_{ji}. \quad (3)$$

The normal to the mounting surface of the TUT defines its main axis direction. The coefficients of sensitivity  $k_i$ ,  $k_{ij}$  include the spurious inclination of the functional axis of the TUT. In general, for a FSM the parasitic components  $X_{2,...,6}$  are small crosstalk to the main component  $F_x$  and proportional to it. So the first term is first order, the second term is second order in  $F_x$ . If the TUT inclines or bends additionally under load, another second-order term arises. The components of sensitivity  $k_i$ ,  $k_{1i}$ ,  $i \neq 1$  rotate with the TUT. This leads to a sinusoidal effect which is eliminated by averaging over three rotational positions in ISO 376. The spread of these values (reproducibility "b" according to ISO 376) provides information about misalignment of the FSM, but also about an ill-defined mounting surface and bending of the TUT. The creep of the TUT,  $c(t)$ , is considered in ISO 376. Due to the creep, timing inaccuracy produces an excess uncertainty.

## 2.2. Contributions to Uncertainty

The stated uncertainty values for normal distribution are given for a probability of 95%. For rectangular distribution, the half interval is stated.

### Traceability of the Weight (mass, gravity, buoyancy) $u_{TW}$

For long-term stability, the balance is provided with internal adjustment weights. These are subject to air buoyancy and its day-to-day change. The air buoyancy (and its variation) can be corrected by the CIPM formula [8]. For the traceability of the weight, an external weight is used. Its mass is adjusted in such a way that its weight corresponds, under standard lab conditions and measured local gravity, to the force of 1 Newton [2]. The mass is known with a relative standard uncertainty of  $1 \cdot 10^{-6}$  (normal, type B), the local gravity due to

tides with  $2 \cdot 10^{-7}$  (rectangular, type B). The neglect of the air buoyancy variations leads to  $3 \cdot 10^{-6}$  (normal, type B). For simplicity this should be assumed first.

$$u_{TW} = 3.3 \cdot 10^{-6} \text{ N}^{-1} \cdot F \text{ (normal, type B)}$$

#### Linearity of the Balance $u_{LB}$

The short scale linearity is not known, but an upper limit can be given. A comparison with a force transducer measurement chain ( $1 \mu\text{N}$  corresponds to  $1 \text{ nV/V}$ ) shows only a statistical noise of  $1 \text{ nV/V}$  (fig. 3) that is known to be equal to the repeatability of bridge amplifier readings after internal calibration. The long scale linearity has been tested by measurement of load/unload cycles with a mass artefact corresponding to a weight of  $0.5 \text{ N}$  at 3 working points shifted about  $0.5 \text{ N}$  to each other. The measured nonlinearity is  $0.28 \mu\text{N}$  per  $0.5 \text{ N}$ . Over the full scale of  $2 \text{ N}$ , all values lie within an interval of  $\pm 0.28 \mu\text{N}$ . The short scale linearity is supposed to be at least three times smaller. Neglect of corrections leads to:

$$u_{LB} = 1 \cdot 10^{-7} \text{ N} + 2.8 \cdot 10^{-7} \cdot F \text{ (rectangular, type B)}$$

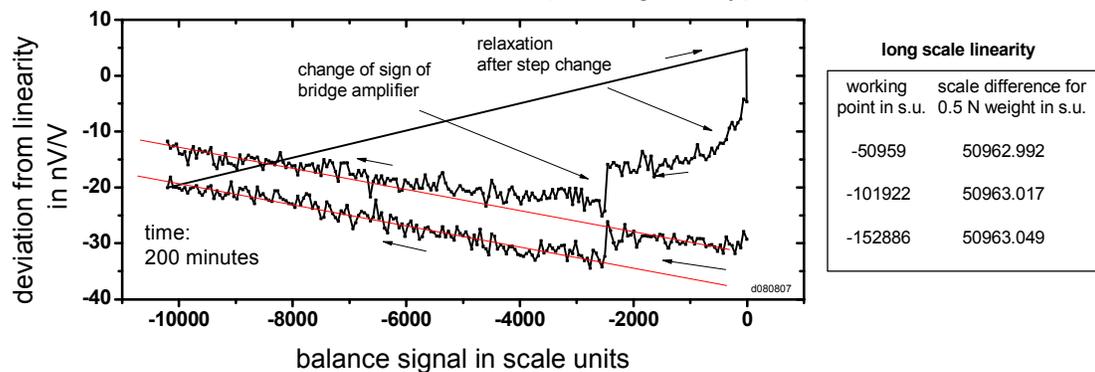


Figure 3: Linearity check. Left: Measurement of a strain gauge transducer chain with a sensitivity of about  $1 \mu\text{N}$  per  $1 \text{ nV/V}$  against the balance. The measurement represents two upward ramps of force. The range is  $100 \text{ mN}$  in steps of  $0.5 \text{ mN}$ . The change of the sign of the bridge amplifier produces an offset of  $6 \text{ nV/V}$ . The noise observed is  $1 \text{ nV/V}$ . The balance signal is negative, as the force acts bottom up. The zero drift of the balance/actuator system is about  $2 \mu\text{N/h}$  and leads to the shift of both cycles. Right: weighing an artefact with different tara artefacts tests the linearity. S.u.: scale units  $\sim$  conventional mg.

#### Contributions by Temporal Development of the Sensitivity of the Balance $u_{TD}$

This contribution can be determined by step-response caused by loading/unloading the balance with a weight under good thermal conditions. For unloading this has previously been published in [2]. For the balance used, this has not changed, even when tested under different conditions of thermal coupling and different values of the deadweights. In figure 5, experimental data are shown. A load change produces an overshoot of  $2.5 \cdot 10^{-6}$  five minutes after the event. In measurements with a waiting time of 30 seconds according to ISO 376, this is a typical time for a load change with a step height of the nominal force  $F_N$  having occurred in the history of the TUT before. The uncertainty has thus more likely the character of a summand in equation (2):

$$u_{TD} = 2.5 \cdot 10^{-6} \cdot F_N \text{ (rectangular, type B)}$$

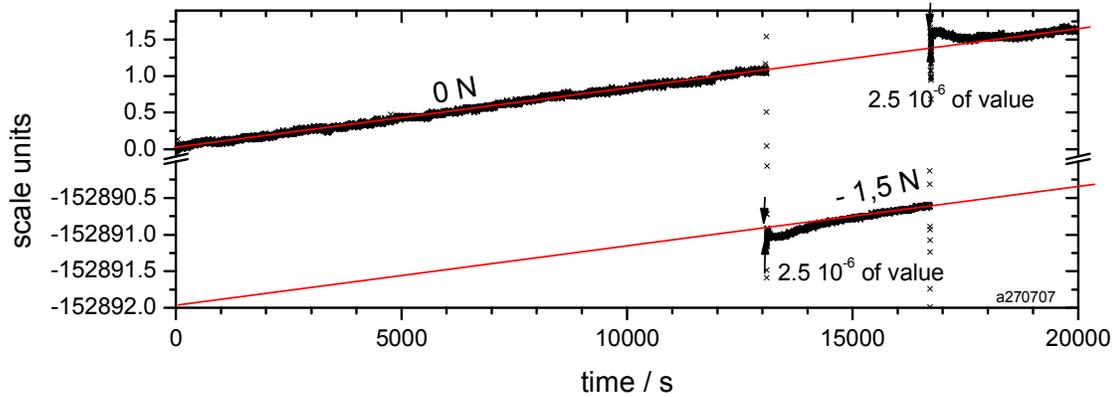


Figure 4: Temporal behaviour of the balance. Scale unit is conventional mg. After four hours of observation time, the tara load of 1.5 N is removed for one hour. The zero-drift stays constant  $3 \mu\text{N/h}$  and halves after one weekend. The step response shows an overshoot of  $2.5 \cdot 10^{-6}$  relative to the step height. The typical time scale is 10 minutes. Thermal stability is better than  $6 \text{ mK/h}$ .

#### Contributions by Zero Drift of the Balance/Actuator System $u_{\text{ZD}}$

This contributes to  $s_0$  in equation 2. Figure 4 shows a typical result of the zero-drift. A few hours before, new wires for the PAU have been installed. On a long temporal scale this value decreases, but might change the sign. The thermal history is also of relevance. Empirically, the following is assumed for the time after the last zero-setting  $T_{\text{zero}}$ :

$$u_{\text{ZD}} = 2 \cdot 10^{-6} \text{ N/h} \cdot T_{\text{zero}} \text{ (normal, type B)}$$

#### Contributions to the Signal of the Balance by Piezo Voltage and Cables

Electrostatic force of the supply voltage of the PAU contributes to  $v_0$  in equation 2. Its effect is shown in figure 5. Full voltage produces a systematic deviation of force of less than  $1 \mu\text{N}$  and a hysteresis which is not measurable (i.e.  $< 0.1 \mu\text{N}$ ). The necessary voltage  $U_{\text{PAU}}$  is proportional to the force and its maximum depends on the stiffness of the TUT. The coefficient which is assumed to be typical yields:

$$u_{\text{VC}} = 1.0 \cdot 10^{-8} \text{ N/V} \cdot U_{\text{PAU}} \text{ (normal, type B)}$$

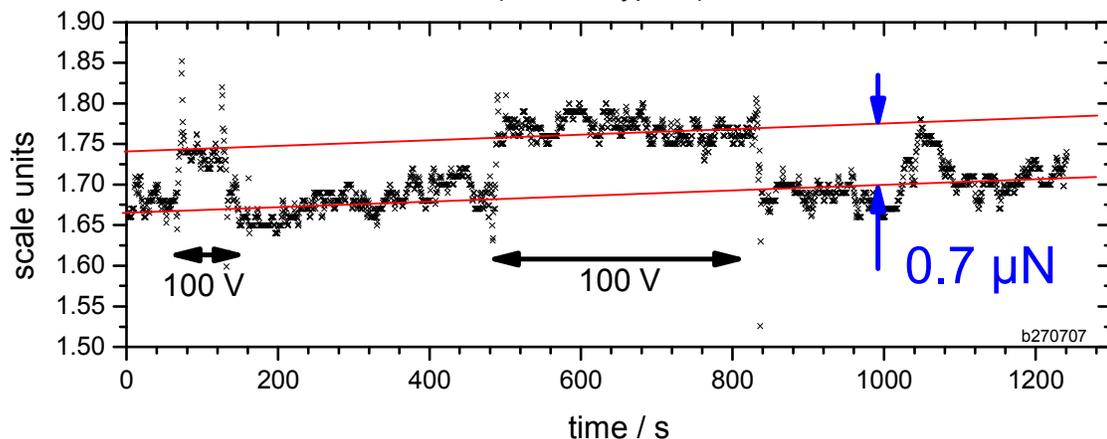


Figure 5: Influence of the electrostatic force. By appropriate wiring of the piezo-actuator unit, the influence of its supply voltage is reduced to below  $1 \mu\text{N}$ .

### Contributions by the Asynchronicity of the Sampling $u_{AS}$

In regulated systems, the actual value of force is time-dependent. This is represented by  $r(t)$  in equation 2. As the ECB measures the actual force synchronously,  $q$  in equation 2 differs from the setpoint. This can and has to be corrected insofar as the correction is small enough. If the transducer chain has very high differential nonlinearity, this is not possible. A residual uncertainty remains, due to different filter characteristics and the dead time of the TUT measurement chain and the ECB. It is dependent on the estimated temporal uncertainty, the average time derivative of force and the number of regulation oscillation periods during averaging. For the filter characteristic used, a time shift of 1/6 s has been observed [2]. This can be assumed as being typical of  $u_t$ . Detailed investigations of the measurement results yield for a settling time of 30 seconds and a typical TUT:

$$u_{AS} = 20 \cdot 10^{-6} \text{ N/s} \cdot u_t, \text{ (normal, type B) for step sizes } \geq 200 \text{ mN}$$

$$u_{AS} = 6 \cdot 10^{-7} \text{ N/s} \cdot u_t, \text{ (normal, type B) for step sizes } \leq 10 \text{ mN}$$

### Uncertainty of Necessary Corrections

To calculate the corrections, the difference between the setpoint and the measured force is multiplied by the estimated sensitivity. If the measurement chain of the TUT exhibits excessive differential nonlinearities on the scale which has the size of the correction, this produces an additional uncertainty. The bridge amplifier used has been adjusted to less than 1 nV/V on each 0.1% interval of its range. This effect can thus be neglected.

### Reduction of the Signal of the TUT by a Misalignment of the Axes

If the axis of force generation is not parallel to the axis of the TUT, the parallel acting component is reduced. This is a cosine effect. For a maximum misalignment of 1 mrad (0.06°), the effect is only  $5 \cdot 10^{-6}$  and is neglected.

### Contributions to the Signal of the TUT by Lateral Force Component

The parasitic behaviour of the TUT can be described by lateral stiffness and sensitivity. For the sample which is shown later in this text its relative radial sensitivity is only 0.2% in comparison to the axial direction. The radial stiffness is ten times higher than the axial one. For the build-up of force during measurement, the TUT has to be deformed over a certain distance. Especially the angular alignment error might lead to additional lateral deformation. Without radial decoupling, a misalignment of 1 mrad would lead to a lateral force component of 1%. The result would be an effect with an amplitude of  $2 \cdot 10^{-5}$  of the actual value, sinusoidally dependent on the rotational position of the transducer. The sensitivity of the balance to lateral forces is estimated by its sensitivity to off-centre load. It is assumed that the centre of the life end of the Roberval mechanism is the pivot. With this estimate, even a 0.25 N lateral force on the PAU unit would not affect more than 0.2  $\mu\text{N}$  in the displayed result of the balance. With radial decoupling, this effect is assumed to be insignificant.

### Other Contributions to the Signal by Lateral Effects $u_{LE}$

Beyond the previous effect, pure second-order effects might occur corresponding to equation (3). As the matrix of sensitivity rotates with the TUT, averaging over rotation eliminates these effects. Measurements complying with ISO376 are repeated in the positions 0 degree, 120 degrees and 240 degrees.

The (relative) reproducibility “ $b$ ” describes its maximum spread at a given load. For a purely sinusoidal effect, its amplitude is statistically about 60.5% of  $b$ . The sensitivity is an average value over these rotational positions. Assuming an uncertainty of the rotational position of  $\pm 5$  degrees (rectangularly distributed) for manual measurement, leads to a standard uncertainty (normal) of the sinusoidal amplitude of 4.2%. The result is:

$$u_{LE} = 0.026 \cdot b / k_x, \text{ (normal, type B).}$$

### 2.3. Combined Uncertainty

The model of uncertainty, equation (2) and (3), is a sum of the contributions stated. To calculate the combined uncertainty with a coverage of 95%, first the rectangularly distributed contributions are weighted with the factor  $2/\sqrt{3}$ . Subsequently, the squares of the contributions are added and the root of the sum is extracted. It should be emphasized that there are no significant contributions of type A. Thus, there is no point-to-point noise in a measurement cycle. The following examples shall illustrate the experiments which are shown later in this text.

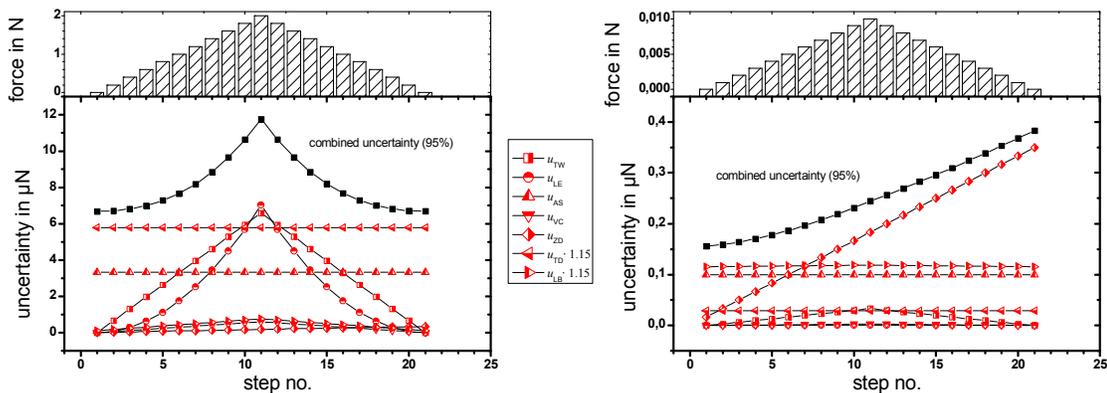


Figure 6: Typical combined uncertainty. Left: Typical measurement cycle with load steps every 30 seconds. The maximum force is 2 N. Right: same with 10 mN maximum force.

#### Example 1:

Force transducer with  $k_x \approx (2 \text{ mV/V}) / (2 \text{ N})$ , ISO 376 measurement, 10 load steps with intervals of 30 seconds each, synchronisation of transducer and ECB known to 1/6 s, nominal force range  $F_N = 2 \text{ N}$ . The observed reproducibility is:  $b = (F / 2\text{N})^2 \cdot 270 \text{ nV/V}$ .

Figure 6 shows the combined measurement uncertainty for the subsequent load steps after averaging over the three rotational positions. The coefficient of sensitivity is the difference between load step 11 and the first value. The resulting combined uncertainty (95%) is 13.5  $\mu\text{N}$ . This is a relative uncertainty of  $6.5 \cdot 10^{-6}$  of the nominal force value.

#### Example 2:

The same, but with a nominal force range of  $F_N = 10 \text{ mN}$ . It is  $k_x \approx (10 \mu\text{V/V}) / (10 \text{ mN})$ . In contrast to the first example, the zero-drift and the nonlinearity of the ECB here dominate the uncertainty budget. The resulting combined uncertainty (95%) for the sensitivity measurement is 0.3  $\mu\text{N}$ . In relation to the nominal force, this is a relative uncertainty of  $3 \cdot 10^{-5}$ .

### 3. Measurements, Simulation and Discussion

For the establishment of this unique FSM, the accuracy has to be proven. There are different strategies for this which rely on measurements carried out with a reliable transfer transducer. The measure to be used is the sensitivity value attained in accordance with ISO 376. The reproducibility depends on the measurement uncertainty, but also on the stability and dependence on hidden parameters of the TUT. The comparison results thus yield an upper limit.

#### 3.1. Prediction of Characteristic by Step Response

The step response function (SRF) of the signal can be measured easily. In a certain way, it describes the creep of the TUT and is known to be in the order of  $1 \cdot 10^{-4}$  of the force step. Information about the creep modelling of torque transducers can be found in [9]. As additionally, the span and the zero signal are temperature-dependent (typ.  $4 \cdot 10^{-5}$  mV/V,  $2 \cdot 10^{-5}$  / °C) the temperature should stay within an interval of  $0.01^\circ\text{C}$  during the measurement. Figure 7 shows the 1 N step response of a TUT with a nominal force range of 2 N. The difference after subtraction of an ideal linear characteristic is shown. The sensitivity used for this is obtained from an ISO 376 measurement with a full range of 2 N. The stability of the ECB under the conditions shown is better than  $10 \mu\text{N}$ .

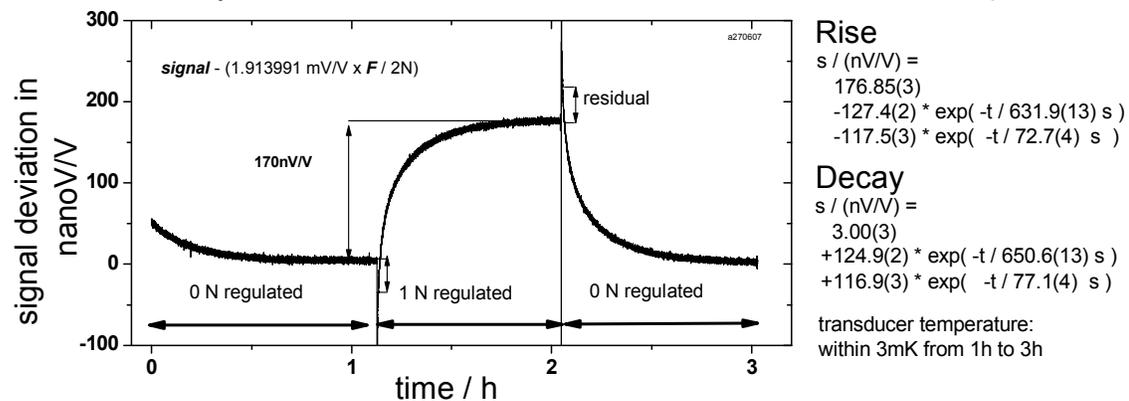


Figure 7: Step response after a load of 1 N. Single values are averaged over 0.2 seconds. The filter of the bridge amplifier has a 0.9 Hz Bessel characteristic. The force is stabilised to the ECB. The noise measured is due to this regulation.

The sensitivity for the large time scale differs from the ISO 376 result by about  $2 \cdot 10^{-4}$ . An analytical fit function of the SRF consists of two exponentials with time constants of about 640 s and 75 s. There are only slight discrepancies between the constants for rising and falling edge, which could be explained by an underlying slight linear drift.

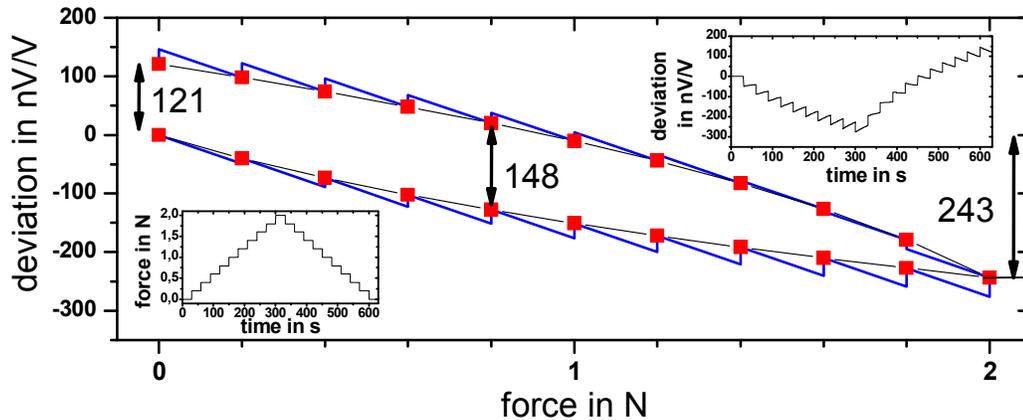


Figure 8: Simulation of measurement. Plotted is the deviation in comparison to a non-creeping transducer.

Assuming a linear behaviour, the signal can be described by a convolution of the derivative of the applied force and the SRF. Figure 8 shows the results for a typical measurement cycle. It uses the experimental data from figure 7. There are four main aspects:

1. The sensitivity for load cycles of 30 seconds is 243 nV/V smaller than the sensitivity measured with infinitely long load cycles.
2. The maximal hysteresis is 148 nV/V.
3. There is a zero residuum of 121 nV/V.
4. The shape is convex-curved.

The reason for these effects is mainly the creep component with the 640s time constant. It is comparable with the duration of the load cycle. It is a negative creep and thus produces a lower signal in the increasing part of the cycle, a higher signal in the decreasing branch with a certain time lag. The aftereffect of preloads or previous cycles reduces the effects described above. They add the effect of their decreasing branch to the next increasing one.

### 3.2. Typical Measurements for 2 N and 10 mN

Figure 9 shows first experimental data. Due to partly missing automation and a lack of experience, this is not the best measurement capability, but it is typical. The comparison between the resulting sensitivity measured by the previous prototype and the actual set-up agrees in  $1.3 \cdot 10^{-4}$  relative. This is a satisfactory agreement, as the transducer is reversed. The shape of the hysteresis and the zero residuum of both measurements are identical to a few nV/V. A comparison with the simulated data of the previous section delivers a good explanation for the values and shape. The spread  $b_{2N}$  due to different rotational positions is enlarged from 120 nV/V to 270 nV/V. With the new set-up, even after realignment of the ECB and the TUT, this value has been reproduced several times. The resulting sensitivity at a full range of 2 N is plotted over the temperature in figure 9. The standard deviation of 9 considered values is 16 nV/V. Taking into account the t-distribution, the values lie, with a probability of 95%, within  $\pm 1.9 \cdot 10^{-5}$  of its mean value. This is three times higher than the measurement uncertainty stated.

Figure 9 also shows a measurement with only 10 mN sub-range. For the first two mounting positions all values lie within 1 nV/V rms. This value had been expected anyway due to the noise and the nonlinearity of the bridge amplifier. The excess deviation of the third position might be induced by a short overload

during rotation. The sensitivity deviates by  $2.8 \cdot 10^{-4}$  relative from the reference value, if it is extrapolated to a full range of 2 N.

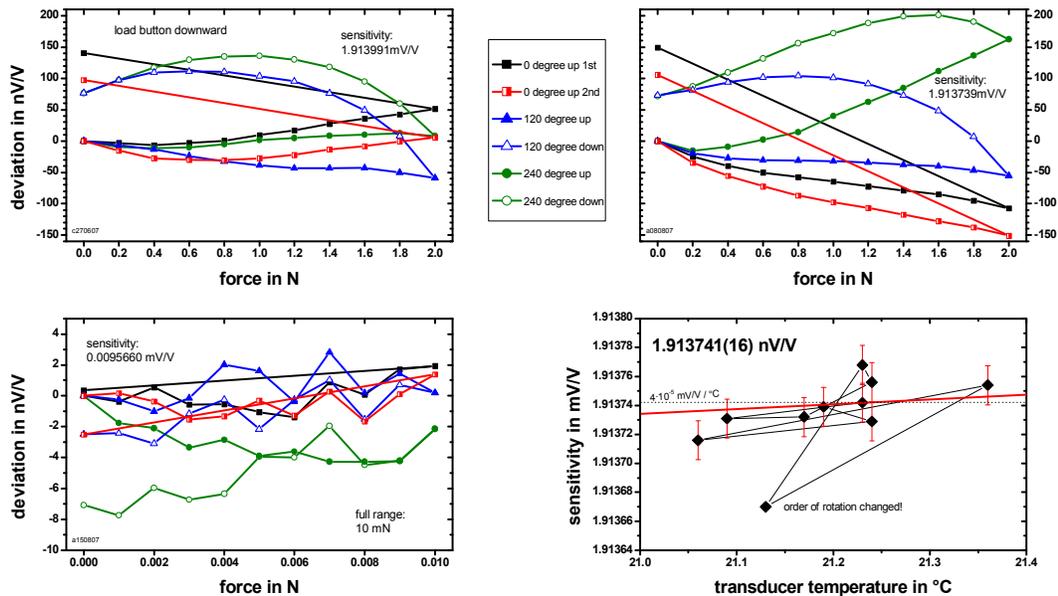


Figure 9: Experimental results. Upper row: left side, ISO 376 measurement with previous prototype, right side, with set-up as described. Lower row: left side, full range reduced to 10 mN, right side, 10 reproduced measurement results like graph above, one value possibly with wrong angle of rotation.

## Conclusion and Outlook:

A new intended force standard machine has been established. It is based on direct comparison with electromagnetically compensated balances. The traceability is guaranteed by substitution with at least one weight and extrapolation by the scale factor. The facility has been built up and tested with an ECB with a full range of about 2 N. The force is generated by a floating-mounted piezo-actuator unit. Corresponding to the law of reciprocal actions, it transmits the same force onto the ECB and the transducer under test.

A model and a detailed list of uncertainty contributions are given. For typical ISO 376 conform calibrations, the combined uncertainty of the sensitivity is calculated. For a nominal range  $F_N$  of 2 N the relative combined uncertainty (95%)  $u_c/F_N$  is  $6.5 \cdot 10^{-6}$ , and for a sub-range  $F_N$  of 10 mN, it is  $3 \cdot 10^{-5}$ , respectively. Even the 1 mN load step exhibits a relative uncertainty below  $5 \cdot 10^{-4}$ . The exact measurement of the transducer's step response opens up the possibility to model the deviations of different time schedules and standards. The hysteresis and zero-residuals observed are well described by the simulation.

For validation purposes, a TUT has been measured in a precursor prototype mounted with load button downwards. The comparison with the later measurements delivers a relative deviation of  $1.3 \cdot 10^{-4}$ . Even after realignment, 9 measurements with load button upwards have been reproduced within an interval of  $\pm 1.9 \cdot 10^{-5}$  (95%) of its mean value. Moreover, here the stability of the TUT might contribute. A measurement of the sub-range of 10 mN is nearly noise-limited by the TUT and its bridge amplifier. The sensitivity deviates by  $2.8 \cdot 10^{-4}$  relatively if it is extrapolated to the full range of 2 N. As this includes the differential nonlinearity of the bridge amplifier, this is plausible.

The facility is equipped with two goniometers and stages. Thus in future the detailed mechanical coupling conditions and their influence can be investigated. Parallel to this there are other efforts at PTB, to investigate the influence of multi-component effects on measurement [10]. The measurement capability could be also extended to measurement in direction of tension by replacement of the decoupling element. The continuous measurement from tension to compression is feasible. For the expansion of the measurement range, ECBs are already provided which have a maximum range of 12 N and 0.2 N. These will be the subject of subsequent investigations.

This work might give general guidance for listing contributions and for the calculation of the combined uncertainty for this class of FSMs. But it also holds good for force reference machines relying on reference transducers.

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