High Speed High Accuracy Multilateration System Based on Tracking Interferometers

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Abstract
The use of tracking interferometers has gained industrial application for the testing and the calibration of measuring and production machines. The concept is based on sequential multilateration [1] and can realize calibration uncertainties in the sub-micron order. Another application for tracking interferometers is the real-time multilateration for accurate and highly dynamic measurement of 3D motion. This paper presents the characteristics of a high speed high accuracy multilateration system based on interferometric displacement measurements.

Introduction
Laser interferometer based multilateration systems have been discussed as a solution to accurate determination of 3D position since the 1970s. A proof of principle has been demonstrated 2000 by Hughes et.al. [2]. Also the Physikalisch-Technische Bundesanstalt (PTB) has developed a system based on commercial available components [3]. An interferometric multilateration system has the following characteristics:

- Direct traceability to the laser wave length
- Highest accuracy due to incorporated Abbe principle
- Self calibration capability

Now a multilateration system has been developed and tested, that combines the previously demonstrated benefits with high dynamic measurement capabilities. This paper describes the system, presents results and reviews possible applications.

The principle of multilateration
The multilateration principle is based on spatial length measurement between a target point and at least three reference points with known coordinates. The multilateration principle is used e.g. in the Global Positioning System (GPS) based on microwave time-of-flight measurement. If four reference points are available, an incremental displacement measurement is sufficient to perform a self calibration of the system [2,3]. The high resolution of an interferometer can be used to realize the displacement measurement in such a system. In the system described here, stable reference points have been realized by the so called Lasertracers [3].
The Lasertracer
A Lasertracer (LT) is a self-tracking laser interferometer developed for high precision metrology applications. A modified Michelson interferometer based on a stabilized HeNe laser beam is used to measure displacement between two reference points. These points are realized by an optical cats-eye reflector on one side and the build-in high precision reference sphere mirror (Form error < 100 nm) on the other (Fig.1). The laser beam follows the optical reflector using a PSD based tracking control while its displacement is continuously measured with nanometer resolution. Compared to other laser tracking systems the LT system due to its design has an exceptional accuracy of the point of rotation. The reference sphere is thermally and mechanically decoupled from the rotating parts of the measurement head. The interferometric setup ensures that the laser beam always measures the displacement directly between the optical centre of the reflector and the centre of the reference sphere. LTs are commercially used for the calibration and testing of high precision coordinate measuring machines and machine tools. The uncertainty of the displacement measurement is U(95%)=0.2 µm + l * 0.3 µm/m (l=measuring distance).

Fig. 1: The Lasertracer: (1) reference sphere, (2) measurement beam, (3) solid stem mounted to base plate

Retro reflectors
A retro reflector reflects an incident laser beam exactly parallel back to the interferometer. Cats-eye reflectors are used because of their wide opening angle and the achievable sub-micron precision of the optical centre. The working principle of a cats-eye reflector is based on the reflection of the incident laser beam on the coated back surface of the reflector, whereby the glass body itself is used to focus the laser beam on that surface. Two types of cats-eye reflectors are used for the multilateration system: reflectors made from two
hemispheres of glass with matching radii and refractive index and full sphere reflectors having an refractive index close to 2 (so called n=2-reflectors).

**Multilateration algorithms**

The following formula describes the basic principle of multilateration:

\[ l_{ij} = \sqrt{(T_{xi} - R_{xj})^2 + (T_{yi} - R_{yj})^2 + (T_{zi} - R_{zj})^2} + \Delta L_i + r_{ij} \]

\( l_{ij} \): distance from Tracer \( i \) to reflector position \( j \) including constant offset \( \Delta L_i \) of LT \( i \)

\( [T_{xi}, T_{yi}, T_{zi}] \): Position of LT \( i \)

\( [R_{xj}, R_{yj}, R_{zj}] \): Position of reflector \( j \)

\( r_{ij} \): residual of length \( l_{ij} \), \( \sum r_{ij} \rightarrow \min \)

This multilateration equation is transformed to a linear form, as described in [1] and solved by least square method. To calibrate the system at least 10 reflector positions are required. Once the system variables are determined, the reflector positions may be calculated by a closed form multilateration algorithm [4]. Alternatively, in the further acquisition of measuring points, the data can also be used to improve the calibration of the system by a total best fit.

Fig. 2: Sample configuration of multilateration system using 4 LT

**Calibration procedure**

The coordinate system of the calibration volume is initially defined by the position and the orientation of one (the primary) of the four LTs. Firstly reflector positions are estimated by the angular measurements of the primary LT and by a roughly estimated distance to the reflector. The tolerances for these first estimates are in the order of 100 mm. During the
calibration process the reflector is moved to positions well distributed in the measurement volume and paused for a short moment, which automatically triggers a measurement. The calibration procedure can be performed hand guided, as the displacement data is triggered with a jitter below 200 ns. Once the system is calibrated, the angular data of the LTs is not used anymore for the 3D evaluation; the reflector positions are calculated solely from the displacement measurements. Consequently, the influence of angular tracking errors is reduced to a cosine error and therefore can be neglected even for dynamic measurements.

**Optimal configurations**

To achieve minimal uncertainties, the locations of the LTs as well as the point cloud for calibration have to be optimized. To make self calibration possible, the four LTs must not be in the same plane. The smallest position offset of a single LT in relation to the plane spanned by the other three is determining the self calibration capability. The calibration point pattern is optimal, if it covers the extreme positions of the working volume in all three directions. Typically, the working volume is limited by the acceptance angle of the used reflector. For practical applications of a multilateration system the acceptance angle of the reflector will often be a limiting the possible locations of the 4 LTs.

**Software**

A multilateration software was developed that supports the calibration procedure, the measurement planning and the static or dynamic point acquisition. The measurement uncertainty management is an integral part of the software in all stages. Basis for the calculation of the measurements uncertainties are analytical calculations used in geodesy as well as Monte Carlo Methods. An outstanding characteristic of the system is the high measurement resolution provided in time and space. For continuous measurement the current implementation is limited to a frequency of 200Hz due to a bottleneck in the USB data transfer. However, in case the internal buffers of the interferometer units are used, an acquisition frequency of up to 500 kHz can be achieved. Simultaneously, the spatial resolution is in the order of <100 nm in a measurement volume of one cubic meter. The achievable accuracy strongly depends on the configuration, but simulations show, that with a proper setup and a laboratory environment with thermal stability of 0.2 K an accuracy of U(95%)=0.5 µm within a cubic meter can be accomplished. Fig. 3 shows a measured 3D trajectory of a decelerated machine head including the overshooting motion resulting from mechanical elasticity of the machine. It illustrates the high spatial resolution of the system even in dynamic mode.
Fig. 3: Trajectory of a decelerated machine head (in y-direction), projected to the yz-plane.

**Validation of the accuracy**

Since the system consisting of 4 LTs, after calibration the system shows an increasing degree of redundancy with every point measured. The internal residuals of the system are an indicator for the consistency of the measured data and an indicator for the limits of accuracy. Fig. 4 shows the distribution of residual errors after more than 30,000 measurements in dynamic mode in a measurement volume of about 1 m$^3$. The standard deviation is only 42 nm.

Fig. 4: Distribution of the residual errors in the over constrained multilateration bundle, based on approx. 30,000 dynamically recorded measuring points. The standard deviation is 42 nm.
The same system was also compared to an independent LT measurement along a fixed line measurement that was oriented differently from the main direction of the multilateration system. The errors observed were below 0.6 µm (Fig. 5). It has to be noted, that the experiments were performed in a non air conditioned environment. It is compliant with the uncertainty calculations. Even better results can be expected in a well controlled environment.

![Deviations to length measurements of control LaserTracer](image)

**Fig. 5:** Results of a comparison between the multilateration system and an independent laser measurement along the measurement beam of a 5th LT

**Summary and Outlook**

The presented system exhibits the unique characteristic of self calibration combined with highest accuracy and dynamic resolution. Data has presented, that validates these characteristics. First multilateration systems have been delivered to customers in research and fundamental metrology. However, possible applications can also been seen in industry, e.g. for the inline production metrology, the manufacturing of large optics and the control of precision machines and robots. Etalon is currently conducting case studies for these applications together with interested parties from research and industry.
References


