A new way to minimize uncertainty in calibration process of force testing machines

Boris Katz

Quality Control Center Hazorea (QCC Hazorea), Israel

Abstract

The results of the calibration of load cells (force transducers) according to ISO 376-1999 are described in table of values where force F (kN) is a function of electric signal R (mV/V) and by analytical approximation of the calibration curve.

The analytical approximation is usually expressed by polynom of 2^{nd} or 3^{rd} degree. The measurement of force in the calibration of testing machines by means of digital unit device DMP-40 (HBM) suppose application of linear interpolation between two adjacent values of signal R.

By means of mathematical calculations it is theoretically proven in which cases the linear interpolation is undesirable and leads to additional interpolation error, and increasing of the value of uncertainty measurement in accordance.

A few studies and experiments of the issue confirm the theoretical conclusions: in some cases minimizing interpolation error and uncertainty measurement in measuring force F as a function of reading the signal R, can be achieved by approximation polynom instead of linear interpolation between two adjacent values. For this purpose QCC Hazorea has developed program called "MABA-2000," which can solve this problem.

Keywords: materials testing machines, measurements uncertainty, interpolation error, calibration curve.

1.Introduction

According to [1] the process of calibration of force testing machines (tension and compression) requires calculation of an expanded uncertainty of the results. For all of the four accuracy classes of testing machines there are recommended maximum values of relative expanded uncertainties [4, table 1]. One of the components for the calculation of combined measurement uncertainties is the value of interpolation deviation.

The Accredited Laboratory Quality Control Center Hazorea (QCC) uses in its current work of calibration of testing machines a set of measurement instruments, which are calibrated according to [2]: load cells and digital unit device (amplifier) DMP-40 (HBM, Germany). The performance of the calibration (comparison of actual force to reading of the testing machine) requires previous definition in the memory of the amplifier tabular function $F = f(R)$ of calibration of the load cell, where F is actual force (kN) and R is electrical signal (mV/V)

2.Discussion

According to [5], values of the force F as a function of electrical signal R are calculated by the method of linear interpolation, i.e. the arc of characteristic curve of the calibration function on a given section between two adjacent points *n* and *n+1* is replaced by linearized characteristic curve (figure 1).

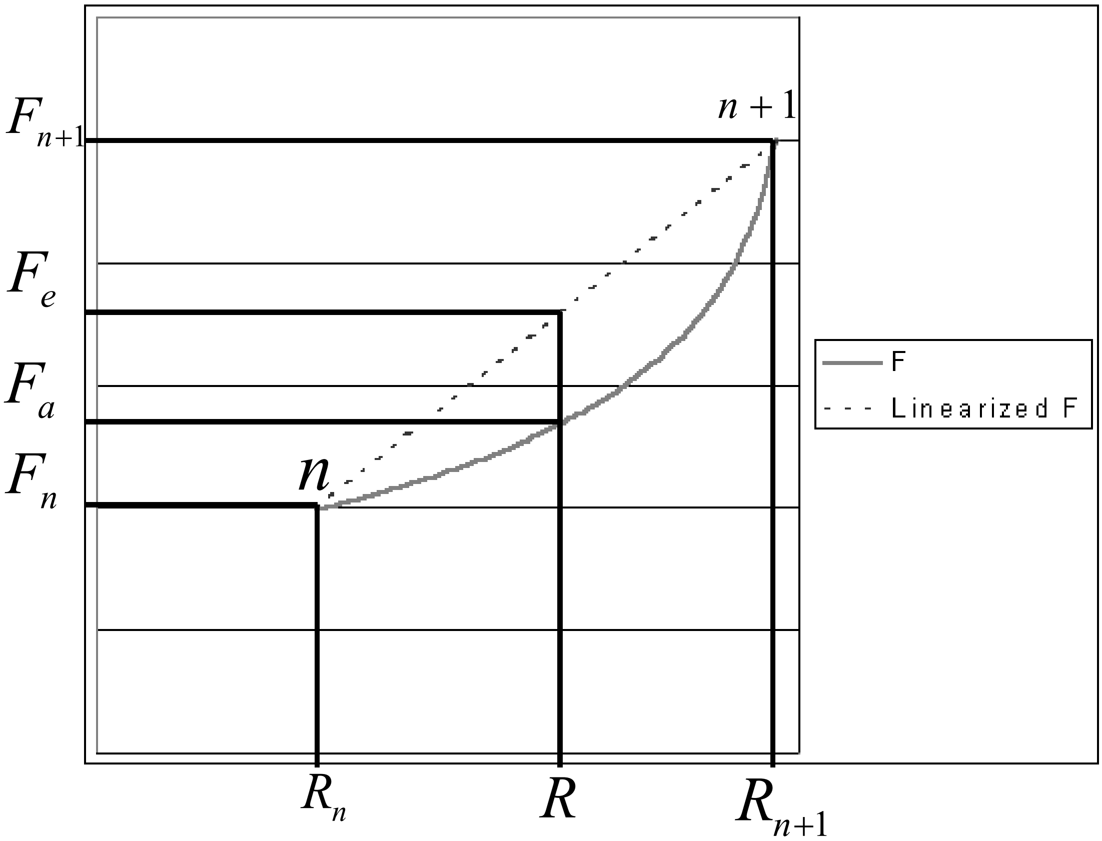

Figure 1 - calibration curve

Multiple experiments and observations being carried out in QCC reveal, that using a linear mathematical model is possible under certain limitations. In some cases, for the purpose of minimizing the value of interpolation deviation $\delta_l = F_a - F_e$ (fig. 1), it is advisable to replace the linear model by a polynomial interpolation.

Let us consider those limitations: The equation of chord, which passes through the points (R_n, F_n) and (R_{n+1}, F_{n+1}):

$$\frac{F - F_n}{F_{n+1} - F_n} = \frac{R - R_n}{R_{n+1} - R_n}$$

This proportion leads to the solution of the equation:

$$F_e = F_n + \delta F_n \frac{R - R_n}{h} \qquad (1)$$

The difference between the exact value of function $f(R)$ and its approximate value, as defined by linear function (1) is:

$$\varphi(R) = f(R) - F_n - \delta F_n \frac{R - R_n}{h} \qquad (2)$$

Where $\delta F_n = F_{n+1} - F_n$, $h = R_{n+1} - R_n$ and R - is the current value of the electrical signal.

In order to find out the solution of function $\varphi(R)$ we must assume that the second derivative of function $f(R)$ in the interval $[R_n, R_{n+1}]$ is continuous (it is always continuous for a polynomial curve) and thus following the in-equation:

$$\left| f''(R) \right| \le M$$

Where M is the maximal value of second derivative function $f(R)$ on the present interval.

The first and the second derivatives of $\varphi(R)$:

$$\varphi'(R) = f'(R) - \frac{\delta F_n}{h}$$

$$\varphi''(R) = f''(R)$$

Therefore

$$\left| \varphi''(R) \right| \le M \qquad (3)$$

For the next discussion we will assume, that the calculated values of F_n and F_{n+1} of the polynom coincide with the values given in the calibration table (at the present assumption we neglect the interpolation error, which according to [2] is known at each calibration point). According to this assumption:

$$\varphi(R_n) = \varphi(R_{n+1}) = 0 \qquad (4)$$

Inside the interval R_n, R_{n+1} let us examine the point $R_{\max}$, at which the absolute value of function $\varphi(R)$ is maximal.

For the extreme analysis of this function we can expand it according to Taylor formula (expansion by degrees):

$$\varphi(R) = \varphi(R_{\max}) + \varphi'(R_{\max})(R - R_{\max}) + \frac{\varphi''(c)}{2}(R - R_{\max})^2 + O\left[(R - R_{\max})^3\right]$$

$$O\left[(R - R_{\max})^3\right] \to 0$$

Where c is the point between current value R and value $R_{\max}$.

Since $\varphi'(R_{\max}) = 0$ the formula can be transformed to the next equation:

$$\varphi(R) = \varphi(R_{\max}) + \frac{\varphi''(c)}{2}(R - R_{\max})^2 \qquad (5)$$

Let us mark as $\overline{R}$ the nearest point to $R_{\max}$ (rather R_n or R_{n+1}). Then, according to (4) and (5) we will accept the next equation:

$$\varphi(R_{\max}) = -\frac{\varphi''(c)}{2}(\overline{R} - R_{\max})^2$$

Since $\left|\overline{R} - R_{\max}\right| \le \frac{h}{2}$ and according to (3):

$$\left|\varphi(R_{\max})\right| \le \frac{\left|\varphi''(c)\right|}{2} \cdot \frac{h^2}{4} \le \frac{Mh^2}{8}$$

As far as the interval of values R_n , R_{n+1} $\left|\varphi(R)\right| \le \varphi(R_{\max})$ is in condition of evaluation linearized interpolation error will be expresses by the in-equation:

$$\left|\varphi(R)\right| \le \frac{Mh^2}{8} \tag{6}$$

While using this formula for determination of the force F as a function of electrical signal R we can establish the cases, in which the linear interpolation leads to additional interpolation error, and increasing of the value of uncertainty measurement in accordance. In such cases, as an alternative, it is recommended to use the approximation polynom of 2^{nd} or 3^{rd} degree, which accompanies each calibration certificate of a load cell.

3. Experiment

The theoretical limitations (6) are confirmed by a number of experiments, which were carried out in QCC for compression load cells. Measurement and error evaluation were compared to another (reference) load cell. The force values mentioned in the tables below are calculated by two different methods:

Table 1 Values of forces for load cell 200 kN

F_{act}	25	30	35	105	110	115	185	190	195
F_{pol}	24.990	29.994	35.005	104.998	109.986	114.988	184.990	189.998	195.000
F_{lin}	24.978	29.984	34.994	105.000	109.991	114.991	184.996	189.996	194.999

Table 2 Values of forces for load cell 1 MN

F_{act}	125	150	175	525	550	575	925	950	975
F_{pol}	125.017	149.946	174.938	525.015	550.053	575.043	925.000	950.006	975.019
F_{lin}	124.741	149.723	174.739	524.831	549.842	574.829	924.792	949.812	974.872

Table 3 Values of forces for load cell 50 kN

F_{act}	6.25	7.50	8.75	26.25	27.50	28.75	46.25	47.50	48.75
F_{pol}	6.2478	7.4926	8.7387	26.2313	27.4868	28.7378	46.2439	47.4866	48.7400
F_{lin}	6.2471	7.4920	8.7384	26.2318	27.4877	28.7391	46.2436	47.4865	48.7400

Where F_{act} - actual force i.e. measured by reference load cell (kN);

F_{pol} - computed force, using 2^{nd} degree interpolation polynom (kN);

F_{lin} - computed force, using linear interpolation (kN).

The data in tables 1 and 2 shows the reduction of interpolation error in cases where the forces are calculated by mean of interpolation polynom of 2^{nd} degree. Reducing of interpolation error is especially prominent in the range of low loads. The data of table 3 convinces that usage of both methods leads to the same results.

4. References

[1]. ISO 7500-1:1999 Verification of static uniaxial testing machines-verification and calibration of force-measuring system.

[2]. ISO 376—1999 Metallic materials- Calibration of force-proving instruments used for the verification of uniaxial testing machines

[3]. EA-10/04 (EAL-G22):1996. Uncertainty of calibration Results in Force Measurement.

[4]. A. Sawla. Uncertainty of measurement in the verification and calibration of the force–measuring systems of testing machines. Proceedings of the 5-th Asia-Pacific Simposium on Measurement of Force, Mass, and Torque (AMPF-2000).

[5]. Hottinger Baldwin Messtechnik. Digital precision measuring amplifier DMP 40, DMP 40S2. Operating manual.

[6]. S.B. Norkin, P.Y. Berry, I.A. Jabin, D.P. Polozkov, M.I. Rozental, H.R. Suleimanova. Elements of computational mathematic. Moskow, 1963